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Wave propagation and imaging in structured optical media

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WAVE PROPAGATION AND IMAGING IN STRUCTURED OPTICAL MEDIA

A Dissertation

Submitted to the Faculty

of

Purdue University

by

Zun Huang

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of

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To my mom and dad.

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SYMBOLS

c	vacuum speed of light
ω	angular frequency
λ_0	free-space wavelength
$\mathcal{F}$	Fourier transform

ABBREVIATIONS

1D	one-dimensional
2D	two-dimensional
3D	three-dimensional
NIM	negative index metamaterial
HMM	hyperbolic metamaterial
IR	infra-red
PC	photonic crystal
IFS	iso-frequency surface
IFC	iso-frequency curve
TM	transverse magnetic
Re	real part
Im	imaginary part
EMT	effective media theory
T-matrix	transfer matrix
PHC	photonic hypercrystal
FWHM	full width at half maximum
CR	conical refraction
FFT	Fast Fourier Transform

ABSTRACT

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Structured optical media, usually characterized by periodic patterns of inhomogeneities in bulk materials, provide a new approach to ultimate control of wave propagation with possible practical applications: from distributed feedback lasers by diffraction gratings, to highly nonlinear performance for super-continuum generation, to fiber-optic telecommunications by microstructured photonic crystal fibers, to invisibility cloaking, to super-resolution imaging with metamaterials etc.

In particular, structured optical media allow to manipulate the wave propagation and dispersion. In this thesis, we focus on engineering the propagation phase dispersion by modulating the compositions and dimensions of the periodic elements. By tailoring the dispersion in momentum space, we can obtain new optical properties of the structured media and show applications in optical imaging.

In this work, we present a novel zeroth-order transmission resonance in hyperbolic medium with a Fabry-Perot geometry, which allows to control the propagation phase by subwavelength elements. This approach can also be extended to periodic structures and be applied to improve the performance of imaging systems. In particular, we show a negative refraction lens based on the photonic hypercrystals, which possesses a nearly constant negative refractive index and can be used to substantially reduce the image aberrations. We apply similar ideas in purely dielectric photonic crystals and demonstrate the phenomenon of conical refraction, which allows a new approach to imaging. In particular, it offers a new method of optical phase retrieval that enables a single simultaneous measurement and guarantees a rapid recovery to the true solution.

1. INTRODUCTION

The rapid development of optics and photonics technology in the past few decades has dramatically revolutionized the global industry and extended their indispensable functionalities and impacts to the society: from the manufacture and inspection of the integrated circuits in every electronic device to advanced medical imaging and diagnostics, from the high-resolution displays on mainstream smartphones and computers to global fiber-optic network and telecommunications etc. Optics have been playing a central role in our daily life.

With the continuous innovations in optical science and photonic engineering, a great potential for even more significant societal influences can be envisioned in the near future. Solar cell powering, for example, could transform the landscape of global energy. In addition, the tremendous capabilities in optical metamaterials and plasmonics will be an essential key to sustain Moore’s Law and support the exponential growth of internet.

Controlling the electromagnetic properties and light propagation in a few newly emerging classes of structured optical media, e.g. metamaterials, represents one of the newest frontiers that just opened up in the last few years but already showed enormous potentials influencing the overall optics and photonics technology. Many innovative science and engineering can be accomplished if we can engineer new materials responding to light over a desired frequency range by e.g. perfectly reflecting, or confining them within a specific range and volume, or guiding them along certain directions, or directing them to perform critical operations. It is with these goals in mind that we present our new studies on structured optical media in this thesis.

The advancement in material science and manufacture technology enables us to transform the theoretical breakthroughs into practical engineering innovations. The exciting and challenging “Treasurer Island” of nanophotonics and metamaterials now

can be explored thanks to the advancement of material science and nano-fabrication technology. In particular, the new capability of imaging nanoscale subwavelength features with accuracy has intrigued many scientists challenge Abbe's diffraction limit. A powerful nanometer-resolution optical imaging device not only offers a dream tool for biologists to encode the life secrets in the atomic-level DNA structures, but also will reforms the photolithography technology that is indispensable to semiconductor manufacture industry. Therefore, in this thesis we paid more attentions and efforts to imaging applications.

1.1 Structured optical media: literature review

Structured optical media, engineered composites with modulated material and geometry configurations in the periodic inhomogeneous elements, have offered a new approach to ultimate control of light propagation with possible practical applications including: distributed feedback lasers by diffraction gratings [1], highly nonlinear performance for super-continuum generation [2], fiber-optic telecommunications by microstructured photonic crystal fibers [3], invisibility cloaking by transformational optical structure [4, 5], super-resolution imaging with metamaterials [6–9] etc.

Metamaterials [10] and photonic crystals (PCs) [11] represent two paradigms of the pioneering innovations in structured optical media, where their electromagnetic properties and the corresponding physics are defined by the relationship between the unit-cell size D and free-space wavelength λ_0 . In PC regime where $D \approx \lambda_0$, the light propagation is featured by energy bandgaps due to Bragg scattering. On the other hand, in metamaterial regimes where $D \ll \lambda_0$, the macroscopic electromagnetic response can be described in terms of averaged permittivity (ϵ) and permeability (μ) tensors. By manipulating the unit-cell components and geometry, a broader degree of controlling the optical response can be achieved.

1.1.1 Metamaterials

Ever since the introduction around 2000, metamaterials represent one of the most active research frontiers promoting the cutting-edge revolutions in material science and photonic technology. Compared with the naturally occurring materials whose electromagnetic characteristics are determined by the properties of atoms and molecules, metamaterials allows us to manipulate the electromagnetic space and wave propagation through the rationally designed micro- or nanostructured “meta-atoms” that are much smaller than the light wavelength. It is this subwavelength feature of the unit-cell and coupling between them that governs the macroscopic electromagnetic properties. Such a powerful control over material parameters offers an extraordinary light-matter interactions with unprecedented optical properties unattainable in nature, such as negative refractive index [12–16], optical magnetism [17–19], zero index of refraction [20–22], inverse Doppler effects [23], perfect absorption [24,25] and enhanced nonlinear optical properties [26,27]. Those fascinating applications of metamaterials include super-resolution imaging [6–9], optical nanocircuits [28], cloaking devices [4,5], ultra-compact resonators [29,30] etc.

The development of nanofabrication techniques have paved the way to realize metamaterials in a broad range of optical frequencies. Initial experimental work on metamaterials concentrated on the double-negative response (simultaneously negative permittivity and permeability) inspired by the highly promising “Superlens” that has shown significant potential to surpass the optical diffraction limit [31]. In recent years, the research focus has shifted to active and tunable metamaterials with simplified structures, which offer more advantages in design and fabrication while preserving their unusual properties and functionalities.

Some typical examples of the nanostructured building blocks for three-dimensional (3D) metamaterials can be found in Ref. [32], where some of them have been used in the pioneering research of investigating negative refraction from microwave to optical frequencies. Recently, the planar structure becomes a more attractive alternative to

achieve non-resonant negative refractive index, such as multilayered semiconductor metamaterials [33] and stacked periodic plasmonic waveguides [34] etc. These planar metamaterials not only simplify the design complexity, but also greatly facilitate the fabrication process.

However, there are still several shortages of metamaterials that may potentially restrict the transition from scientific innovations to the real-world applications. First, to qualify as a feasible “material” or device, the structure must contain some densely packed 3D unit-cells to approach the bulk medium limit [32]. This increases technical difficulties in both design and implementation. Second, a broadband working frequency shall be enabled and needs to be compatible with “conventional” detection devices. Third and most importantly, the absorption loss inherent from the metallic elements should be carefully tailored in a reasonable fashion. For most significant applications such as negative refraction imaging, this loss needs to be kept as low as possible. In metal based metamaterials the absorption losses are inevitable, whereas all-dielectric structures excluding the metallic components may offer a potential solution in selected frequency regimes [35].

1.1.2 Photonic crystals

Photonic crystals are periodic composites consisting of artificial “lattices” — structured materials with dimensions close to the wavelength of light. Natural PCs can be found in the form of structural coloration. Opals, for example, have naturally periodic microstructures of hydrated amorphous silica, which can diffract the light to display an iridescent color.

The initial study of PC can be traced as early as 1887 when Lord Rayleigh studied a one-dimensional (1D) periodic multilayers of dielectric stacks (known as Bragg mirror) and demonstrated the bandgaps (stop bands) [36]. However, it was until 1987, about 100 years later, the term of “photonic crystal” was formally given in the two milestone papers by Eli Yablonovich and Sajeev John [37, 38].

The electromagnetic wave in PCs experiences propagating modes and photonic bandgaps, which results in several exotic optical phenomena, such as inhibited spontaneous emission, low-loss waveguiding and highly reflective omni-directional mirrors [11]. In general, the photons inside a PC are governed by the periodic dielectric functions of its constituents in a similar way to electrons in semiconductor or crystal solids that are controlled by periodic ionic potentials. Therefore, we can refer the concepts of Brillouin zones and Bloch wave in semiconductor theory to define the characteristics of electromagnetic wave in PCs.

Depending on the dimension, PCs can be categorized into three types with different structures [40]. The fabrication complexity is increased with the number of dimension, which may directly affect the feasibility in practical applications.

The 1D PC is often composed of stacked periodic layers of different materials, where the photonic bandgaps can be managed in a single direction. The Bragg grating is a typical example which has found significant applications in distributed feedback laser [1].

Two-dimensional (2D) PCs are fabricated by drilling holes in a dielectric substrate or constructing nano-rod arrays within a bulk medium. Photonic crystal fiber is one of the most extensively applied commercial examples, which finds broad applications in fiber-optic communication, fiber lasers, high power transmission, polarization control, gas sensing and nonlinear devices [3]. In particular, a photonic bandgap fiber with multiple hollow cores is capable of boosting the transmission speed to a state-of-art record of 255 Tbps, about 30% faster than commercial fibers [39].

The most challenging type is 3D PC in terms of fabrication complexity. Various structures associated with 3D PCs have been established [11] including: spheres in a diamond lattice, woodpile, a stack of two 2D PCs etc. The 3D photolithography and etching techniques have been the most popular fabrication tool to construct these different structures.

Recently, some anomalous phenomena have been reported in PC with tailored band diagrams. For example, an absorption-free, negative-refraction lens without negative refractive index was realized by a 2D air-hole-silicon PC [42].

1.2 Phase-space dispersion and engineering

1.2.1 The iso-frequency surface

Structured optical media provide a tremendous capability to imitate and study the physical process of novel materials. One special design freedom is the control over the phase-space dispersion characterized by iso-frequency surface (IFS), the surface of allowed wave vectors for a constant frequency. This phase-space dispersion is important in understand the electromagnetic radiation properties in a complex composite. By engineering the IFS in structured media, we can explore unconventional wave dynamics and optical properties.

We consider a uniaxial non-magnetic material with the permittivity tensor $\bar{\epsilon}(\vec{r}) = \text{diag}(\epsilon_{xx}, \epsilon_{yy}, \epsilon_{zz})$, where $\epsilon_{xx} = \epsilon_{yy} = \epsilon_{\parallel}$ and $\epsilon_{zz} = \epsilon_{\perp}$. Here, the subscripts $\parallel$ and $\perp$ indicates components parallel and perpendicular to the principal axis, respectively. For a transverse magnetic (TM) polarized plane wave propagating in this structure, the IFS can be derived directly by solving the Maxwell's equation in Fourier (or spatial frequency) domain, which is given by

$$\frac{k_x^2 + k_y^2}{\epsilon_{\perp}} + \frac{k_z^2}{\epsilon_{\parallel}} = \frac{\omega^2}{c^2}, \quad (1.1)$$

where ω is the angular frequency and c is the speed of light in vacuum.

While closed IFS is ordinary for an isotropic material ($\epsilon_{\parallel} = \epsilon_{\perp}$), the otherwise anisotropy is extraordinarily magnified when the two principal permittivity components possess opposite signs, i.e. $\epsilon_{\parallel}\epsilon_{\perp} < 0$. There, the IFS is dramatically changed from a simple sphere (or ellipsoid if $\epsilon_{\parallel}\epsilon_{\perp} > 0$ and $\epsilon_{\parallel} \neq \epsilon_{\perp}$) to an open hyperbola, leading to a new class of metamaterials – hyperbolic metamaterial (HMM). A wide range of optical singularities and exotic phenomena have been found in HMM, such

as negative refraction [43–47] and hyperlensing [7], diverging photonic density of states [48], strong enhancement of spontaneous emission [49] and thermal hyperconductivity [50, 51] etc.

In the metamaterial regime, the optical property and wave dynamics are determined by subwavelength features of unit-cell components. Therefore, such an extreme anisotropy can be obtained by stacked multilayered structure. Considering the simplest two-component multilayered composite, the homogenized dielectric response can be described by effective media theory (EMT) [52] as

$$\epsilon_{\parallel} = \frac{\epsilon_1 d_1 + \epsilon_2 d_2}{d_1 + d_2}, \quad (1.2)$$

$$\epsilon_{\perp} = \frac{d_1 + d_2}{d_1/\epsilon_1 + d_2/\epsilon_2}, \quad (1.3)$$

where ϵ_1 and ϵ_2 are complex electric permittivities of each component layers, and d_1 and d_2 are the corresponding layer thicknesses, respectively. By modulating the optical parameters (ϵ_1, ϵ_2) and material dimensions (d_1, d_2) , we can achieve the desired IFS including the extreme case of hyperbola if $\text{Re}[\epsilon_{\parallel}] * \text{Re}[\epsilon_{\perp}] < 0$.

Fig. 1.1 shows two different types of IFSs in a 1D anisotropic metamaterial structure with different configurations of the double-layer unit-cell. For simplicity, the absorption loss corresponding the imaginary part of permittivity is omitted here. As can be seen, the values of wave vectors are confined in ellipsoid IFS corresponding to purely dielectric components, whereas the open hyperbola IFS allows propagating modes to exist for unbounded values of wave vectors.

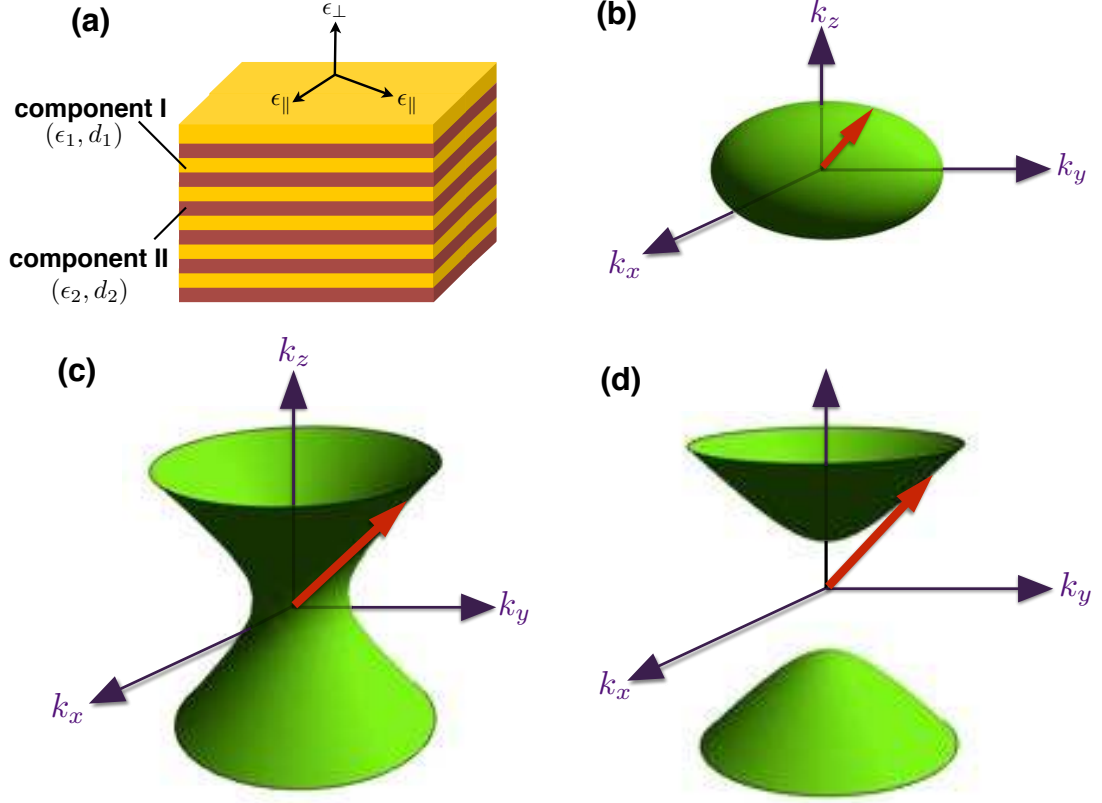


Fig. 1.1. (a) Schematic illustration of a planar 1D uniaxial metamaterial composed of two-component nanolayer unit-cells. (b) Closed ellipsoid shape of IFS for anisotropic configuration ($\epsilon_\parallel \epsilon_\perp > 0$) with both two component layers being dielectrics. (c-d) Two examples of open hyperbola shape of IFS (with different opening directions) for extreme anisotropic configuration ($\epsilon_\parallel \epsilon_\perp < 0$) and the component layers are metal and dielectric. The allowed values of wave vectors (denoted by red arrows) are limited to the IFS boundary in (b) but unbounded in (c-d)

1.2.2 Transfer matrix method in periodic system

The EMT provides a fast determination of macroscopic optical properties of metamaterials. However, when the spatial dispersion in each element boundary becomes dramatic [53] and the unit-cell dimension is approaching the subwavelength limit,

EMT fails to adequately describe the exact metamaterial modes. Therefore, we need a more rigorous transfer matrix (T-matrix) [54, 55] method to address these issues.

The matrix definition and theory originate from linear algebra. Onsager [56] first used the matrix transformation method in crystal optics to solve the partition function of a 2D Ising model. Shortly afterwards this matrix method was introduced in studying the thin-film optical properties [57] and dimer solutions [58]. Recently, the matrix theory has been extensively applied in optics, quantum mechanics, signal and image processing, topology mathematics and modern economics etc. A general theory of using the T-matrix method to solve the periodic structured media can be found in [55, 59].

Below, we describe a 2×2 T-matrix formulation for a 1D two-component multilayer structure as shown in Fig. 1.2. The dielectric permittivity and layer thickness of each component layer are denoted by $\epsilon_{1(2)}$ and $d_{1(2)}$, and the unit-cell size is $L = d_1 + d_2$. It is worth noting that the T-matrix method can also be effectively applied in non-periodic complex systems.

We consider a TM polarized electromagnetic plane wave propagating inside the periodic system. The magnetic field $H_y(x)$ parallel to y -axis consists of a right-traveling wave and a left-traveling wave that can be written as

$$H_y(x) = R_j e^{ik_x^{(j)}x} + L_j e^{-ik_x^{(j)}x} = A_j(x) + B_j(x), \quad (1.4)$$

Here R_j and L_j are constants in the j -th component layer, $A_j(x)$ and $B_j(x)$ represent the field amplitude of waves traveling in $x+$ and $x-$ directions, and $\pm k_x^{(j)}$ is the x components of wave vector in j -th layer that is given by

$$k_x^{(j)} = \sqrt{\epsilon_j(\omega/c)^2 - k_z^2}, j = 1, 2, \quad (1.5)$$

where k_z is the conserved z component of wave vector parallel to layer surface. The associated electric field parallel to z -axis $E_z(x)$ can be obtained from Maxwell's equations as

$$E_z(x) = \frac{k_x^{(j)}}{\omega\epsilon_1} \left[L_j e^{-ik_x^{(j)}x} - R_j e^{ik_x^{(j)}x} \right]. \quad (1.6)$$

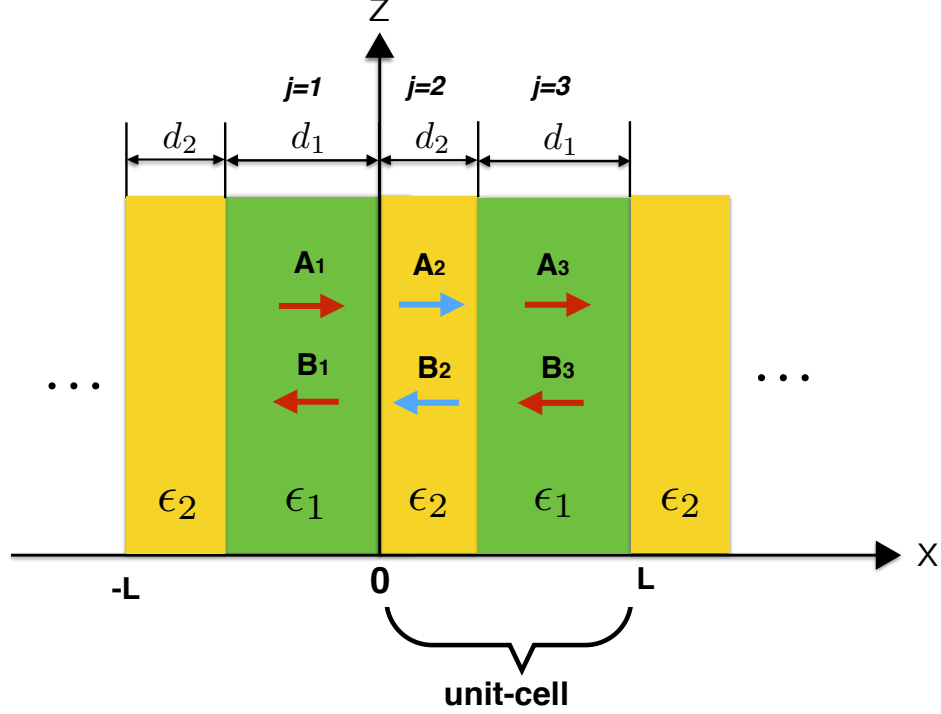


Fig. 1.2. Schematic illustration of a periodic multilayered structure consisting of two-component unit-cell. The T-matrix $\mathbf{T}_0$ of the unit-cell transfers the input magnetic field amplitude vector (A_1, B_1) at $x = 0^-$ to output amplitude vector (A_3, B_3) at $x = L^-$. Here A_j denotes the field amplitude of waves traveling to $x+$ direction, and B_j denotes those traveling to $x-$ direction.

Applying continuity conditions for both electric and magnetic field parallel to surface at the two interfaces $x = 0, d_2$, we get the following boundary equations:

$$R_1 + L_1 = R_2 + L_2, \quad (1.7)$$

$$\frac{k_x^{(1)}}{\epsilon_1}(R_1 - L_1) = \frac{k_x^{(2)}}{\epsilon_2}(R_2 - L_2), \quad (1.8)$$

$$R_2 e^{ik_x^{(2)} d_2} + L_2 e^{-ik_x^{(2)} d_2} = R_3 e^{ik_x^{(1)} d_2} + L_3 e^{-ik_x^{(1)} d_2}, \quad (1.9)$$

$$\frac{k_x^{(2)}}{\epsilon_2}(R_2 e^{ik_x^{(2)} d_2} - L_2 e^{-ik_x^{(2)} d_2}) = \frac{k_x^{(1)}}{\epsilon_1}(R_3 e^{ik_x^{(1)} d_2} - L_3 e^{-ik_x^{(1)} d_2}). \quad (1.10)$$

If R_j and L_j are shown in column vectors, we can transform the above equations into simplified matrix multiplications, as shown below:

$$\mathbf{D}_1 \begin{bmatrix} R_1 \\ L_1 \end{bmatrix} = \mathbf{D}_2 \begin{bmatrix} R_2 \\ L_2 \end{bmatrix}, \quad (1.11)$$

$$\mathbf{D}_2 \mathbf{P}_2 \begin{bmatrix} R_2 \\ L_2 \end{bmatrix} = e^{ik_x^{(1)}L} \mathbf{D}_1 \mathbf{P}_1^{-1} \begin{bmatrix} R_3 \\ L_3 \end{bmatrix}, \quad (1.12)$$

where $\mathbf{D}_j$ and $\mathbf{P}_j$ ($j = 1, 2$) are given by

$$\mathbf{D}_j = \begin{bmatrix} 1 & 1 \\ \frac{k_x^{(j)}}{\epsilon_j} & -\frac{k_x^{(j)}}{\epsilon_j} \end{bmatrix}, \quad (1.13)$$

$$\mathbf{P}_j = \begin{bmatrix} e^{ik_j^{(j)}d_j} & 0 \\ 0 & e^{ik_j^{(j)}d_j} \end{bmatrix}. \quad (1.14)$$

Accordingly we can find the relationship between (R_3, L_3) and (R_1, L_1) as

$$\begin{bmatrix} R_3 \\ L_3 \end{bmatrix} = e^{-ik_x^{(1)}L} \mathbf{P}_1 \mathbf{D}_1^{-1} \mathbf{D}_2 \mathbf{P}_2 \mathbf{D}_2^{-1} \mathbf{D}_1 \begin{bmatrix} R_1 \\ L_1 \end{bmatrix}. \quad (1.15)$$

The T-matrix $\mathbf{T}_0$ relating the input (at $x = 0^-$) and output (at $x = L^-$) electromagnetic field amplitudes in a unit-cell is defined by

$$\begin{bmatrix} A_3(L) \\ B_3(L) \end{bmatrix} = \mathbf{T}_0 \begin{bmatrix} A_1(0) \\ B_1(0) \end{bmatrix}, \quad (1.16)$$

with the following relations:

$$\begin{bmatrix} A_3(L) \\ B_3(L) \end{bmatrix} = e^{k_x^{(1)}L} \begin{bmatrix} R_3 \\ L_3 \end{bmatrix}. \quad (1.17)$$

Note that at $x = 0$, $[A_1(0), B_1(0)] = (R_1, L_1)$. Combining Eq. (1.15 - 1.17) we can obtain the formula of $\mathbf{T}_0$ for a two-component unit-cell, given by

$$\mathbf{T}_0 = \mathbf{P}_1 \mathbf{D}_1^{-1} \mathbf{D}_2 \mathbf{P}_2 \mathbf{D}_2^{-1} \mathbf{D}_1. \quad (1.18)$$

This T-matrix of a unit-cell in a periodic system is useful as we can retrieve the Bloch wave number (k_n) from its eigenvalues. The Bloch wave number is a

crucial physical quantity implying the key features of a periodic system, such as the characteristic photonic modes and bandgaps in PCs, absorption length in plasmonic omposites, the effective index of refraction and wave dynamics in metamaterials etc. In particular, we can obtain the phase-space dispersion as shown by the IFS.

The Bloch wave number k_B in a periodic system with periodicity L can be calculated from the equation $\cos(k_B L) = \text{tr}\{\mathbf{T}_0\}$, where tr denotes the trace of the T-matrix. In a lossless system, it is given by

$$k_B = \frac{\cos^{-1}(0.5\text{tr}[\mathbf{T}_0])}{L}. \quad (1.19)$$

For a lossy system, the sign of $\text{Re}[k_B]$ is important as it indicates the propagation directions of light. A exact solution of Bloch wave number with losses included is shown as

$$k_B = \text{Sgn}\{\text{Im}[\cos^{-1}(0.5\text{tr}[\mathbf{T}_0])]\} * \frac{\cos^{-1}(0.5\text{tr}[\mathbf{T}_0])}{L}. \quad (1.20)$$

In a multilayered periodic composite, the Bloch wave number is usually represented by the normal-to-plane wave vector k_n which is a function of the tangential (or parallel-to-plane) wave vector k_t . Such a “ $k_n \sim k_t$ ” relationship gives arise the 3D IFS or iso-frequency cruve (IFC) in 2D.

We can also extend the T-matrix for a unit-cell to the whole periodic system and study the reflection and transmission in a real scenario. In the following example, we consider a planar multilayered system composed of N unit-cells surrounded by two bulk media ϵ_0 and ϵ_s as shown in Fig. 1.3. The optical waves are incident from $x < 0$.

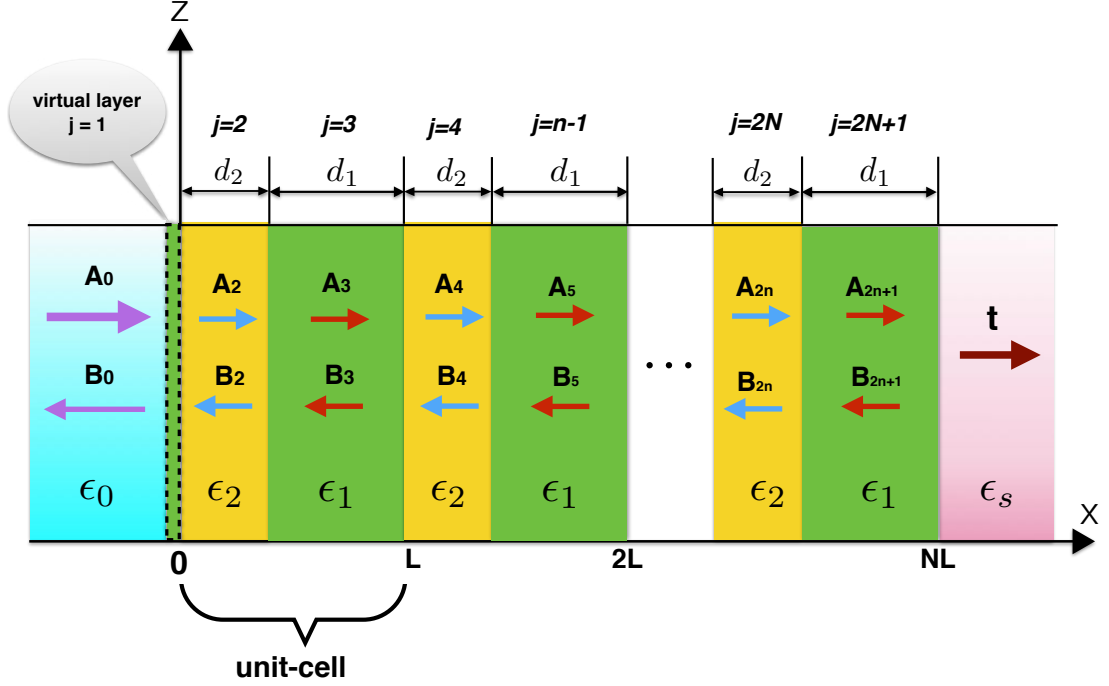


Fig. 1.3. Schematic illustration of a periodic multilayered system bounded by two bulk media of ϵ_0 and ϵ_s . The periodic system consists of N unit-cells with each layer parameters given. The dashed boxes at $x = 0$ denotes the virtual layer $j = 1$ with zero thickness, which is added to facilitate the T-matrix calculation.

To facilitate the calculation of T-matrix, we introduced a virtual “buffer” layer of ϵ_1 with zero thickness at the entrance facet ($x = 0$) of the periodic system. This layer preserve the complete form of T-matrix as extended from a single unit-cell to the whole composite: $\mathbf{T}_N = \mathbf{T}_0^N$.

The T-matrices at the two boundaries of the composite including the “buffer” layer are

$$\mathbf{T}_{01} = \mathbf{D}_1^{-1} \mathbf{D}_0, \quad (1.21)$$

$$\mathbf{T}_{1s} = \mathbf{D}_s^{-1} \mathbf{D}_1. \quad (1.22)$$

The overall T-matrix $\mathbf{T}_{\text{tot}}$ can be obtained simply by

$$\mathbf{T}_{\text{tot}} = \mathbf{T}_{1s} \mathbf{T}_0^N \mathbf{T}_{01}. \quad (1.23)$$

Since this T-matrix relates the input and output field magnitude as

$$\begin{bmatrix} t \\ 0 \end{bmatrix} = \mathbf{T}_{\text{tot}} \begin{bmatrix} A_0 \\ B_0 \end{bmatrix} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \begin{bmatrix} A_0 \\ B_0 \end{bmatrix}, \quad (1.24)$$

where T_{11}, T_{12}, T_{21} and T_{22} are 4 matrix elements of $\mathbf{T}_{\text{tot}}$. We can obtain the reflection and transmission coefficient r and t for the whole system given by

$$r = \frac{B_0}{A_0} = -\frac{T_{21}}{T_{22}}, \quad (1.25)$$

$$t = \frac{T_{11}T_{22} - T_{12}T_{21}}{T_{22}} = \frac{1}{T_{22}}. \quad (1.26)$$

The corresponding reflectivity and transmittance are $R = |r|^2, T = |t|^2$.

1.2.3 Hyperbolic dispersion

There is a special class of optical materials featured by the exotic hyperbolic dispersion, which originates from the opposite signs in one or two of the principal components of their electric or magnetic effective tensor. These anisotropic materials with hyperbolic dispersion used to be classified as HMM since many unusual properties in the artificially structured composites were found, such as negative refraction [43–47] and hyperlensing [7], diverging photonic density of states [48], spontaneous emission enhancement [49] and thermal hyper-conductivity [50, 51] etc.

Unlike the isotropic negative polarization response to electric field in metal electrons, the dielectric permittivity tensor in hyperbolic medium only shows negative components in one or two spatial directions. Therefore one can achieve the hyperbolic dispersion by confining the free-electron motions to only specified directions. This anisotropy can be realized by metamaterials composed of stacked alternating metallic and dielectric layers, which is one of the most commonly used structure in experiments.

Typical structures of HMM with different configurations can be found in Ref. [60]. Other than the most popular planar interleaved layers of metal-dielectric, some

more complicated structures are also fabricated, including curved cylindrical layers, nanorod arrays, fishnet multilayers, nanopyramids etc.

Other than the artificially engineered HMM, a wide range of natural materials capable of exhibiting hyperbolic dispersion has been discovered recently [61] covering a broad electromagnetic frequency bands. Different with the negative response of free-electrons in a specified direction, the naturally hyperbolic dispersion arise from the substantial anisotropy in phonons (e.g. sapphire at mid-IR [62]) and effective electron masses (e.g. Bismuth [63]). This new class of natural hyperbolic materials provides a competitive alternative to HMM made of metal-dielectric multilayers or other complex composites, and offers attractive potentials in some practical applications of nanophotonic devices, such as super-resolution imaging and thermal photovoltaics.

1.2.4 Summary

In summary, engineering the IFS in structured media has opened a new avenue to explore new optical properties and control the light propagation. There are now well established methods to fabricate artificial optical media with phase-space dispersion “to order” – which brings an entirely new toolset to nanophotonics and optics in general. It is with this objective in mind that we introduce some novel applications using this new capability in this thesis.

1.3 Overview of the thesis

This thesis is divided into 6 chapters with the following logics.

Chapter 1 gives a literature review of structured optical media as well as the important physical definitions and major technical methods used.

Chapter 2 presents an example of manipulating the propagation phase with a zeroth-order resonant unit-cell as the core of a Fabry-Perot hyperbolic resonator.

Chapter 3 extends the zeroth-order unit-cell to periodic structure, leading to a new class of photonic hypercrystals that combines properties of both photonic crystals

and hyperbolic materials. A novel negative refraction lens is developed based on the photonic hypercrystals to address the improvement of imaging performances.

Chapter 4 applies the similar idea of engineering phase space dispersion on a planar dielectric photonic crystal and demonstrates a conical refractive flat lens with a cone-shaped iso-frequency surface. This can be applied to a new approach of imaging.

Chapter 5 introduces a new method of optical phase retrieval using the conical refraction imaging, following the previous work in Chapter 4.

Chapter 6 summarizes the thesis and discusses the future work.

2. ZERO-ORDER RESONANT UNIT-CELL

2.1 Introduction

In metamaterial-based photonic devices, the size of unit-cell has significantly limited the general system performance. Hyperbolic materials, which exist in natural form, offer an alternative way to address the device miniaturization problem without the unit-cell size restrictions. In particular, some of these natural hyperbolic materials with working optical frequencies enable us to control the unit-cell elements with arbitrarily small size.

Thanks to the rapid development in material science and nanotechnology, optical hyperbolic metamaterials (HMM), nanostructured artificial composites with different signs in dielectric functions along two orthogonal directions, can now be fabricated and have been one of the most active research frontiers in the last 15 years [7–9, 33, 43–51, 64–69]. A wide range of new physical phenomena have been discovered in those highly anisotropic systems including: subwavelength light confinement by waveguides [47], 3D resonant cavities [64], enhanced energy transport and radiation [50, 51] and novel quantum-electrodynamics [48, 49] etc. These new phenomena have provided lots of striking applications, such as far-field super-resolution imaging with Hyperlens [7–9], enhanced spontaneous emission [49] and new stealth technology [65].

Generally, these new discoveries are rooted from two key features in hyperbolic materials: (1) negative group velocity and all-angle negative refraction inherent in a “metal” type hyperbolic materials [33, 44–47]; (2) the extraordinary capability of supporting propagating waves with unlimited wave numbers [7], leading to a broadband enhancement of phonic density of states [66].

Owing to these features, light confinement beyond the diffraction limit can be achieved. For example, three-dimensional nanopillars hyperbolic metamaterial

(HMM) allows extremely subwavelength scale of resonant cavities [64]. However, practical applications of such devices are still unfeasible and technically challenging, since most energy in these high-order modes is exponentially dissipated resulting in very little out-coupling.

Nevertheless, there is another important feature of hyperbolic materials which is usually neglected: the negative phase velocity supported in a “dielectric” type of hyperbolic materials. When a positive refraction occurs at the interface of hyperbolic medium, the Poynting vector of refracted wave is in opposite direction of the refracted wave vector. As a result, negative phases are accumulated in transmitted waves, giving arise to a new zeroth-order transmission resonance in subwavelength scale, which is similar to resonators or cavities with negative refractive index [29]. However, our subwavelength resonator is different with from the purely high-order hyperbolic modes [64], since light can be refractively coupled to the surrounding (even lossy) media (e.g. thin metallic films). Therefore, we can obtain an arbitrary control of radiative out-coupling under this new resonance.

In this chapter, we introduce a new optical resonator based on the zeroth-order resonance in a hyperbolic material configured with Fabry-Perot geometry. Such a resonator combines the merits of subwavelength dimensions and strong out-couplings, with potential applications in ultracompact waveguides and nanolasers.

2.2 Zeroth-order resonance in hyperbolic medium configured as a Fabry-Perot resonator

For simplicity, let's consider a TM-polarized electromagnetic plane wave incident upon a planar interface between an isotropic dielectric and a hyperbolic medium (Fig. 2.1(a)). The latter has a positive normal permittivity component $\epsilon_y > 0$ and negative in-plane dielectric permittivity component $\epsilon_x < 0$. The magnetic field of the incident plane wave is given by

$$\mathbf{B} = B_0 e^{i(k_x x + k_y y - \omega t)} \hat{\mathbf{z}}, \quad (2.1)$$

and the associated electric field $\mathbf{E}$ and Poynting vector $\mathbf{S}$ can be directly derived from Maxwell's equations:

$$\mathbf{E} = B_0 \frac{c}{\omega} \left[\frac{k_x}{\epsilon_y} \hat{\mathbf{y}} - \frac{k_y}{\epsilon_x} \hat{\mathbf{x}} \right] \exp(ik_x x + ik_y y - i\omega t), \quad (2.2)$$

$$\mathbf{S} = |B_0|^2 \frac{c^2}{4\pi\omega} \left[\frac{k_x}{\epsilon_y} \hat{\mathbf{x}} + \frac{k_y}{\epsilon_x} \hat{\mathbf{y}} \right]. \quad (2.3)$$

We use the iso-frequency curve in Figure 2.1(b) to show the directions of wave vector $\mathbf{k}$ (corresponding to the phase velocity) and Poynting vector $\mathbf{S}$ (corresponding to the energy flow) in two adjacent media. Taking into account that the transmitted energy flux (represented by Poynting vector $\mathbf{S}_t$) stays away from the boundary, and the tangential (in-plane) momentum k_x at the planar interface is preserved, the refraction is normally *positive* [13, 44, 45, 47]. However, the refracted wave vector $\mathbf{k}_t$ in this example essentially stays at opposite direction comparing with the incident one. Consequently, a *negative* phase will be accumulated when light is propagating in this hyperbolic medium. This will introduce unusual consequences for interference and the transmission resonances in such a system.

For a planar Fabry-Perot resonator, if the regular active “core” medium (of thickness D) is filled with an “dielectric” type HMM with negative permittivity ϵ_m , and the two bounding high-reflective mirrors are replaced with thin metal conducting barriers (see Fig. 2.2(a)), the resonance condition in half of a round-trip can be approximated by

$$k_n D + \phi_r = \pi m, \quad (2.4)$$

where k_n is the normal-to-plane wave number in the core medium, ϕ_r is the reflection phase from the HMM core to the metal barriers, and $m = 0, 1, \dots$ is an integer. For a dielectric core medium, the Fabry-Perot resonance (Eq. (2.4)) implies $m > 0$ and that the core thickness needs to be larger than at least half of light wavelength, i.e. $D \geq 0.5\lambda_0$.

However, this behavior is completely different if one use the hyperbolic medium as the resonator core. In particular, for a metallic-type hyperbolic material (negative

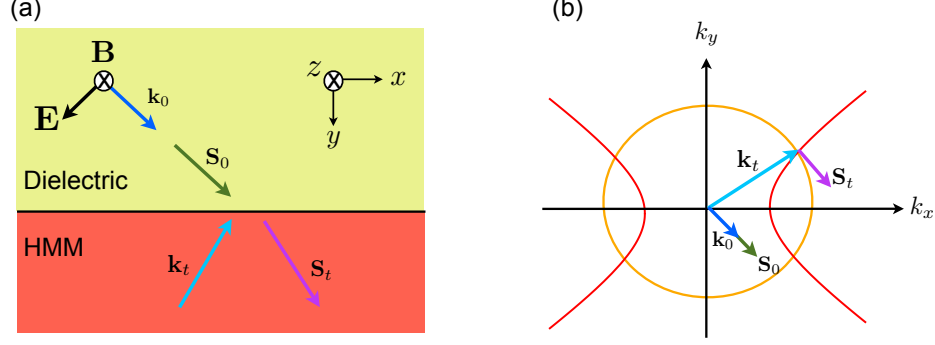


Fig. 2.1. (a) Illustration of energy flow (represented by Poynting vector $\mathbf{S}$) and phase velocity (corresponding to wave vector $\mathbf{k}$) for a plane wave striking onto the interface of a dielectric (yellow color) and hyperbolic (red color) media. The latter has a positive normal-to-plane ($\epsilon_y > 0$) permittivity and negative in-plane permittivity ($\epsilon_{x,z} < 0$) components. In the isotropic dielectric medium, the wave vector $\mathbf{k}_0$ and Poynting vector $\mathbf{S}_0$ have the same directions. However, those in HMM (wave vector $\mathbf{k}_t$ and Poynting vector $\mathbf{S}_t$) are dramatically different. (b) The closed circular IFC for an isotropic dielectric and open hyperbolic IFC for an HMM with $\epsilon_x < 0$, $\epsilon_y > 0$, and the wave vectors and Poynting vectors described in panel (a). Note that the refracted wave vector $\mathbf{k}_t$ is in the opposite direction of incident $\mathbf{k}_0$.

in-plane permittivity) that allows a negative phase velocity, i.e. $k_n < 0$, a zeroth-order resonance corresponding to $m = 0$ is enabled from Eq. (2.4). When the penetration depth of the metal barrier is smaller than its thickness d , for this zeroth-order ($m = 0$) resonance we can approximate the length of HMM core by

$$\begin{aligned} \left. \frac{D}{\lambda_0} \right|_{\text{res}} &= \frac{-\phi_r}{k_n \lambda_0} \\ &= \frac{1}{2\pi \sqrt{\epsilon_x - \frac{\epsilon_x}{\epsilon_y} \epsilon_d (\sin \theta)^2}} \text{Arg} \left[\frac{-\epsilon_m \sqrt{\epsilon_x - \frac{\epsilon_x}{\epsilon_y} \epsilon_d (\sin \theta)^2} + \epsilon_x \sqrt{\epsilon_m - \epsilon_d (\sin \theta)^2}}{\epsilon_m \sqrt{\epsilon_x - \frac{\epsilon_x}{\epsilon_y} \epsilon_d (\sin \theta)^2} + \epsilon_x \sqrt{\epsilon_m - \epsilon_d (\sin \theta)^2}} \right], \end{aligned} \quad (2.5)$$

which allows a peak transmission at deep subwavelength geometry (of D). As illustrated in Fig. 2.2, the transmission is shown as a function of HMM core length D

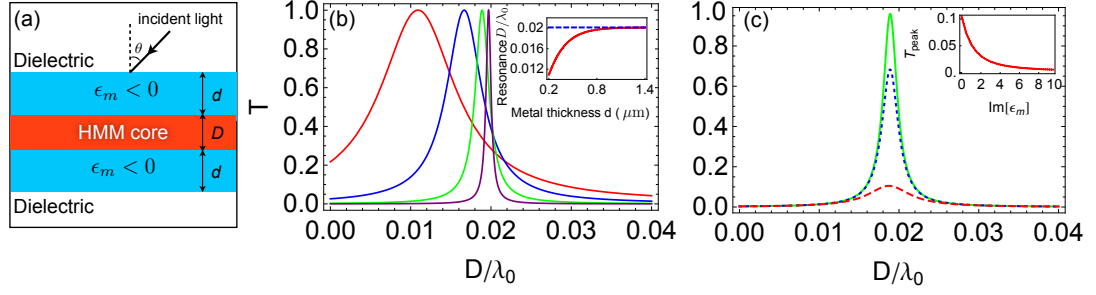


Fig. 2.2. (a) The Fabry-Perot resonator with a HMM core and metal conducting layers ($\epsilon_m < 0$) resembling the reflectors. (b) The transmission coefficients as a function of the HMM core length for different metal layer thicknesses d : 800 nm (violet curve), 600 nm (green curve), 400 nm (blue curve), 200 nm (red curve). The dielectric permittivity function are: (1) $\epsilon_x = -2$ and $\epsilon_y = 1$ in HMM core; (2) $\epsilon_m = -10$ in conducting metal barriers. The incident angle of TM-polarized plane wave is $\theta = 60^\circ$ with a free-space wavelength $\lambda_0 = 10 \mu\text{m}$. The dashed line in the inset plot shows the resonance length of HMM core predicted from Eq. (2.5). (c) The transmission coefficients for a lossy HMM core that has different imaginary parts in the permittivity function: $\epsilon_x = -2 + 0.002i$, $\epsilon_y = 1 + 0.001i$ (solid green curve); $\epsilon_x = -2 + 0.02i$, $\epsilon_y = 1 + 0.01i$ (dotted blue curve); $\epsilon_x = -2 + 0.2i$, $\epsilon_y = 1 + 0.1i$ (dashed red curve). A single metal barrier thickness is fixed at $d = 600$ nm. The values of peak transmission coefficient as the function of material losses (the imaginary parts of permittivity) in metal barriers $\text{Im}[\epsilon_m]$, when the core permittivity function is fixed at $\epsilon_x = -2 + 0.2i$, $\epsilon_y = 1 + 0.1i$.

(in units of λ_0). Fig. 2.2(b) shows the peak resonance position shifts with different metal barrier thickness d assuming no lossless are included. The inset in panel (b) gives the predicted resonance position as a function of d based on Eq. (2.5). As expected, the resonance length of core medium is quickly approaching to the theoretical prediction by Eq. (2.5) when increasing the metal barrier thickness d . Fig. 2.2(c) assesses the influence of material losses in both the HMM core and in the metal barriers, since the substantial absorption exists in all different types of HMMs. Although such absorption decreases the transmission intensity as expected, the length of core medium corresponding to zeroth-order resonance is still subwavelength and virtually stays unchanged with respect to that of “lossless” case.

2.3 Zeroth-order resonance in a multilayer HMM core

The unit-cell size of 3D optical metamaterials is usually comparable to the wavelength in the component materials [14–16, 70]. On the contrary, planar metamaterials benefited by the advanced fabrication technologies such as molecular-beam epitaxy [71] and chemical-vapor deposition [72] can shrink this size to the scale of less than one-tenth of the wavelength [33]. Therefore, the electromagnetic response in hyperbolic metamaterials with planar geometry (especially in the mid-IR range [33]) can be described by effective medium theory (EMT) [10] which approximates the effective dielectric permittivity tensor in terms of its conductive permittivity (ϵ_c) and dielectric permittivity (ϵ_d) components. In particular, for a planar HMM with 1:1 ratio of conducting and dielectric layer thickness, the in-plane (ϵ_n) and normal-to-plane permittivity (ϵ_τ) are given by

$$\epsilon_n(\omega) = \frac{\epsilon_c(\omega) + \epsilon_d(\omega)}{2}, \quad (2.6)$$

$$\epsilon_\tau(\omega) = \frac{2\epsilon_c(\omega)\epsilon_d(\omega)}{\epsilon_c(\omega) + \epsilon_d(\omega)}. \quad (2.7)$$

As a comparison, we also use the standard and more rigorous T-matrix method to calculate the transmission coefficient of the system, taking into account the ac-

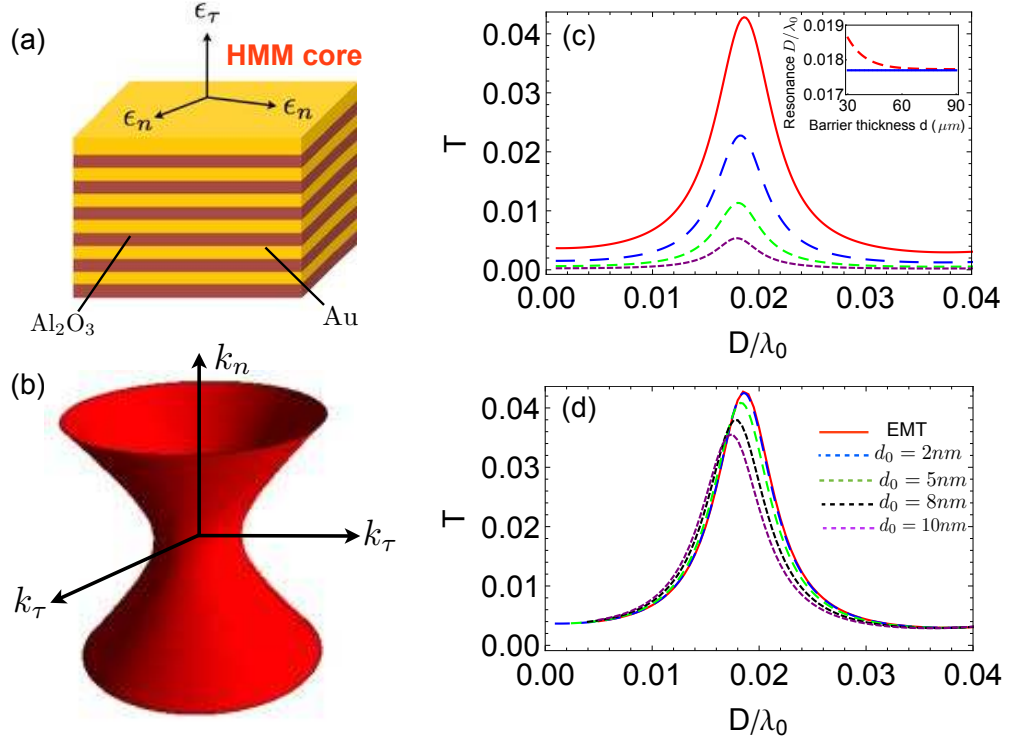


Fig. 2.3. (a) The HMM ($\epsilon_n > 0$ and $\epsilon_\tau < 0$) structure containing 1:1 alternating layers of Au- Al_2O_3 as the core of the Fabry-Perot resonator. (b) The IFC of the multilayered HMM core in panel (a) at $\lambda_0 = 4 \mu\text{m}$. (c) Transmission diagram of the Fabry-Perot resonator (with actual material losses included) consisting of the HMM core and different metal (Au) barrier thicknesses: $d = 30 \text{ nm}$ (solid red curve), $d = 35 \text{ nm}$ (dashed blue curve), $d = 40 \text{ nm}$ (dashed green curve) and $d = 45 \text{ nm}$ (dashed purple curve). The inset shows the theoretically predicted resonance length of the HMM core. The incident angle in this example is $\theta = 60^\circ$. (d) The transmission diagram based on calculations using T-matrix (dashed curves) and EMT (solid red curve). The multiple curves show the calculated results for different single component layer thicknesses (d_0) in HMM: $d_0 = 2 \text{ nm}$ (dashed blue curve), $d_0 = 5 \text{ nm}$ (dashed green curve), $d_0 = 8 \text{ nm}$ (dashed black curve), and $d_0 = 10 \text{ nm}$ (dashed purple curve). The metal barrier thickness here is set as $d = 30 \text{ nm}$.

tual component layer thicknesses of HMM – see Figs. 2.3(b) and 2.3(c). Our results validates the 0th-order transmission resonance still exist even in the deep subwavelength length of HMM core ($D \approx 0.02 \lambda_0$). In particular, when the component layer thickness is very small (less than $0.125 \lambda_0$), the results quantitatively well match the theoretical predications by EMT.

In the current work, we only show the zeroth-order resonance in a metallic-type HMM that has a negative in-plane permittivity ϵ_n . However, such resonance can also be found in the other dielectric-type HMMs where $\epsilon_n > 0$ and $\epsilon_\tau < 0$. The metal conducting barriers in the opposite case, however, need to be replaced by dielectric materials, which may be helpful to improve the magnitude of transmission. We also anticipate by optimizing the materials and dimensions, a larger transmission with improved Q-factor of such subwavelength cavity can be achieved.

2.4 Conclusions

In conclusion, we discovered a novel zeroth-order transmission resonance in a hyperbolic optical resonator with Fabry-Perot structure, which allows a new approach to confining light propagation in subwavelength cavity with relatively high out-coupling. Such a resonance inherently originates from the negative phase velocity in hyperbolic materials. Our device is not only robust to dissipative losses and structural imperfection, but also compatible with planar metamaterial technology.

3. IMAGING WITH STRUCTURED OPTICAL MEDIA: PHOTONIC HYPERCRYSTALS

3.1 Introduction

The curved refraction lens, usually made out of transparent dielectrics, represents one of the most extensively applied optical elements in imaging devices, such as microscopy and telescope. However, this lens has now reached its fundamental limit, such as spherical aberration and diffraction-limited resolution. In order to enhance the imaging performance, alternative approaches of optical imaging have been recently discussed and proposed [31, 73–77]. Among many of these new ideas, metamaterials that are rationally designed composites with periodically structured subwavelength components [10] have played an essential role. For example, one of the most attractive studies is a planar negative refraction lens – Veselago lens [78] (also known as super lens or perfect lens, or Pendry’s lens [31]), which was first introduced by Veselago [78] in 1968. This lens is known for its striking potential in super-resolution imaging, which has excited the extensive studies of metamaterials ever since 2000 [6, 31, 79]. Another crucial nature of Veselago lens is the planar geometry, which can be applied in sub-diffraction-limit photolithography [6] as well as large-scale nano-patterning of integrated chipsets [80, 81].

The negative index metamaterials (NIMs) have initiated the early studies and opened the avenue to experimental realization of Veselago lens [15, 82–86]. However, there are enormous technical challenges in fabrication, and the critical resonant losses for simultaneously negative permittivity and permeability region are extremely high [87], making the NIMs unfeasible in practical applications. Other than NIMs, some non-magnetic methods have been investigated, such as transparent photonic crystals (PCs) [42, 88–92], periodically coupled plasmonic waveguides [34, 93–96] and

hyperbolic metamaterials (HMMs) [45, 97–99]. With these new methods, the device structures are much simplified and the dissipation losses in the materials are also significantly reduced.

However, an aberration-free image for most of these non-magnetic imaging methods is still not attainable since the resulting refractive index is not a constant [93, 97]. Consequently, the performance of corresponding alternative imaging systems (e.g. [97]) is significantly hindered.

A new approach of Veselago lensing is proposed in this chapter based on photonic “hypercrystals” (PHCs) that can substantially reduce the spherical aberrations [100–102]. The PHCs are essentially structured optical media combining the properties of both hyperbolic materials and photonic crystals [103–107]. We show the imaging of point sources to verify the functions of our device as a proposed Veselago lens and discuss the structure defects on the imaging performances.

3.2 Veselago lens realized by photonic hypercrystals

The PHC is an artificial composite that extends the zeroth-order resonant unit-cell introduced in Chapter 2 to a periodic structure while maintaining the subwavelength dimension (see Fig. 3.1). This structure exhibits some common features of hyperbolic materials and photonic crystals in this novel composite. First, PHC allows negative refraction that is inherent in a metallic-type HMM as one of its major components. Second, the allowed bands and forbidden bandgaps in k -space still persist in the subwavelength regime which is a direct result of Bragg scattering, as shown in Fig. 3.1. Third, the Bragg scattering of high- k waves in HMM can excite strong localized subwavelength surface modes, resulting in substantially reduced dissipation losses compared with the regular surface plasmons [100]. Those essential features lead to an intriguing application in optical imaging. In particular, we can use a planar PHC to establish a new type of Veselago lens which can improve the imaging performances in both PCs and HMMs.

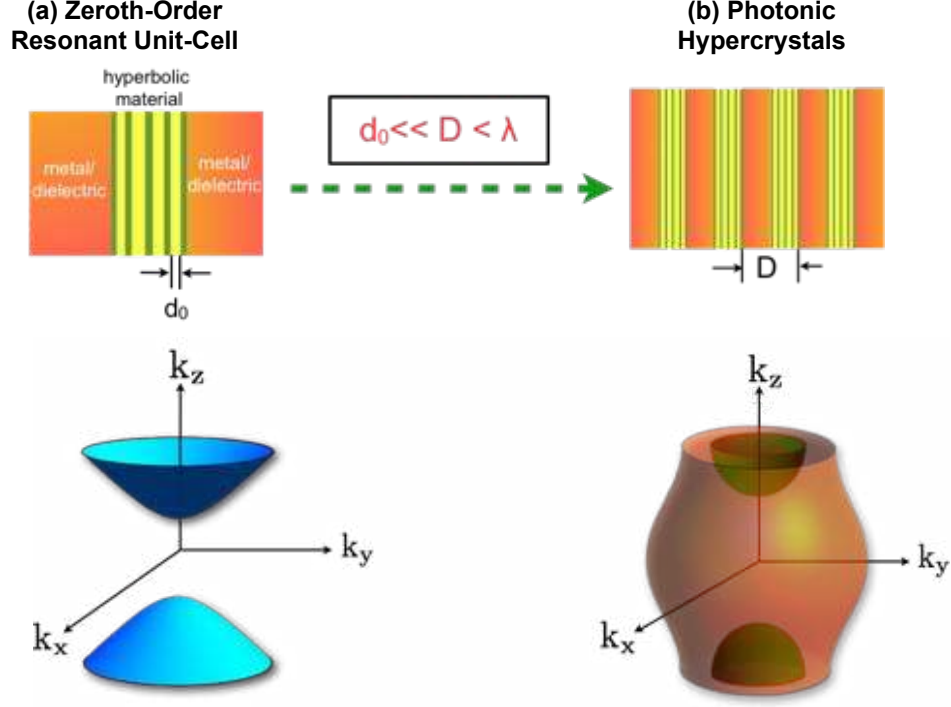


Fig. 3.1. Schematic illustrations of the structures and iso-frequency surfaces (IFSs) of the zeroth-order resonant unit-cell (panel a) and the extension to PHC (panel b), in the lossless limit. (a) The planar layered structure of hyperbolic medium (as the core) in a unit-cell and its IFS, where each component layer thickness is in much smaller than the wavelength; (b) The periodic structure of a planar PHC and its IFS. The green hyperboloid indicates the first (also the widest) propagating band in PHC, which is surrounded by translucent orange outline as the second propagating band.

Here, we use multilayers of Si and sapphire (Al_2O_3) to implement the planar PHC as shown in Fig. 3.2(a). The sapphire is a natural hyperbolic medium at $20 \mu\text{m}$ [62], and the IFC of sapphire is shown in Fig. 3.2(c).

Negative refraction can always be observed in both PHC and a metallic-type ($\epsilon_r > 0$ and $\epsilon_n < 0$) [45, 97–99] hyperbolic medium, however, the latter cannot focus all the homocentric incident beams from an illumination source or object. As shown in Fig. 3.3(b), those refracted beams with large incident angles become defocused at

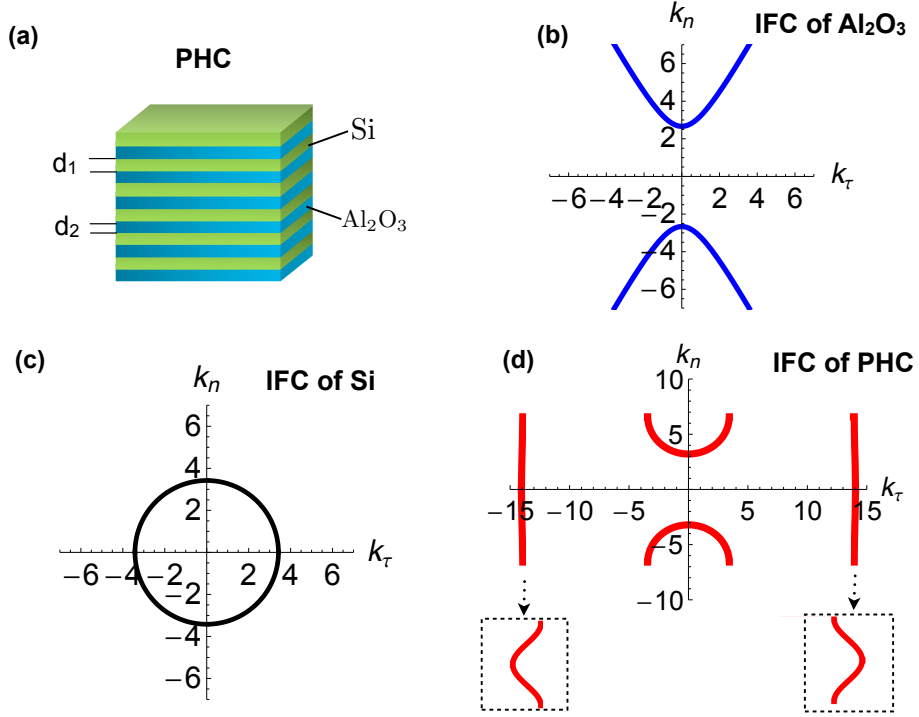


Fig. 3.2. (a) Schematic illustration of a planar PHC consisting of multi-layered Si and sapphire (Al_2O_3), with the optimal component layer thicknesses given by $d_1(\text{Si}) = 1 \mu\text{m}$ and $d_2(\text{Al}_2\text{O}_3) = 0.47 \mu\text{m}$ at free-space wavelength $\lambda_0 = 20 \mu\text{m}$. (b - d) The iso-frequency curves (IFCs) of Al_2O_3 (blue hyperbola in panel (b)) as hyperbolic medium, Si (black circle in panel (c)) as dielectric medium, and PHC (red curves in panel (d)) at $20 \mu\text{m}$. The inset dashed boxes illustrate the enlarged view of two narrow propagating bands. Copyright of this figure belongs to Ref. [101].

the interface between the HMM and an isotropic dielectric [97]. On the contrary, the PHC allows all the refracted beams to be focused to one point when the associated IFC becomes a half circle (Fig. 3.3(c)). Therefore, the image aberration problems can be addressed if the PHC is used as a Veselago lens.

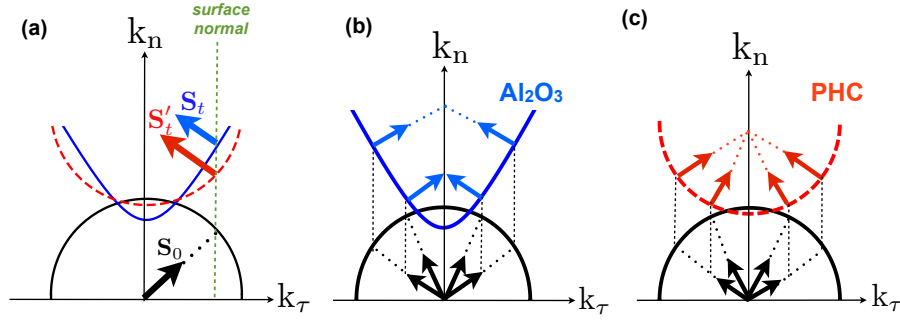


Fig. 3.3. Schematics illustration of IFCs of Si (black solid circle) and sapphire (blue solid curve) as natural hyperbolic medium, PHC (red dashed curve) as the proposed Veselago lens, for a TM polarized incident beam. The Poynting vectors of the incident and refracted beams are indicated by bold arrows. $\mathbf{S}_0$ represents the Poynting vector of plane waves incident from Si. $\mathbf{S}_t$ and $\mathbf{S}'_t$ denote those vectors with refracted beams in sapphire and PHC, respectively. At the interface between Si and both sapphire and PHC, as shown in panel (a), negative refraction occurs. This is indicated by the same positions of both incident and refracted Poynting vectors with regard to the surface normal. Panel (b) shows a partial focusing in sapphire as not all of the refracted rays are directed to the same point. However, a nearly perfect light focusing by PHC is observed in panel (c) where all the refracted rays are pointing to the center of the semi-circle. Copyright of this figure belongs to Ref. [101].

3.3 Material parameter optimization in PHC for Veselago lensing

The Veselago lens requires a constant index of refraction matching the dielectric environment. Apparently, all the refracted rays at the interface of Si and PHC have to be focused to a single point, otherwise a perfect focusing in this planar system cannot be achieved. Therefore, we modify the first and also the wide propagating band in IFC to obtain a semi-circle shape with its curvature matching that of Si. Specifically, we need optimize the component layer thicknesses (d_1 and d_2) of PHC so

that the IFC can be reshaped to be a half-circle (Fig. 3.3(c)). The refractive index relative of PHC in such a geometry is given by Snell's law as

$$n^* \equiv \frac{\sin(\theta_t)}{\sin(\theta_{\text{inc}})} = n_0 \kappa, \quad (3.1)$$

where n_0 denotes the index of refraction in the surrounding dielectric medium, and κ represents the dimensionless curvature of the first band in IFC given by

$$\kappa = \frac{d^2 k_n}{dk_\tau^2}. \quad (3.2)$$

The Veselago lensing condition is satisfied only if the curvature of IFC κ matches n_0 , leading to the constant negative value of n^* :

$$n^* = -1. \quad (3.3)$$

To achieve this condition in a planar PHC, two boundary conditions need to be satisfied with regard to the “opposing” ends of incident angle range, i.e. $\theta_{\text{inc}} = 0^\circ$ and $\theta_{\text{inc}} = 90^\circ$.

For the first boundary condition, the curvature κ of IFC at normal incidence ($\theta_{\text{inc}} = 0^\circ$) where the in-plane wavenumber $k_\tau = 0$ has to match that of the surrounding dielectric:

$$\left. \frac{d^2 k_n}{dk_\tau^2} \right|_{k_\tau=0} = \frac{1}{n_0}. \quad (3.4)$$

To ensure Eqn. (3.3) at $\theta = 90^\circ$, we use the bandgap of PHC in momentum(k) space to enforce the IFC at this angle (edge of first band) to be orthogonal to the k_τ axis so that the refraction angle $\theta_t = -90^\circ$ – see Fig. 3.3(c). This gives us the second boundary condition:

$$k_n|_{k_\tau=n_0 \frac{\omega}{c}} = \frac{\pi}{d_1 + d_2}, \quad (3.5)$$

where ω is the angular frequency, c is the vacuum speed of light, and “ $d_1 + d_2$ ” is the period of Brillouin zone in k -space.

With these two boundary conditions, we can solve the optimal thickness parameters of the dielectric and HMM components given their effective permittivity functions. For the Si-Al₂O₃ planar PHC, the optimal parameters at $\lambda_0 = 20 \mu\text{m}$ are given in Fig. 3.2.

3.4 Improving the strong image aberrations in HMM

Hyperbolic medium with positive in-plane permittivity ($\epsilon_\tau > 0$) and negative normal-to-plane permittivity ($\epsilon_n < 0$) allows negative refraction [45, 97–99]. Nevertheless, it cannot focus homocentric incident beams from all different incident angles since the refracted angles in such hyperbolic medium are dependent on the incident angles – see Fig. 3.4(a). This behavior is however completely different in PHC if the first band is close to a semi-circle. As can be seen in Fig. 3.4(c), all the refracted angles (θ_t) in PHC are almost equal to the incident ones (θ_{inc}), indicating a perfect focusing of all refracted beams and thereby a constant refractive index independent of incident angles. Therefore, a planar hyperbolic medium used to image a point source results in an imperfect focusing (Fig. 3.4(a)) which is equivalent to the effect of spherical aberration. However, with a planar PHC, the homocentric waves coming from a point source can be perfectly focused, yielding an aberration-free images. Even the material losses are present in sapphire, a nearly perfect focusing is still observable – see Fig. 3.4(b).

The value of relative refractive index n^* for a planar PHC with optimal d_1 and d_2 is almost constant for all incident angles θ_{inc} (Fig. 3.4(d)), while for hyperbolic medium (e.g. sapphire) this index strongly depends on incident angles θ_{inc} . Therefore, a nearly-perfect focusing can be obtained by HPC as a Veselago lens. For very large incident angles (corresponding to larger tangential wave number k_τ), the refracted rays are partially deflected from the focus. Nevertheless, most energy of incoming waves is transferred by this Veselago lens and recovered near the predicted exit focus.

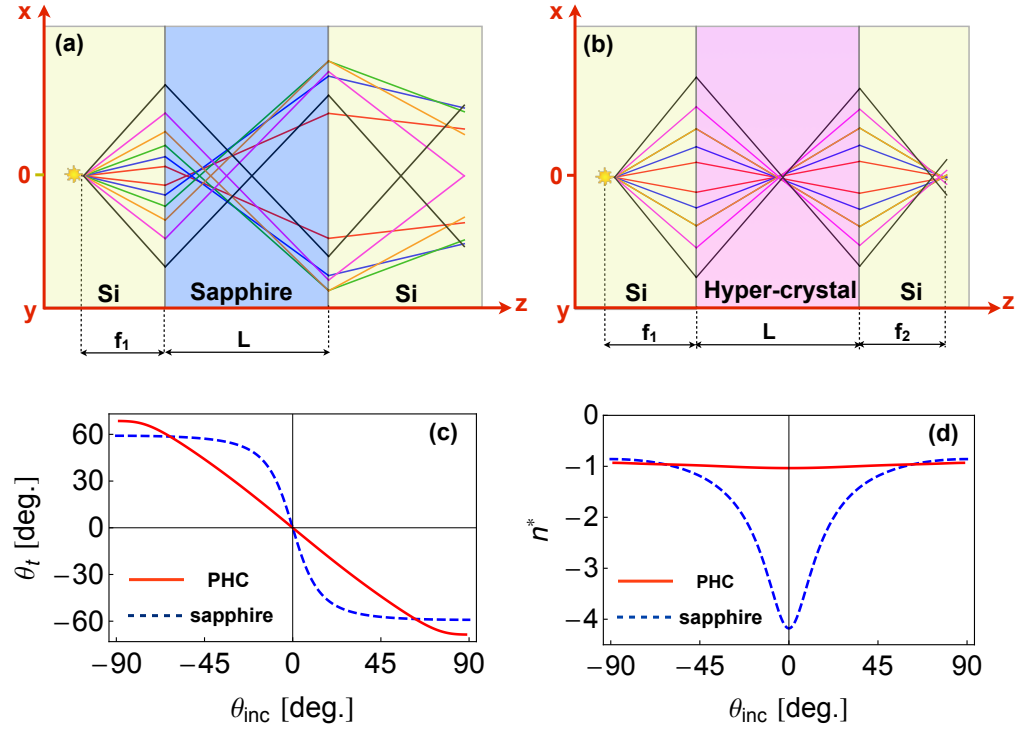


Fig. 3.4. (a-b) Illustration of the ray trajectories of light emanating from a point source and refracted by two different slab media: (a) sapphire (Al_2O_3) as a natural hyperbolic medium at $\lambda_0 = 20 \mu\text{m}$; (b) PHC used as a Veselago lens. The PHC contains 70 Si- Al_2O_3 unit-cells with a total thickness of $L = 105 \mu\text{m}$. Each component layer has optimal thickness: $d_1(\text{Si}) = 1 \mu\text{m}$, $d_2(\text{Al}_2\text{O}_3) = 0.47 \mu\text{m}$. In both (a) and (b) the light source is located at $f_1 = 51.45 \mu\text{m}$ from the slab. Only partial focusing by sapphire is observed in panel (a), whereas all the incident beams are nearly perfectly focused inside and outside the PHC in panel (b). (c-d) The (incident) angular dependance of refraction angle (θ_t) and relative refractive index (n^*) for a TM-polarized plane wave entering the Al_2O_3 (blue dashed) and PHC (red solid) from Si. The index n^* is shown as a function of incident angle θ_{inc} . Copyright of this figure belongs to Ref. [101].

We also assess the impact of material dispersion by changes of wavelength to the refractive index n^* of PHC. Three different wavelengths λ_0 were chosen with the optimal layer thickness d_1 and d_2 . As shown in Fig. 3.5. the values of n^* are still

close to -1 with very small variations for different incident angles. As a matter of fact, the deviations of n^* from -1 is comparable with higher values for large θ_{inc} even when the actual losses are present.

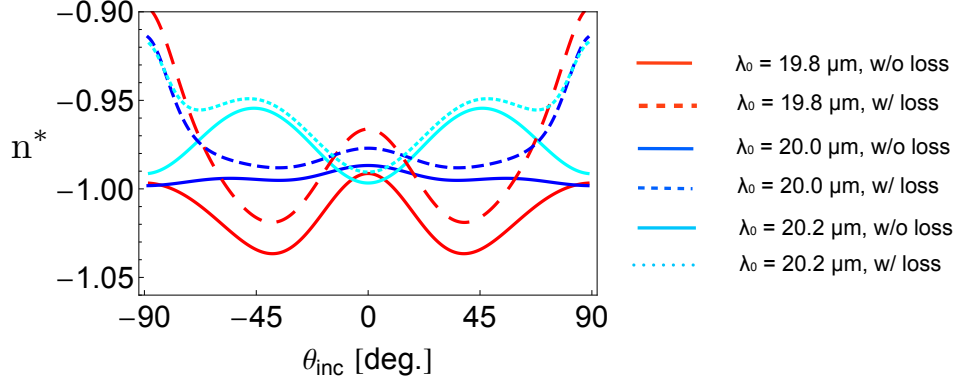


Fig. 3.5. The effect of material dispersion on the relative refractive index n^* of PHC, which is calculated for different incident angle θ_{inc} for three wavelengths: 19.8 μm , 20.0 μm and 20.2 μm . The actual material losses are included in dashed curves and excluded in solid curves. Copyright of this figure belongs to Ref. [101].

Depending on the curvature of tailored first propagating band of PHC, the value of n^* may vary with different θ_{inc} . For a perfect half circular shape and the curvature at every point (corresponding to different θ_{inc} or k_{τ}) shall be equal to that of the IFC for the surrounding dielectrics. However, the modification of IFC for PHC is always “imperfect”, which causes the slightly varying n^* . Even though an ideal IFC of semi-circle is barely attainable in PHC, the values of n^* is still close to a constant with different θ_{inc} . The substantial improvement of imaging performance thus can be achieved.

In the next two sections, we use the COMSOL *Multiphysics* and Finite Element Method [108] to demonstrate the negative refraction using Gaussian beam in Fig. 3.6, as well as the imaging of point line sources in Fig. 3.7 and 3.8.

3.5 Negative refraction demonstration in PHC

We use the same example of a planar PHC consisting of multilayers of Si and Al_2O_3 at the wavelength $\lambda_0 = 20 \mu\text{m}$ as shown in Fig. 3.2 (a-b). With optimal component layer thicknesses satisfying Eqn. (3.4 – 3.5), COMSOL *Multiphysics* was used to simulate the propagation of a TM polarized Gaussian beam with 45° incident angle. As can be seen in Fig. 3.6, the directions of the Poynting vectors for the reflected and refracted beams are almost anti-parallel, which verifies the negative refraction $\theta_t = -\theta_{\text{inc}}$ at the interface between Si and PHC even with losses considered.

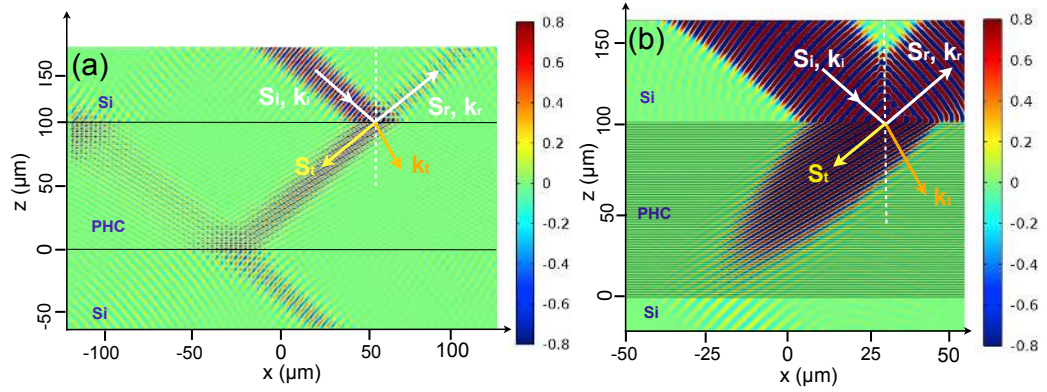


Fig. 3.6. (a-b) Numerical demonstrations of negative refraction for a Gaussian beam propagating from Si and striking upon the PHC without (a) and with (b) actual absorption losses. The paraxial Gaussian beam is TM polarized and has a beam waist of $20 \mu\text{m}$. The beam incident angle is 45° . The beam source is excited from the scattering boundary condition at the top, and perfect matching layers are applied at all other boundaries. On the interface between PHC slab and Si, a negative refraction angle equal to the incident one is clearly shown, as one can observe from the Poynting vectors of incident ($\mathbf{S}_i$), refracted ($\mathbf{S}_t$) and reflected ($\mathbf{S}_r$) beams. The component layer thicknesses of PHC are $d_1(\text{Si}) = 1 \mu\text{m}$, $d_2(\text{Al}_2\text{O}_3) = 0.47 \mu\text{m}$. Copyright of this figure belongs to Ref. [101].

3.6 Imaging with the PHC-based Veselago lens

We use the Optics and Wave Module of COMSOL *Multiphysics* to compute the full simulation of the imaging with a single line point source using a planar Si-Al₂O₃ PHC proposed as the Veselago lens. As shown in Fig. 3.7, the line source is located in Si and above the PHC around $22.05 \mu\text{m} \approx 1.1\lambda_0$. This line source is TM polarized which can be generated by a magnetic current source. The PHC contains 30 unit-cells with a total thickness around $45 \mu\text{m}$. Each component layer thicknesses of Si and Al₂O₃ is the same as used in Fig. 3.7.

The Veselago lensing was successfully demonstrated by the “double-focus” phenomenon in the simulation results in Fig. 3.7. All the incident beams generated by the line source are negatively refracted and focused by the Veselago lens, forming two foci inside and outside the PHC slab at the predicted positions. Even in the circumstance that PHC possesses actual material losses, the Veselago lensing effect is still validated as shown in Fig. 3.7(b).

We use the full width at half maximum (FWHM) of the intensity distribution $I_f(x)$ along x-axis at the predicted image plane to evaluate the imaging resolution. When the substantial material losses are absent, the resolution of Veselago lens is shown to approach the classical diffraction limit. As shown in Fig. 3.7(a), the FWHM at image plane is around $0.15\lambda_0 \approx \lambda_0/(2n_0)$, where n_0 is the refractive index of Si (~ 3.4). The image intensity function $I_f(x)$, however, becomes broadened when the actual material losses are included in PHC.

Based on the above observations, the optical resolution of our proposed Veselago lens is still restricted by the classical diffraction limit. In our example, this resolution (represented by the minimum FWHM of the intensity function at external focus) is around half of the wavelength in the surrounding dielectric medium. Such a result indicates that our PHC device doesn’t amplify the evanescent waves carrying the subwavelength information. We can further improve this resolution by coupling the evanescent wave to degenerate surface plasmons.

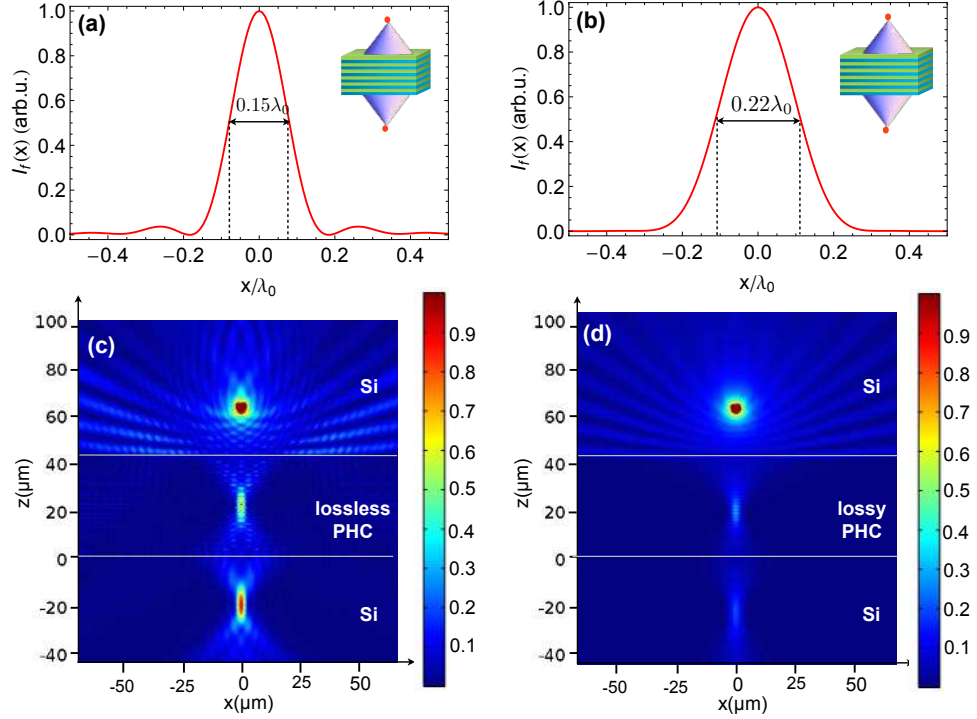


Fig. 3.7. The numerical simulations of a single line source imaging by a planar Si-Al₂O₃ PHC proposed as Veselago lens. The line source is TM-polarized and located in Si which surrounds the PHC slab. Each component layer thickness of PHC is optimized at $\lambda_0 = 20 \mu\text{m}$. (a-b) The intensity distribution $I_f(x)$ that is normalized at the predicted position of external focus without (a) and with (b) material loss included. In the lossless example (panel (a)), the FWHM of $I_f(x)$ approaches the diffraction limit in Si, whereas in lossy case it is slightly broadened. (c-d) The intensity distributions of simulated electromagnetic field in both lossless (c) and lossy (d) PHC examples. Both internal and external foci are clearly observed in both cases, validating the Veselago lensing. The PHC contains 30 unit-cells of total thickness $44.1 \mu\text{m}$. Its top and bottom boundaries are indicated by two grey horizontal lines. Note the size of line source appears to be larger than that of the two foci, which is a tradeoff of vision defects when chopping the overall intensity range in order to increase the image contrast. Copyright of this figure belongs to Ref. [101].

Next, we show the simulation results of two line point source imaging in Fig. 3.8. These two TM polarized line sources are located at same distance from PHC

around $22.05 \mu\text{m}$, and are separated by $\Delta H = 0.4\lambda_0$. Again, the Veselago lensing phenomenon is demonstrated by the complete separation of the two intensity peaks in the lossless case at the predicted two foci planes inside and outside the PHC. This separation distance equals to that of the original value of ΔH in the lossless example. When the material loss is considered, the two peaks are partially overlapped in the center but still show a pronounced separation with the same distance of ΔH .

Noted that strong intensity oscillations are observed at the entrance plane (top surface) of the PHC slab in lossless case – as shown in Fig. 3.7(c) and 3.8(c). These oscillations are generated by the coherent interference between the incident and reflected waves, and they disappear at the exit plane (bottom surface) of PHC. These interference patterns however are diminished when the material losses are considered, which dramatically reduces the intensity contrast between the maxima and minima – see Fig. 3.7(d) and 3.8(d).

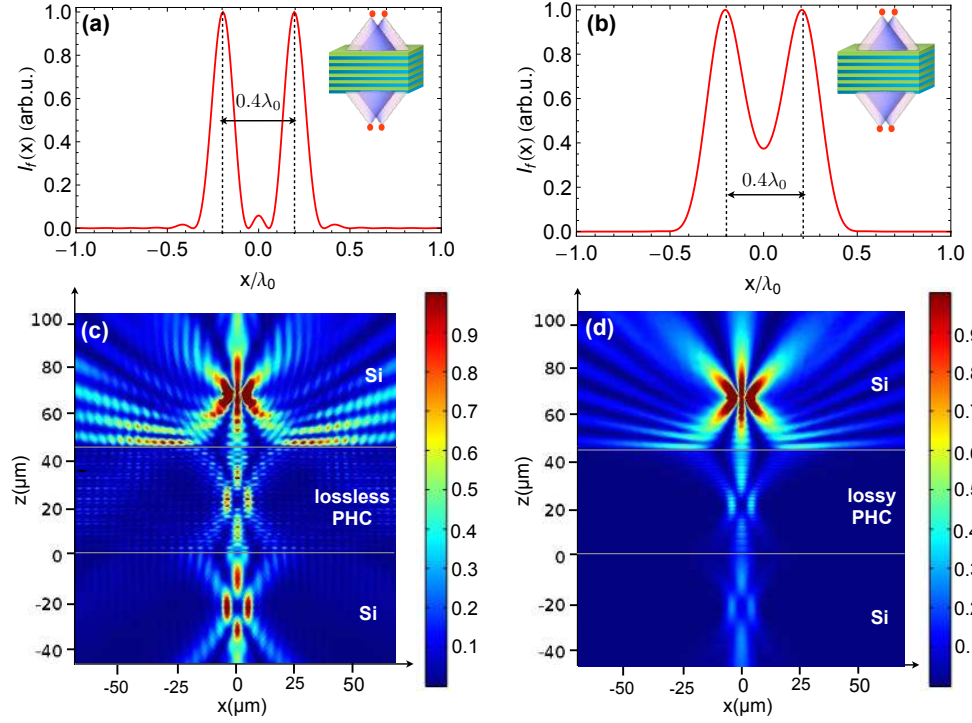


Fig. 3.8. The numerical simulations of two line source imaging by a planar Si-Al₂O₃ PHC proposed as Veselago lens. The component layer thicknesses of PHC are optimized at $\lambda_0 = 20 \mu\text{m}$ at $\lambda_0 = 20 \mu\text{m}$. The two line sources are TM polarized and located in Si about $22.05 \mu\text{m}$ above the PHC slab. The separation distance between them is $\Delta H = 0.4\lambda_0$. (a-b) The intensity function $I_f(x)$ that is normalized at the predicted external focus, without (a) and with (b) actual material losses. In the lossless example (a), the two peaks of the image intensity are completely isolated while they are only partially separated if losses are present (b). (c-d) The intensity distributions of simulated electromagnetic field in both lossless (c) and lossy (d) PHC examples. Both internal and external foci are clearly observed in both cases, validating the Veselago lensing. Copyright of this figure belongs to Ref. [101].

3.7 The material disorder effect on the performance of Veselago lens

The imaging performance of Veselago lens significantly rely on the device fabrication technique and quality. To evaluate the effect of device defects, especially the

dimension disorders of single component layer, we calculate the variations of relative refractive index n^* of the PHC based Veselago lens with small perturbations in the optimal component layer thickness. Fig. 3.9 shows the values of n^* are still comparable to that of the “optimal system” even the maximum variations in thickness are about 40 nm (corresponding to 10% relative change).

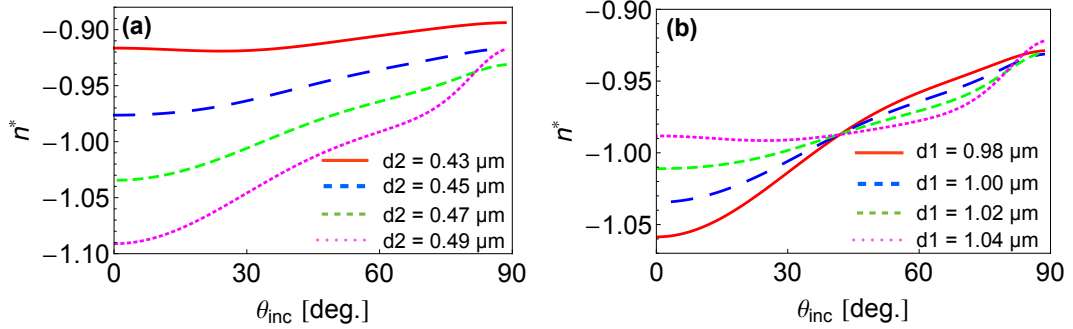


Fig. 3.9. The effects of material dimension variations on the refractive index (n^*) for the Si-Al₂O₃ planar PHC at $\lambda_0 = 20\mu\text{m}$. (a) The values of n^* with different Al₂O₃ layer thickness d_2 when the Si component layer thickness d_1 is fixed at $1\mu\text{m}$. (b) The values of n^* as a function of d_1 when d_2 is fixed at $0.47\mu\text{m}$. Copyright of this figure belongs to Ref. [101].

3.8 Conclusions

In conclusion, we developed a new type of Veselago lens based on the planar PHC, which can be implemented by multilayered dielectric and hyperbolic materials. Our device shows a nearly-constant refractive index with substantially improved imaging performances. We also showed the simulations of line source imaging, and analyzed the affecting factors on the imaging performances. Many engineering applications can be related with our device, such as the photomask design in laser lithography and the hot-spot inspection in Si-based integrated circuits.

4. CONICAL REFRACTION AND IMAGING BY STRUCTURED OPTICAL MEDIA

4.1 Introduction

Conical refraction (CR), first predicted by Sir. Hamilton in 1832 and shortly afterwards demonstrated by Lloyd [109, 110], represents one of the most successful theoretical predictions in science. This optical phenomenon states that a single or a bundle of collimated beams splits into a hollow cone when traveling along one of the optical axes in a birefringent (biaxial) crystal. For example, a narrow collimated Gaussian beam entering a biaxial crystal along one of its optical axes spreads into a hollow cone inside the crystal (rigorously referred as internal CR) and exits as a concentric ring-shaped cylinder shell - see Fig. 4.1(a). Conversely, a hollow cone of light beam converging on the surface of a biaxial crystal slab becomes collimated along the optical axis inside the crystal, and emerges as an expanding light cone outside (Fig. 4.1(b)). This is typically known as external CR. A detailed description of CR using geometrical optics can be found in Ref. [111].

The theory of wave optics provides a technical approach to describe CR. Belsky and Khapalyuk [112] developed a diffraction theory for the electromagnetic wave propagating through a biaxial crystal. Berry and Jeffery [113] improved their work and treated the light propagation through the crystal as a linear transformation relating the transverse field of input and output beams. They obtained a paraxial solution of the electromagnetic field that can be evaluated numerically at any transverse plane. The later experimental work demonstrated very close agreement with such numerical evaluations [114].

As one of the earliest optical singularities in history [115] and first application of Hamilton's phase space concept, CR has played an important role in accelerating

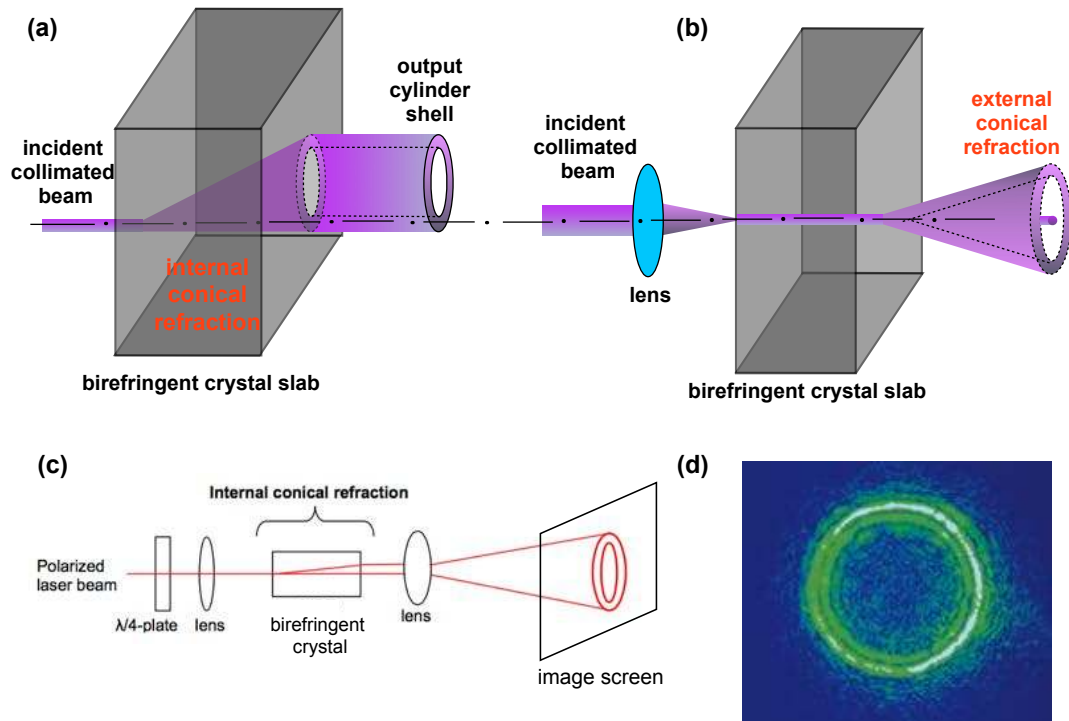


Fig. 4.1. Schematic illustrations of the CR and experimental observation. (a-b) The geometrical optics description of internal (a) and external (b) CRs produced by a collimated beam and a birefringent crystal. The front and back surface of the crystal is aligned both perpendicular to one of the optical axes. (c) A simple experimental set-up to observe the internal CR. (d) The internal CR image of double concentric rings generated by a $\text{KGd}(\text{WO}_4)_2$ crystal and 532 nm laser beam [132].

the acceptance of transverse wave nature of light, and promoting the development of theoretical optics [114]. Recently, there are some renewed interests in CR owing to several demonstrated possibilities of the effect in manipulating the amplitude, phase, and polarization of light beams [116–120], and a few intriguing applications in lasing [121], particle trapping [122–124], and free-space optical communications [125]. Fine experimental work on CR has been successfully completed and compared with

theory by a single [114] or multiple cascade [126–128] biaxial crystals, and honeycomb lattices [129, 130].

CR realized by birefringent crystals is fundamentally important in modern photonic devices as most of known crystals are biaxial. However, the criteria of biaxial crystals for practical applications of CR are extremely high. The crystal must have an undisturbed molecular structure with high optical transparency that is free of dispersion. In addition, the refractive indices along all three major optical axes must have very equidistant values [131]. Other requirements include stable chemical and thermo-mechanical properties.

In fact, only a few specimens of natural aragonite (as used by Lloyd [110]) can satisfy all these requirements. That’s probably the reason why until almost 160 years later after the first observation, practical applications of CR starts emerging and attracting researchers’ attentions thanks to the advancement in artificial crystal cutting and growing technologies.

Here, we present a new method to achieve CR by planar structured optical media. Our approach emphasizes on tailoring the phase dispersion rather than the inherent biaxial feature of natural or artificial crystals. It not only significantly reduces the difficulties in fabrication but also allows a new approach to imaging. In particular, it offers a new approach to optical phase retrieval without interferometric or holographic settings (see next chapter). A novel “negative” (focusing) CR in photonic hypercrystals is also discussed.

4.2 Conical refraction by engineering the phase-space dispersion

4.2.1 General theory

The behavior of electromagnetic waves in a given matter can be described by its dispersion - an interplay between the wave energy (frequency) and angular momentum (wave vector). For a monochromatic (iso-frequency) wave, the dispersion in purely propagation phase (or momentum) space, i.e. the iso-frequency surface (IFS, 3D)

or iso-frequency curve (IFC, 2D), illustrates the wave propagation characteristics, such as how the wave is reflected and refracted when encountering an interface. This phase space dispersion is important to understand and explain the electromagnetic radiation properties and features in a complex composite, such as metamaterials [10] and photonic hypercrystals (PHCs) [100, 101]. Other than most of the well-known examples of IFSs, e.g. circular for isotropic dielectrics or metals, hyperbola for hyperbolic materials, there is a special case which hasn't drawn much attention - the cone-shaped IFS (or two lines of IFC).

We illustrate this cone-shaped IFS and the associated impacts on wave propagations in Fig. 4.2. A linear relation of IFC between two orthogonal wave vectors k_τ and k_n can be found in Fig. 4.2 (b). There, $k_{x(y)}$ represents the in-plane wave vector parallel to the interface between the air and CR medium, and k_z is the normal-to-plane wave vector perpendicular to the interface. As shown in the two-dimensional linear IFC in Fig. 4.2(b), any incoming beam that is infinite in x or y direction with oblique incident angle (corresponding to a non-zero k_x or k_y), the refracted Poynting vectors $\mathbf{S}$ (normal to the IFC) all point to the same direction. Therefore, for any incident beams with different in-plane momenta (corresponding to different incident angles), only two possible opposite refraction angles (θ_t and $-\theta_t$) are allowed. As a result, in the special case of a collimated beam propagating from air and along the normal to the CR medium, two refracting rays will be produced at the entrance interface with opposite refraction angles (in 2D) forming a hollow cone (in 3D) inside the CR medium. At the exit facet of the CR medium, double concentric-ring beams will be observed - see Fig. 4.2 (c). This indicates that CR can be realized by a conical IFS (or linear IFC).

4.2.2 Conical refraction in 1D photonic crystal

To obtain the CR, we consider modulating the dimensions and compositions of the periodic elements in structured media (e.g. photonic crystals (PCs) and metama-

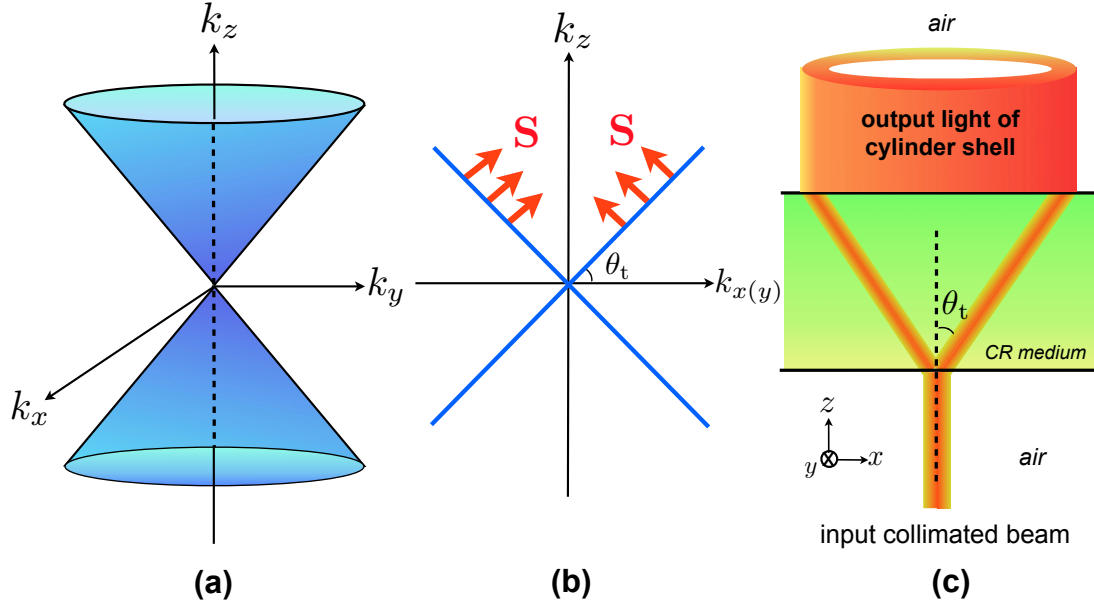


Fig. 4.2. (a) The cone-shaped IFS associated with the uniaxial CR medium. (b) Linear IFC relating two orthogonal wave vector k_z and $k_{x(y)}$. The Poynting vectors ($\mathbf{S}$) which are normal to IFC all point to the same refraction angle (θ_t or $-\theta_t$) for any single oblique incidence ($k_{x(y)} \neq 0$). For normal incidence where $k_{x(y)} = 0$, these two opposite refraction angles ($\pm\theta_t$) both exist, indicating CR occurs inside the medium. (c) Schematic illustration of CR in a uniaxial medium for a normal incident beam that is infinite in y direction.

terials) to tailor the associated IFC. This is a powerful tool that leads to discovery of many unconventional optical properties and potential important applications, such as enhanced thermal radiation effect [133], negative refraction [34, 42, 89, 90, 96–98], subwavelength imaging [7–9, 134] etc.

We consider the simplest case of a one-dimensional (1D) PC with the unit-cell composed of two dielectric materials. Fig. 4.3(a) shows an example of a planar Si-SiO₂ multilayered PC, with the permittivities of Si and SiO₂ component layers given

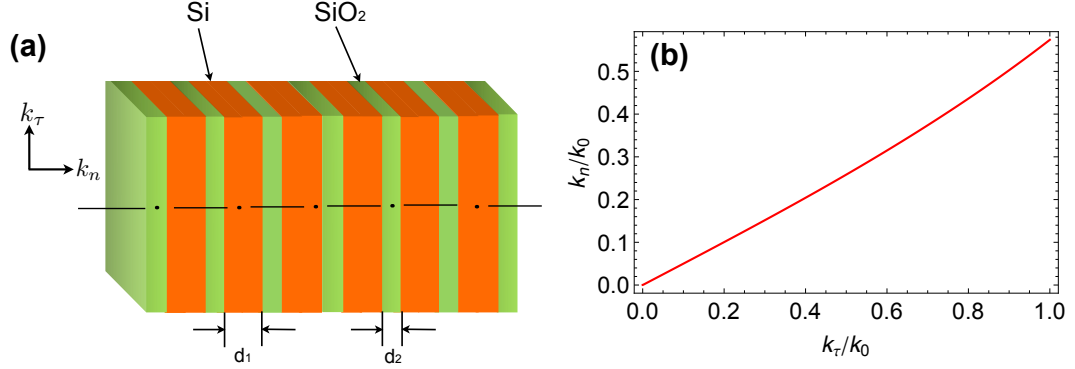


Fig. 4.3. (a) The schematic structure of a planar PC composed of periodic alternating layers of Si and SiO₂. The permittivities of Si and SiO₂ are chosen in the low-absorption spectra (from visible to infrared) as $\epsilon_1 = 11.7$ and $\epsilon_2 = 2.25$. The propagation Bloch wave vector k_n is parallel to the principal optical axis of PC indicated by the dash-dot line. (b) The linear IFC of the Si-SiO₂ planar PC with optimal component layer thicknesses $d_1(\text{Si}) = 0.204015 \lambda_0$ and $d_2(\text{SiO}_2) = 0.116587 \lambda_0$, where λ_0 is the free-space wavelength and k_0 denotes the free-space wave number. The slope of IFC near $k_\tau = 0$ is approximately 0.5, corresponding to a refraction angle of $\theta_t \approx 26.6^\circ$.

by ϵ_1 and ϵ_2 . The Bloch propagating wave vector k_n (normal-to-plane) in this system can be calculated from the standard T-matrix method based on in unit cell [11]:

$$\cos(k_n D) = \cos(k_{1n} d_1) \cos(k_{2n} d_2) - \frac{1}{2} \left(\frac{\epsilon_1 k_{2n}}{\epsilon_2 k_{1n}} + \frac{\epsilon_2 k_{1n}}{\epsilon_1 k_{2n}} \right) \sin(k_{1n} d_1) \sin(k_{2n} d_2). \quad (4.1)$$

Here d_1 and d_2 are the component layer thicknesses of Si and SiO₂ respectively, $D = d_1 + d_2$ is the total thickness of a unit cell. k_{1n} and k_{2n} denote the normal-to-plane wave vectors in Si and SiO₂ component layers given by

$$k_{1n} = \sqrt{\epsilon_1 \left(\frac{\omega}{c} \right)^2 - k_\tau^2} \quad (4.2)$$

and

$$k_{2n} = \sqrt{\epsilon_2 \left(\frac{\omega}{c} \right)^2 - k_\tau^2}, \quad (4.3)$$

where k_τ is the in-plane (parallel to surface) wave vector, ω is the angular frequency and c is vacuum speed of light.

For a linear IFC (corresponding to a conical IFS), the two orthogonal wave vectors k_n and k_τ are directly proportional to each other:

$$k_n = s * k_\tau, \quad (4.4)$$

where the constant coefficient s represents the slope of the linear function indicating the refraction angle ($s = \tan(\theta_t)$). When k_τ is very small which is close to the normal incidence case, Eq. (4.1) can be expanded by Taylor polynomials in terms of k_τ as

$$\cos(k_n D) = 1 - sk_\tau^2/2, \quad (4.5)$$

$$\cos(k_{1n} d_1) = \cos(\sqrt{\epsilon_1} d_1 \frac{\omega}{c}) + \sin(\sqrt{\epsilon_1} d_1 \frac{\omega}{c}) \frac{cd_1 k_\tau^2}{2\omega\sqrt{\epsilon_1}}, \quad (4.6)$$

$$\cos(k_{2n} d_2) = \cos(\sqrt{\epsilon_2} d_2 \frac{\omega}{c}) + \sin(\sqrt{\epsilon_2} d_2 \frac{\omega}{c}) \frac{cd_2 k_\tau^2}{2\omega\sqrt{\epsilon_2}}. \quad (4.7)$$

Substituting the above Eqs. (4.5-4.7) in Eq. 4.1 and dropping the higher-order Taylor polynomial terms, we can obtain the following equation:

$$1 - sk_\tau^2/2 = A_1 - A_2 k_\tau^2, \quad (4.8)$$

where A_1 and A_2 are positive constant coefficients. A_1 is exactly the right-hand-side of Eq. 4.1 when $k_\tau = 0$:

$$A_1 = \cos(\sqrt{\epsilon_1} d_1 \frac{\omega}{c}) \cos(\sqrt{\epsilon_2} d_2 \frac{\omega}{c}) - \frac{1}{2} \left(\sqrt{\frac{\epsilon_1}{\epsilon_2}} + \sqrt{\frac{\epsilon_2}{\epsilon_1}} \right) \sin(\sqrt{\epsilon_1} d_1 \frac{\omega}{c}) \sin(\sqrt{\epsilon_2} d_2 \frac{\omega}{c}). \quad (4.9)$$

At normal incidence when $k_\tau = 0$, A_1 and A_2 are required to satisfy $A_1 = 1$ and $A_2 < 0$, which leads to the analytical solution of the mathematical relationship between d_1 and d_2 . For an optimal condition of maximum slope s , we can obtain an numerical approximation of d_1 and d_2 , which gives the a quasi-linear IFC as shown in Fig. 4.3 (b).

To further improve the linearity of IFC, we consider a more complicated configuration of the uniaxial multilayered PC where the unit-cell consists of four alternating dielectric layers of Si and SiO₂. Each component layer thickness in the new configuration is denoted by d_1 (Si), d_2 (SiO₂), d_3 (Si) and d_4 (SiO₂), and these values are obtained in a similar way as shown above. As a comparison, we show the IFCs of both two-layer and four-layer unit-cell configurations in Fig. 4.4.

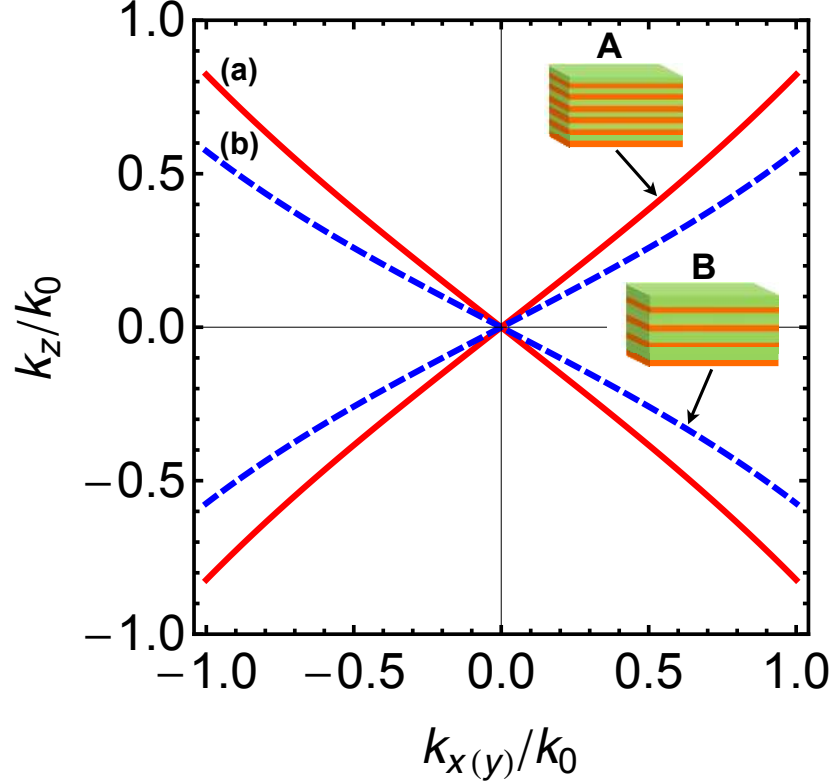


Fig. 4.4. The IFCs of a planar multilayered PhC of two different unit-cell configurations (**A** and **B**). The red solid curve (a) shows the IFC of configuration **A**, where the thicknesses of component layers are: $d_1(\text{Si}) = 0.204 \lambda_0$, $d_2(\text{SiO}_2) = 0.1166 \lambda_0$. The blue dashed curve corresponds to the IFC of configuration **B**, where the four alternating dielectric layer thicknesses are: $d_1(\text{Si}) = 0.086 \lambda_0$, $d_2(\text{SiO}_2) = 0.068 \lambda_0$, $d_3(\text{Si}) = 0.086 \lambda_0$, $d_4(\text{SiO}_2) \approx 0.0677 \lambda_0$.

4.3 Imaging based on conical refraction

Based on the material and geometric configurations of the 1D planar PC as shown in Fig. 4.3, we study the imaging using the CR. A single TM-polarized paraxial Gaussian beam incident from air is considered, which propagates along the surface normal of the PC slab. The latter is composed of 50 two-layer unit-cells of a total thickness $L \approx 16\lambda$. The full wave propagation through the PC and electromagnetic

field distribution can be calculated using Fast Fourier Transform (FFT) [135]. The analytical approach to obtain the imaging results is described below.

The transmission coefficient of the 1D “air-PC-air” system can be obtained from a 2×2 T-matrix [55]. For a TM-polarized incident plane wave, this coefficient $\mathbf{t}$ is given as a function of k_τ :

$$\mathbf{t}(k_\tau) = \frac{1}{B_1(k_\tau)\exp[ik_n(k_\tau)L] + B_2(k_\tau)\exp[-ik_n(k_\tau)L]} \quad (4.10)$$

$$= \frac{\exp[-ik_n(k_\tau)L]}{B_1(k_\tau)\{1 + \frac{B_2(k_\tau)}{B_1(k_\tau)}\exp[-i2k_n(k_\tau)L]\}}. \quad (4.11)$$

Here B_1 and B_2 are complex functions of k_τ satisfying $|B_2/B_1| \leq 1$, thus we can expand Eq. 4.11 in Taylor series by

$$\mathbf{t} = \frac{1}{B_1} \sum_{n=0}^{\infty} \left(-\frac{B_2}{B_1}\right)^n \exp[-i(2n+1)k_n L]. \quad (4.12)$$

The integer n represents the order of output concentric-ring beams as “replicas” of incident Gaussian beams – see Fig. 4.5. The corresponding accumulated phase through the PC slab for each replica is given by $(2n+1)k_n L$.

This transmission coefficient $\mathbf{t}(k_\tau)$ can be treated as a filter or Fourier transfer function, which modifies both the spectral amplitude and phase of the input signal. Details of $\mathbf{t}(k_\tau)$ for a double-layer unit-cell Si-SiO₂ PC slab with 100 unit cells are shown in Fig. 4.6. For any given input optical signal with an electromagnetic field distribution $I_{\text{in}}(x)$, the corresponding imaged field (output signal) $I_{\text{out}}(x)$ at arbitrary distance of $z > 0$ can be obtained by Fourier transform:

$$I_{\text{out}} = \mathcal{F}^{-1}\{\mathcal{F}[I_{\text{in}}(x)] * \mathbf{t}(k_\tau)\}. \quad (4.13)$$

where $\mathcal{F}$ is the Fourier transform between the real-space x and the spatial frequency domain k_τ .

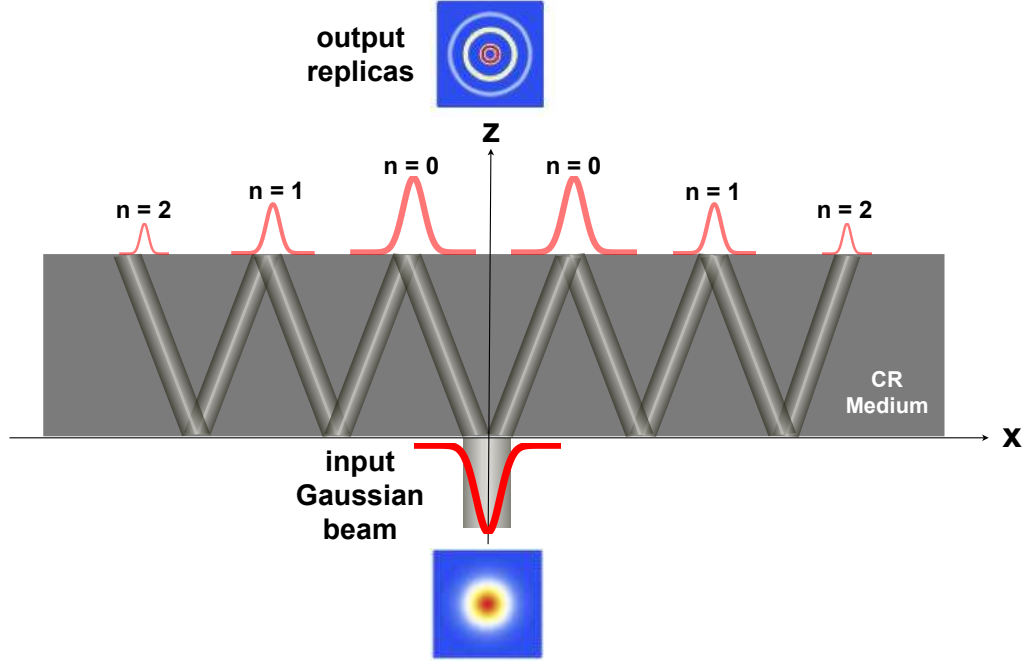


Fig. 4.5. Schematics of a Gaussian beam incident from air and propagating through the CR medium of PC slab. Multiple output replicas emerge in the order denoted by n . The insets show the two-dimensional views of the input beam (bottom) and output concentric ring images (top).

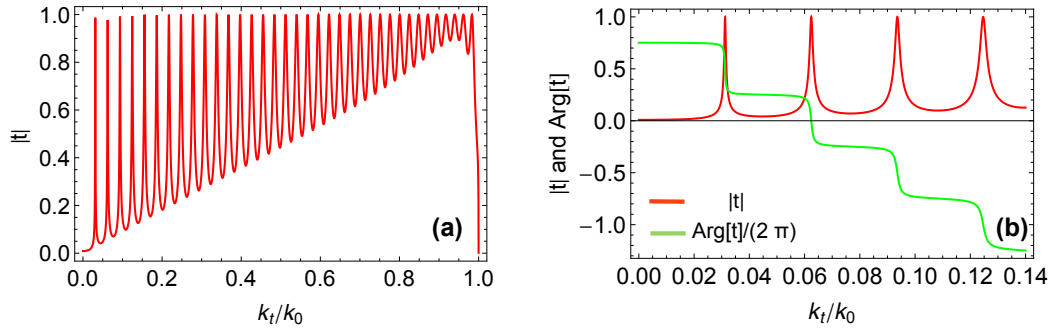


Fig. 4.6. (a) The amplitude of transmission coefficient $|t|$ as a function of in-plane wave vector k_τ for the Si-SiO₂ PC of 100 unit cells. $|t|$ is featured by multiple narrow resonance peaks. (b) The phase ($\text{Arg}[t]$) and amplitude ($|t|$) of the transmission coefficient in a selected small range of $0 < k_\tau < 0.14$, where dramatic phase change occurs at each resonance peak.

Fig. 4.7 shows the CR imaging results computed by FFT. Clearly a series of light cones were formed inside the PC due to the CR and multiple internal reflections at the PC boundaries are observed. At the exit facet of the PC slab, several concentric ring-shaped beams as the output replicas are shown. These numerical results successfully demonstrate the CR in the PC possessing a linear IFC. It is also noticeable that the output beams are slightly diverging, resulting in multiple expanded cylindrical light shells. This is due to the finite transverse size of Gaussian beams and the positive refraction at the interface between air and the PC slab.

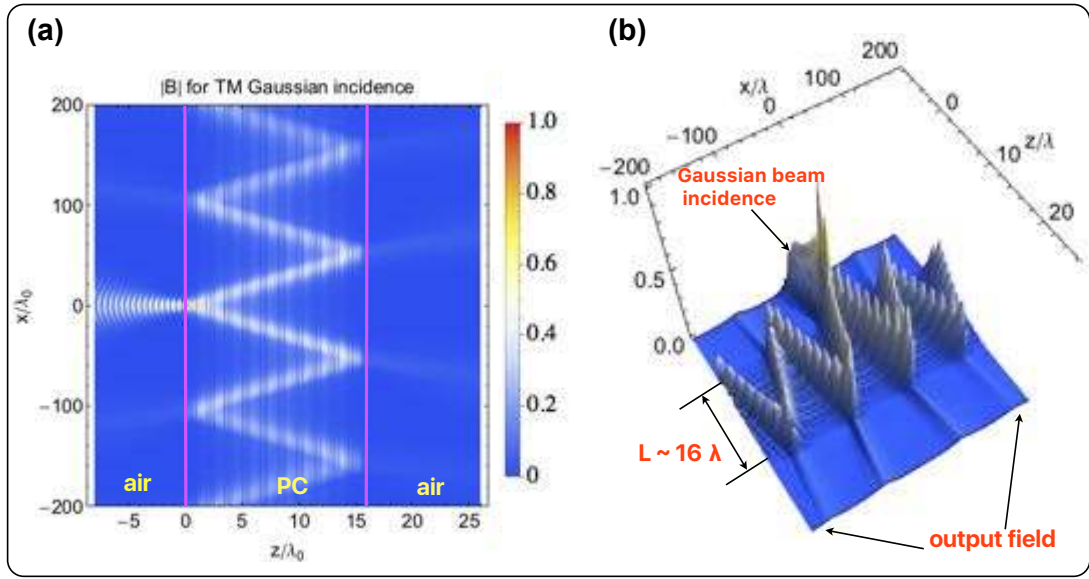


Fig. 4.7. CR by a PC slab and electromagnetic field distributions obtained by FFT. (a) The magnetic field profile inside and outside PC when a TM polarized Gaussian beam is striking onto the PC surface along z -axis. The Gaussian beam width is $5\lambda_0$ and the beam waist is located at $z = 0$. The PC slab contains 50 unit cells of Si-SiO₂ with its boundaries indicated by the purple lines. (b) 3D magnetic field amplitude distributions in the whole space, which clearly show the CR inside PC and the output field as several cylinder beam shells.

Next, we show the CR images of multiple input Gaussian beams as a more complex optical signal in Fig. 4.8. Several replicas of the input signal are generated at the

output with different intensity distributions. The locations of these replicas match well with the theoretical prediction, and their intensity distributions resemble well the input beam profile. However, those replica of higher orders ($n \geq 1$) display significantly distortions due to the multiple internal reflections between the PC boundaries and air.

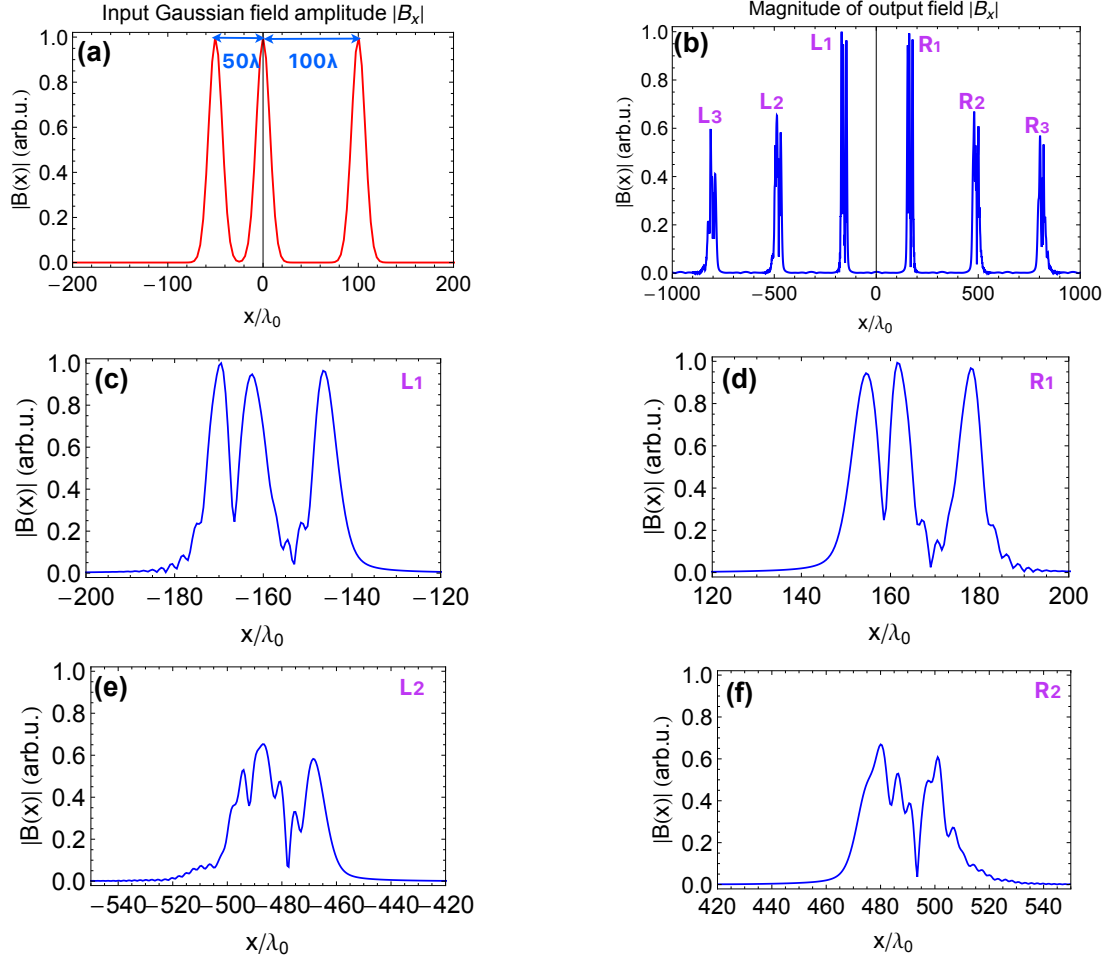


Fig. 4.8. Imaging of multiple TM-polarized Gaussian beams by CR in a PC slab containing 1000 unit-cells of Si-SiO₂. (a) Normalized magnetic field amplitude $|B(x)|$ distributions of three incident Gaussian beams at the entrance of PC. All the Gaussian beams have the same beam width of $10\lambda_0$, and are separated by $50\lambda_0$ and $100\lambda_0$. (b) Images of first 6 replicas of input beams from $x=0$ at the exit facet of the PC slab, which are denoted by L_1 , L_2 , L_3 , R_1 , R_2 and R_3 . The first two replicas L_1 and R_1 corresponding to the order number $n = 0$ are separated by a distance around $320\lambda_0$, which agrees well with theoretical prediction. (c-f) Magnetic field amplitude distributions of the first 4 replicas (L_1 , L_2 , R_1 and R_2). Replicas L_1 and R_1 resemble the input beams quite well, while the others show significant distortions.

Compared with conventional holographic imaging, these concentric ring shape replicas due to CR imaging could offer a new approach to hologram generation without using interferometry. It can also be used as a new method of sensing changes in input signal.

4.4 Negative CR with photonic hypercrystals

Negative refraction has been found in PHCs [101] because of the introduction of hyperbolic medium as one of the periodic elements. Therefore, if we investigate the conical IFS (or linear IFC) in the PHC following the method introduced in section 4.2.2, a new type of “negative” CR could be achieved. Such a refraction can be verified by the output beam characteristics. Instead of diverging concentric ring-shaped light shells, the output beams will be converged to form multiple foci. The focus length from the surface of PHC is determined by the negative refraction angle, which can be found from the corresponding IFC.

Unfortunately, the inevitable absorption losses present in all the hyperbolic medium (either from the phonon resonances in natural hyperbolic materials [62] or the metallic elements in metamaterials [10]) cause significant imperfections and distortions to the conical IFS, thus significantly reducing the overall imaging performances. There is another interesting feature of such negative CR that may resolve the image aberration issue in curved lens when moving close to the target. These will be discussed in future work.

4.5 Conclusions

In summary, we present a new method to achieve the CR based on engineering the dispersion in phase space. Specifically, when the IFS of a structured optical medium is tailored to be a cone shape, we can achieve a CR (and potentially negative “focusing” CR in PHC). The classical approach to realize CR relies on the highly polished biaxial crystals of which only a few can meet the rigorous research or industrial standards.

Our method offers a simple 1D planar structure compatible to current nanofabrication techniques, indicating the potential practical applications. We will show one of the applications in optical phase retrieval in next chapter.

5. OPTICAL PHASE RETRIEVAL USING CONICAL REFRACTION IMAGING

5.1 Introduction

Solving the phase retrieval problem is crucial in various fields of science and engineering, including X-ray crystallography [136–138], neutron radiography [139], electron microscopy [140–142], wavefront sensing [143], astronomy [144], coherent diffractive imaging [145], optical imaging [146], non-optical surface roughness diagnosis [147], and range sensing [148]. The quest of phase retrieval in optics naturally arises due to the fact that all known optical detection devices (e.g. charge-coupled device cameras, spectrometers and photosensitive films) cannot measure the phase of light directly since the high oscillation frequency of electromagnetic field is far beyond their capacity.

Conventional phase retrieval methods utilize interferometer or holographic imaging [149, 150] to decode phase information by analyzing the interference (fringe) patterns, or an algorithmic procedure that reconstructs the phase of a complex signal from the observation of its Fourier transform modulus [151–153]. The latter doesn't rely on the use of a holographic reference wave or any other sophisticated far-field interferometry set-ups.

However, two major technical difficulties are generally encountered in the algorithmic phase retrieval, which may either significantly reduce the performance or stagnate the iterative algorithm. First, both the image and Fourier (corresponding to far-field) intensity recordings at different planes are required [152] which in practical measurement is technically challenging [146]. The Fourier magnitudes are usually incomplete, low-frequency information missing, and corrupted by noise. In addition, the Fourier magnitude detected by each pixel of the detector is in fact an integration of photons

coming from different angles, which differs from the uniform sampling of diffraction signal in numerical simulations. Second, from the mathematical prospective, the general algorithmic phase retrieval problem (here we consider the least-square minimization) is inherently a nonlinear and non-convex problem whose solution requires an appropriate initialization of the phase guess. Unfortunately, whether there exists a stable recovery method is still unknown, and to the best of our knowledge there are no theoretical guarantees that it can converge to a global minimum. Based on the above significant constrains, an alternative approach needs to be explored to address these concerns.

In this chapter, we propose an alternative approach to phase retrieval, taking advantage of structured optical media as a powerful tool of manipulating electromagnetic waves in terms of conical refraction (CR). By tailoring the phase-space dispersion in structured optical media, such as photonic crystals (PCs), one can achieve the CR imaging and reconstruct the phases of both the object and images. Our method only involves a single intensity measurement and guarantees a fast and precise solution of retrieved phases.

5.2 Linear transformation theory in CR medium

Metamaterials [10] and photonic crystals [11] have shown the potential to develop novel optical materials and devices with intriguing properties and applications, such as negative refractive index [15, 16, 82, 101], invisibility cloaking [4, 5] and super-resolution imaging [7, 9, 79]. In particular, they can provide a platform of performing mathematical operations (e.g. spatial differentiation and integration) on electromagnetic waves [154]. One of the simplest mathematical operations is linear transformation, e.g. a linear function in spatial frequency space. Compared with the dominate nonlinear behavior in enormous optical elements and phenomena, the CR in biaxial media (such as birefringent crystals) is shown by wave theory as a linear transformation of the transverse electromagnetic field of optical beams [113]. Essentially, such

a linear feature of CR can be applied to address some constraints in the “nonlinear” algorithmic phase retrieval problem and shows great advantages and simplifications.

Observations of CR imaging for a single incident angle are usually accomplished with the bulky and finely polished biaxial crystals [109, 110]. In contrast to this behavior, all-angle CR can be realized by a uniaxial planar PC composed of periodic multilayers of dielectric materials. By modulating the configurations in a unit-cell of the PC, such as the component dielectric permittivities and layer thicknesses, one can tailor the corresponding iso-frequency surface (IFS) to be a cone which enables CR inside. The size of our proposed device is also adjustable from a few microns to millimeters, which is advantageous for imaging small objects such as biological samples.

This cone-shaped IFS associated with the uniaxial CR medium is given in Fig. 4.1, where the linear transformation in momentum space can be interpreted by the iso-frequency curve (IFC) relating two orthogonal wave vectors k_z and $k_{x(y)}$. There, $k_{x(y)}$ represents the in-plane wave vector parallel to the interface between the air and CR medium, and k_z is the normal-to-plane wave vector perpendicular to the interface.

As an example of uniaxial CR medium, we consider a planar multilayered PC possessing the cone-shaped IFS. The unit-cell of the PC slab consists of four dielectric layers corresponding to the configuration **A** in Fig. 4.2, and the PC slab contains 100 unit-cells of total length $D = 30.8 \lambda_0$. We show the numerical simulations of wave propagation of a single paraxial Gaussian beam in this PC slab that is infinite in the y direction, as illustrated in Fig. 5.1. The incident Gaussian beam is traveling along the optical axis of the planar PC slab (i.e. surface normal). As shown in Fig. 5.1 (a). At the entrance interface ($z=0$), the Gaussian beam splits into two separate refracted beams and continues to experience multiple reflections and refraction at the boundaries of the PC slab. These multiple refractions at the exit interface produce several “replicas” of the original input Gaussian beam with different magnitudes. Our simulation verifies that this uniaxial PC does enable the CR inside, which can be used to image the input optical signals and recover the phases.

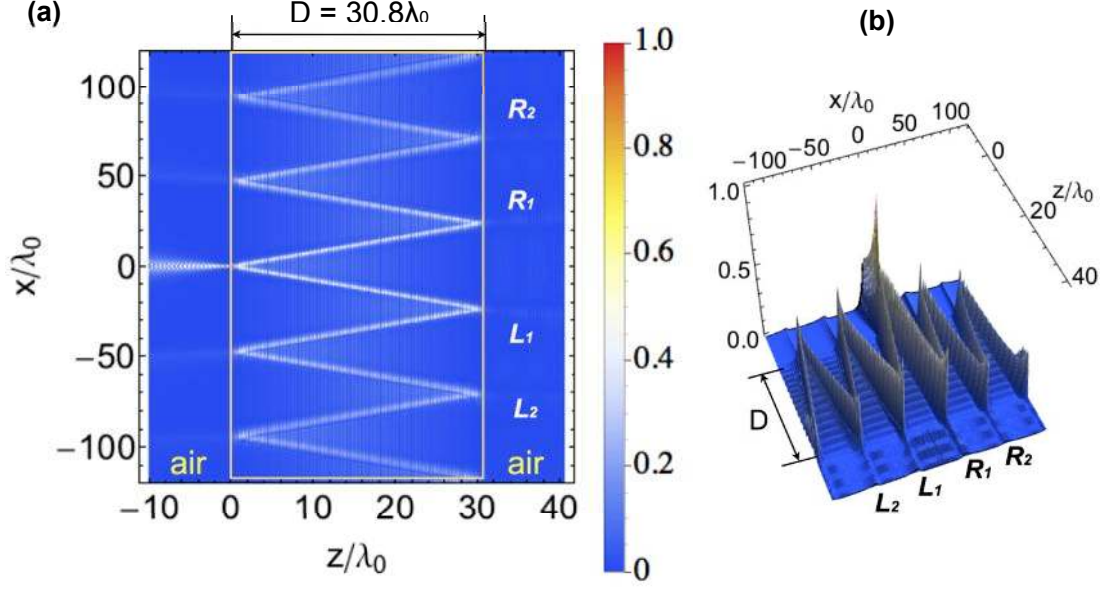


Fig. 5.1. (a) The 2D numerical simulations of a Gaussian beam propagating in a PC slab (indicated by gray box) as the uniaxial CR medium. The TM polarized Gaussian beam diameter ($1/e^2$ diameter) is $7 \lambda_0$. The PC slab consists of 100 unit-cells of “Si-SiO₂-Si-SiO₂” alternating layers, corresponding to a total thickness of $D = 30.8 \lambda_0$. Details of the configuration for the PC slab can be found in Fig. 4.4. The conical refraction angle in this example is $\theta_s \approx 36.9^\circ$. At the $z = D$ plane, the distance between the two replica centers is $2D \tan(\theta_s) \approx 46.26 \lambda_0$, which is very close to the simulation result. (b) The normalized 3D electromagnetic field amplitude distribution with the same PC slab and Gaussian beam incidence. Both simulations (a-b) were calculated by Fast Fourier Transform of the magnetic fields in real and spatial frequency spaces.

Before we continue with the phase retrieval using the CR imaging, we want to emphasize that it is the unique cone-shaped IFS in momentum (or spatial frequency) space that enables the linear transformation between the input and output beams, as well as the adjacent replicas. Such linear correlations significantly reduce the complexities in the inherent “nonlinear” algorithmic phase retrieval problem, which greatly facilitate the practical phase retrieval procedure. For instance, the additional far-field intensity information is no longer required which eliminates all the possible technical issues in the experiment and diminishes the risk of sampling errors in numerical simulation. In addition, all the image measurements can be carried out simultaneously at one plane. Most importantly, a priori knowledge of the linear transformation system allows us to analytically obtain a proper initialization of phase approximation so that a successful convergence to the true solution is always guaranteed.

Below, we describe our approach in case when the incident field does not depend on the y direction. While the general case of the incident wave that nontrivially depends on both x - and y - coordinates can be described in a similar manner, the resulting expressions are much more complex, which complicates the understanding of the proposed concept. Furthermore, the study of the effectively one-dimensional pattern is also important in its own right. In fact, it is the 1D case that is the most difficult problem from the point of view of the convergence of the standard phase retrieval procedure, to the point of possible non-existence of the unique solution [155].

In the following phase retrieval procedure, we consider four adjacent output replicas (L_1 , L_2 , R_1 and R_2 , see Fig. (5.1)) as part of the CR images generated by a complex input optical signal and the PC slab. Our goal is to reconstruct the phase (and amplitude) profile of input signal with only the knowledge of real-space replica amplitude distributions at the exit facet of the PC slab. These replica amplitude distributions are to resemble the measured signals at the image plane.

First, we attempt to recover the phase distributions of replica L_1 and R_1 , whose spectral (spatial frequency) function $H_1(k_x)$ is the direct transformation of the input signal $I_0(k_x)$ given by

$$H_1(k_x) \equiv I_0(k_x) * \mathbf{t}(k_x, n=0) = I_0(k_x)T_0(k_x), \quad (5.1)$$

where $T_0(k_x)$ is the spectral transfer function for the replicas of order $n=0$ as shown in Eq. 4.12. For simplicity, here we only show how to retrieve the real-space phase distribution of R_1 based on the amplitude distributions of the two adjacent replicas R_1 and R_2 . The spectral function of R_1 only corresponds to half part of $H_1(k_x)$ when $k_x < 0$. The real-space phase of L_1 can be extracted in a similar manner.

The initial approximation to the phase of R_1 can be obtained analytically using the quasi-linear transformation function $L(k_x)$ in tangential momentum (k_x) domain given by

$$L(k_x) \simeq (a_0 + a_1 k_x) \exp[i2\pi b_0 k_x], \quad (5.2)$$

which connects the spectral functions $F_{1,2}(k_x)$ of R_1 and R_2 by

$$F_2(k_x) = L(k_x)F_1(k_x). \quad (5.3)$$

Here a_0 and a_1 are linear constants of the amplitude function $|L(k_x)|$, and b_0 (in unit of λ_0) is the slope constant corresponding to the phase of $L(k_x)$. Such a linear relation implies a first-order differential equation between the real-space amplitudes of R_1 and R_2 given by

$$f_2(x) = \left(a_0 - ia_1 \frac{d}{dx} \right) f_1(x + b_0), \quad (5.4)$$

where f_1 and f_2 are the real-space distributions of R_1 and R_2 , i.e. Fourier transforms of F_1 and F_2 : $f_{1,2}(x) = \int F_{1,2}(k_x) \exp(i2\pi x k_x) dk_x$. The value of b_0 in fact corresponds to the distance between the centers of R_1 and R_2 , which is an immediate effect of the conical IFS.

Here $f_{1,2}(x)$ can be expressed by separated amplitude $A_{1,2}(x)$ and phase $\phi_{1,2}(x)$: $f_1(x) = A_1(x)\exp[i\phi_1(x)]$ and $f_2(x) = A_2(x)\exp[i\phi_2(x)]$. Taking $A_{1,2}(x)$ and $\phi_{1,2}(x)$ into Eq. (5.4) gives a first-order differential form of $\phi_1(x)$:

$$\frac{d\phi_1(x)}{dx} = -\frac{a_0}{a_1} + \frac{1}{a_1 A_1(x)} \sqrt{A_2^2(x - b_0) - a_1^2 \left(\frac{dA_1(x)}{dx} \right)^2}. \quad (5.5)$$

Accordingly, ϕ_1 is obtained by integrating Eq. (5.5), leading to

$$\phi_1(x) = -\frac{a_0 x}{a_1} + \int_0^x \frac{ds}{a_1 A_1(s)} \sqrt{A_2^2(s - b_0) - a_1^2 \left(\frac{dA_1(s)}{ds} \right)^2}. \quad (5.6)$$

We then use $\phi_1(x)$ given by Eq. (5.6) as the initialization of phase function in the Gerchberg-Saxton phase retrieval algorithm ([152], also see the Appendix) to reconstruct the final phase distribution and accordingly the entire real-space or spectral profile of R_1 . Since ϕ_1 is very close to the true phase distribution, the convergence to a precise solution is guaranteed. Finally, the spectral function of input optical signal $I_0(k_x)$ (and its phase) can be obtained by performing the inverse transformation directly on the complete spectral function of $H_1(k_x)$:

$$I_0(k_x) = H_1(k_x)[T_0(k_x)]^{-1}. \quad (5.7)$$

5.3 Example of optical phase retrieval using CR imaging

In the following example, we use the same PC slab as given in Fig. 5.1 as our uniaxial CR medium. The spectral transformation functions of $T_0(k_x)$ and $L(k_x)$ as well as the linear fits in both amplitude and phase are given in Fig. 5.2 respectively. The constant coefficients a_0 , a_1 and b_0 for the linear fit of $L(k_x)$ are retrieved in a small range of $0 < k_x/k_0 < 0.1$ for a better numerical accuracy. A bundle of 10 random Gaussian beams (paraxial, TM polarized) are to simulate the complex input optical signal as illustrated in Fig. 5.3(a). The calculated replica amplitudes and phases for L_1 , L_2 , R_1 and R_2 based on the theoretical model are shown in Fig. 5.3(b).

We show the retrieved phases of replica L_1 and R_1 , as well as the recovered input signal spectral function in Fig. 5.4. As can be seen in Fig. 5.4(a), our retrieved

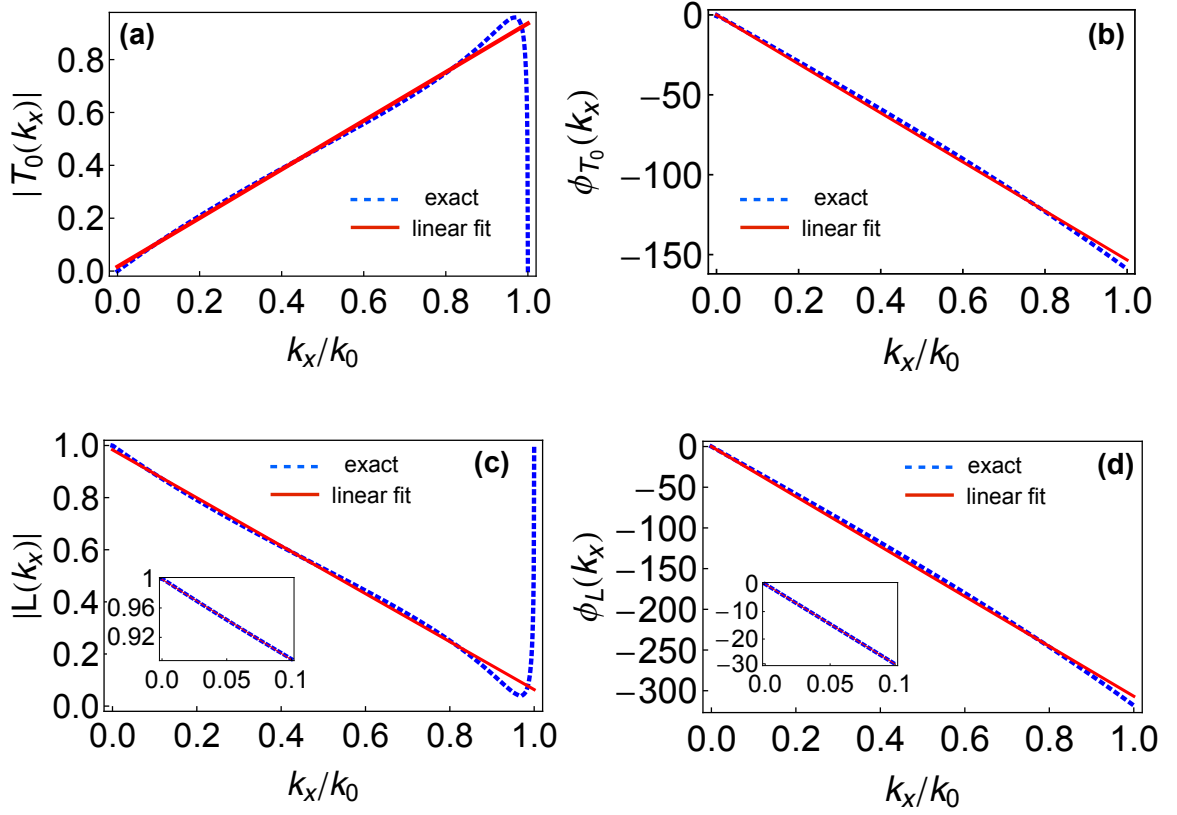


Fig. 5.2. Schematics of the linear fits for the spectral transformation functions $T_0(k_x)$ (a-b) and $L(k_x)$ (c-d) in both amplitudes and phases. A more accurate fit range of $0 < k_x/k_0 < 0.1$ (as shown by the insets in (c-d)) is selected to obtain the closest linear parameters for $L(k_x)$: $a_0 = 0.999$, $a_1 = -1.094$ (k_0), and $b_0 = -46.29$ (λ_0). Note $|b_0|$ is very close to the replica separation distance $2L \tan(\theta_s) \approx 46.26 \lambda_0$.

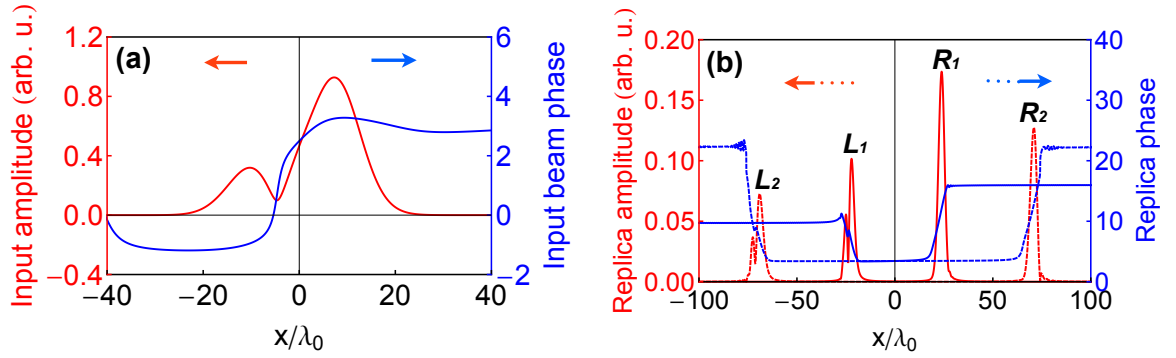


Fig. 5.3. (a) The total amplitude (red curve) and phase (blue curve) distributions of 10 random Gaussian beams as the input optical signal that are infinite in the y direction and characterized by random amplitudes in $[0, 1]$, random phases in the interval $[-\pi, \pi]$, random $1/e^2$ beam diameters in $[8\lambda_0, 14\lambda_0]$ range, and random center positions in $[-10\lambda_0, 10\lambda_0]$. (b) Magnitude and phase distributions of the four replicas used in our phase retrieval example. The solid curves represent the amplitude (red) and phase (blue) of L_1, R_1 . The dashed curves correspond to amplitude (red) and phase (blue) of L_2, R_2 .

replica phases show an excellent agreement with the exact distributions in the major signal region of $-40\lambda_0 < x < 40\lambda_0$, with the relative errors around 5%. The deviations of retrieved phases can be attributed to the bandwidth limitation for the quasi-linear spectral transformation function. They can be further improved if additional nonlinear terms (quadratic and cubic etc.) are included in the numerical modeling of the transformation function in Eq. (5.2). The final results of retrieved spectral amplitude and phase of the input signal are shown in Fig. 5.4 (b), which also match very well with the exact distributions as indicated by the inset of relative errors.

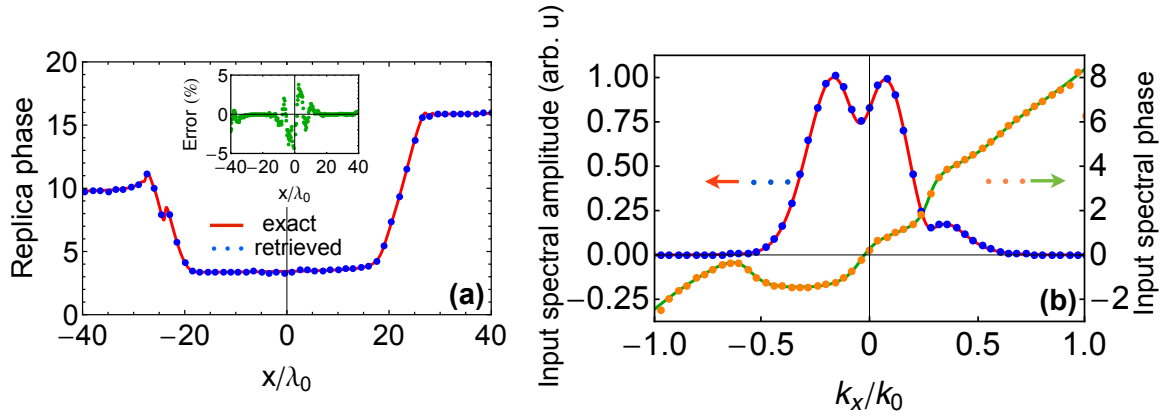


Fig. 5.4. (a) Comparison of the retrieved replica phases (blue dots) with the exact distributions (solid red). The inset shows the relative errors. (b) Reconstructed spectral amplitudes (blue dots) and phases (orange dots) of the input optical signal and compared with exact distributions (solid red: amplitude, solid green: phase).

5.4 Conclusions

In conclusion, we demonstrated that CR imaging can be used to retrieve optical phase information. The advancement of structured optical media such as metamaterials and PCs provide a new platform to achieve CR, which so far can only be

observed by finely polished birefringent crystals. As an example, we show the phase reconstruction of optical signals using a planar multilayered dielectric PC.

Compared with traditional phase retrieval methods relying on elaborate interferometers or the algorithmic approach that requires detections of both image and far-field intensities, our method only needs a single simultaneous measurement of CR image and guarantees a rapid recovery to the true solution. The planar feature of our device is compatible with current nano-fabrication and deposition techniques, which can find broad applications in the community of phase retrieval, optical imaging and signal processing.

6. CONCLUSIONS AND FUTURE WORK

From the zeroth-order transmission resonance in nanostructured Fabry-Perot core medium, to the negative refraction with a constant refractive index in photonic hypercrystals, to the conical refraction in a uniaxial periodic photonic structure, to a new prototype proposal of optical phase retrieval based on conical refraction imaging, we have presented several examples of structured optical media exhibiting tremendous capabilities in light manipulation and wave propagation control through the powerful platform of phase-space dispersion engineering.

All these work have convenient compatibility with the current nano-fabrication technology due to the planar nature of those proposed devices. Therefore, we anticipate an endeavor in experimental verification of the theoretical work will be carried out shortly, which eventually accelerates the practical industrial applications. This concludes one front of the future work.

Simultaneously, there is another work to be pursued on “negative” conical refraction which is beyond all the previous theoretical predictions. Due to the optical singularities of hyperbolic medium as part of the periodic elements in photonic hypercrystals, and the demonstrated capability of negative refraction, one can imagine a cone-shaped iso-frequency surface in photonic hypercrystals undeniably allows the the conical refraction goes to negative (or focusing). As a result, the location of replica images previously parallel to the medium surface will now switch to perpendicular to the single principal optical axis. Such an interesting phenomenon might lead to a new approach to far-field super-resolution imaging.

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APPENDIX

A. GERCHBERG-SAXON PHASE RETRIEVAL ALGORITHM

One of the earliest and most well-known algorithmic phase retrieval methods was proposed by Gerchberg and Saxon [152] (GS) based on alternative corrections (projections). Some literature also call it error reduction algorithm. This method is used to reconstruct the phase of a complex signal or object from the intensity measurements on two different planes - the real-space (image) plane and Fourier-space (diffraction) plane. A general illustration of this algorithm is given in the following Fig. A.1.

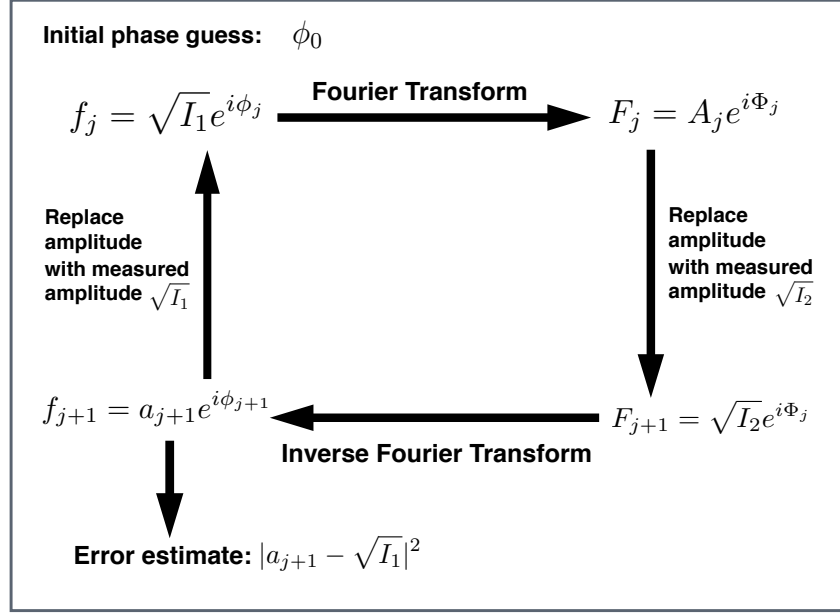


Fig. A.1. Schematic illustration of the Gerchberg-Saxon iterative algorithm for phase retrieval.

The GS algorithm iteratively imposes corrections of real-space intensity I_1 and Fourier intensity I_2 to acquire a phase function that is infinitely close to the exact distribution given if the algorithm is perfectly converged. The estimated iteration

error of the real-space magnitude (similar formula for Fourier space) in the j th loop is defined by

$$\epsilon_j = |a_j - \sqrt{I_1}|^2, \quad (\text{A.1})$$

where I_1 is the measured real-space amplitude and a_j is the derived real-space amplitude by Fourier transform.

The general procedure of GS algorithm can be shown in the following:

(1) **Initialization:** choose an initial phase guess ϕ_0 (e.g random distribution) to establish the real-space function $f_1 = \sqrt{I_1}e^{i\phi_0}$;

(2) **Iteration step** ($j = 1, 2, 3 \dots$) :

- a) Fourier transform f_j to obtain $F_j = A_j e^{i\Phi_j}$;
- b) Keep current Fourier phase Φ_j but correct the Fourier amplitude with measured one: $F_{j+1} = \sqrt{I_2} e^{i\Phi_j}$;
- c) Inverse Fourier transform F_{j+1} to obtain $f_{j+1} = a_{j+1} e^{i\phi_{j+1}}$;
- d) Keep current real-space phase ϕ_{j+1} but correct the real-space magnitude with measured $\sqrt{I_1}$.
- e) Go back to a) until the sum of iteration error ϵ_j is below a certain bound.

Clearly, the GS algorithm is significantly dependent on the initial phase guess. Without an appropriate approximation of the phase, this algorithm may not converge at all. On the other hand, even if the algorithm is converging as can be shown from the monotonically decreasing iteration error function, a stable recovery to the true solution is not guaranteed since this error may converge to a local minimum. Therefore, one has to find an appropriate initial phase guess before applying the algorithmic approach in phase recovery practice, which brings inevitable risk and high uncertainty of exact convergence. To address this problem, we propose a new method in Chapter 5 that doesn't require the initial phase guess. Instead, we offer an analytical solution to the object phase distribution so that a stable recovery to true result is always enabled.

VITA

VITA

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US Patent (pending):

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Peer-reviewed Journal Publications:

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2. Z. Huang and E. E. Narimanov, “Optical Imaging by Photonic Hyper-Crystals: Veselago Lens and beyond”, [Invited] Journal of Optics, 16, 114009 (2014).
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Conference Proceedings:

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2. Z. Huang and E. E. Narimanov, “Veselago Lens by Photonic Hyper-crystals”, Frontier in Optics and Laser Science Conference, Tucson, USA, October 2014.
3. Z. Huang and E. E. Narimanov, “Zeroth-Order Transmission Resonance in Hyperbolic Materials”, Conference of Lasers and Electro-Optics, San Jose, USA, June 2013.