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Using interactive multiobjective methods to solve DEA problems with value judgements

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Abstract

Data envelopment analysis (DEA) is a performance measurement tool that was initially developed without consideration of the decision maker (DM)'s preference structures. Ever since, there has been a wide literature incorporating DEA with value judgements such as the goal and target setting models. However, most of these models require prior judgements on target or weight setting. This paper will establish an equivalence model between DEA and multiple objective linear programming (MOLP) and show how a DEA problem can be solved interactively without any prior judgements by transforming it into an MOLP formulation. Various interactive multiobjective models would be used to solve DEA problems with the aid of *PROMOIN*, an interactive multiobjective programming software tool. The DM can then search along the efficient frontier to locate the most preferred solution where resource allocation and target levels based on the DM's value judgements can be set. An application on the efficiency analysis of retail banks in the UK is examined. Comparisons of the results among the interactive MOLP methods are investigated and recommendations on which method may best fit the data set and the DM's preferences will be made.

Keywords: Data envelopment analysis; Interactive multiobjective programming; Performance measurement

1. Introduction

Data envelopment analysis (DEA) is a nonparametric frontier estimation methodology based on linear programming for measuring the relative efficiency of a set of comparable decision making units (*DMU*s) that posse shared functional goals. Travares [1] in his extensive bibliography report of DEA has found a plethora number of DEA publications, last counted at 3203, of which there were 1259 journal papers published and over 50 books written up on DEA. The growing popularity of DEA over the years is reflected by the many enhancements to the original methodologies of Charnes, Cooper and Rhodes (*CCR*) [2] and Banker, Charnes and Coopers (*BCC*) [3] and an increasing number of successful DEA applications in the industry.

The original DEA model does not include a decision maker (DM)'s preference structure or value judgements while measuring relative efficiency, with no or minimal input from the DM. Value judgements are defined by [4] as "logical constructs, incorporated within an efficiency assessment study, reflecting the DM's preferences in the process of

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assessing efficiency". To incorporate DM's preference information in DEA, various techniques have been proposed such as the goal and target setting models of [5–8] and weight restrictions models including imposing bounds on individual weights [9], assurance region [10], restricting composite inputs and outputs, weight ratios and proportions [11], and the cone ratio concept by adjusting the observed input–output levels or weights to capture value judgement to belong to a given closed cone [12,13]. Alternative approaches include [14] whose model adopts unobserved *DMU*s, derived from pareto-efficient observed *DMU*s, and which incorporate value judgements; [15] also integrates preference information into a modified DEA formulation, while [16] uses hypothetical *DMU*s to represent preference information. However, all the above-mentioned techniques would require prior articulated preference knowledge from the DM, which in most cases can be subjective and difficult to obtain.

In manufacturing or service organizations, decision making can become more complex and often inherently uncertain, more so due to multiple attributes and conflicting objectives. Multiobjective programming methods such as multiple objective linear programming (MOLP) are techniques used to solve such multiple criteria decision making (MCDM) problems. An appealing method to incorporate preference information, without necessary prior judgement or target setting, is the use of an interactive decision making technique that encompasses both DEA and MOLP. Golany [5] first proposed an interactive model combining both of these approaches where the DM will allocate a set of level of inputs as resources and be able to select the most preferred set of level of outputs from alternative points on the efficient frontier. Post and Spronk [17] combined the use of DEA and interactive multiple goal programming where preference information are incorporated interactively with the DM by adjusting the upper and lower feasible boundaries of the input and output levels. Then, [18] showed that there are synergies from both DEA and MOLP, and showed that the DEA formulation is structurally similar to the reference point approach of the MOLP formulation. Further papers by [19–21] introduced the concept of value efficiency analysis (VEA) that effectively incorporate preference information in DEA.

In a similar vein, the aims of the paper are to establish an equivalence model between DEA and MOLP, and show how a DEA problem can be solved interactively without any prior judgement by transforming it into an MOLP formulation.

The main interactive multiobjective methods in the literature such as the tradeoff method of [22], Tchebychev method by [23], the step method by [24], the gradient projection method by [25] and the reference point methods by [26–28] would be used to solve DEA problems.

The combination of several interactive methods in the solution process has also been used previously like for example [29], where for the same preferences (reference levels), different methods based on reference point approach or classification are used. In this same line, Kaliszewski [30] insists in the necessity to combine different interactive methods under a common methodology. Furthermore, relationships between reference point techniques and local tradeoffs are analysed in [31]. There, relations among different types of information requested from the DM (e.g., reference points and local tradeoffs) are studied and such preferences are found which would produce the same solution, starting from the same previous solution. They can be regarded as equivalent pieces of information in the sense that they produce the same solution.

To carry out our purpose, we will use *PROMOIN* [32], an interactive multiobjective programming software. The DM can then interactively search along the efficient frontier to locate his most preferred solution (MPS). As all the Pareto optimal solutions are equally desirable and cannot be ordered completely, a DM can express his preferences in relation to conflicting objectives and is able to identify the most preferred one among them, also known as the MPS. Once the MPS is determined, better estimates of efficiency are calculated with more precise resource allocation and target levels that incorporate the DM's value judgements.

The remainder of the paper is organized as follows. The next section provides a brief description of the DEA techniques and introduces the basic DEA models. In the subsequent sections, the equivalence model between DEA and MOLP, and the general formulation for our proposed model is shown. The next section describes briefly the various interactive multiobjective programming methods. An application on the performance measurement of retail banks in the UK is examined with an analysis of results comparing the various interactive MOLP methods. Concluding remarks are provided in the last section.

2. DEA models

DEA is a linear programming method developed by [2] as a performance measurement technique for evaluating the relative efficiencies of a set of comparable organizational units or *DMU*s.

Suppose an organization has n *DMU*s ($j = 1, \ldots, n$), produces s outputs denoted by y_{rj}, the rth output of DMU_j for $r = 1, \ldots, s$ and consumes m inputs denoted by x_{ij}, the ith input of DMU_j for $i = 1, \ldots, m$. The formulation of the *CCR DEA* primal model is given by

$$\begin{aligned} \min \quad & h_0 = \sum_{i=1}^{m} v_i x_{ij_0} \\ \text{s.t.} \quad & \sum_{i=1}^{m} v_i x_{ij} - \sum_{r=1}^{s} u_r y_{rj} \geqslant 0, \quad j = 1, \ldots, n, \\ & \sum_{r=1}^{s} u_r y_{rj_0} = 1, \quad u_r, v_i > 0 \ \text{ for all } r, i, \end{aligned} \tag{1}$$

u_r is the weight parameter for output r and v_i the weight parameter for input i. $e_0 = 1/h_0$ denotes the optimal technical efficiency score with a possible range of $0 \leqslant e_0 \leqslant 1$ obtained from solving the DEA model. The efficiency score of $e_0 = 1$ shows the *DMU* as technical efficient and $0 < e_0 < 1$ reveals the presence of technical inefficiency. Here, each *DMU* can be evaluated by setting itself as the optimal objective function of DMU_0 and is allowed freedom by the DEA model in assigning the set of output weights u_r and input weights v_i, which will render the *DMU* as efficient as possible. In other words, the efficiency measure e_0 is optimized within the constraints for each of the n *DMU*s. In the output-orientated *CCR* primal model (1), the weighted outputs are fixed to unity and the weighted inputs minimized. The output weights u_r and input weights v_i are adjusted accordingly to generate an efficiency score.

While the *CCR* primal model can generate both an efficiency score and the optimal output weights u_r and input weights v_i, the *CCR* dual model can be used to generate not only the efficiency score but also the composite inputs and outputs that the observed DMU_0 should benchmark against. The output-orientated *CCR* dual model is given as follows:

$$\begin{aligned} \max \quad & h_0 = \theta_{j0} \\ \text{s.t.} \quad & \theta_{j0} y_{rj_0} - \sum_{j=1}^{n} \lambda_j y_{rj} \leqslant 0, \quad r = 1, \ldots, s, \\ & \sum_{j=1}^{n} \lambda_j x_{ij} \leqslant x_{ij_0}, \quad i = 1, \ldots, m, \\ & \lambda_j \geqslant 0 \quad \text{for all } j. \end{aligned} \tag{2}$$

In the output-orientated *CCR* dual model (2), for each observed DMU_0 an imaginary composite unit is constructed that outperforms DMU_0. λ_j is the reference weight for DMU_j ($j = 1, \ldots, n$) and $\lambda_j > 0$ means that DMU_j is used to construct the composite unit for DMU_0. The composite unit consumes at most the same inputs as DMU_0 and produces outputs that are at least equal to a proportion θ_{j0} of the outputs of DMU_0. The parameter θ_{j0} indicates by how much DMU_0 has to proportionally increase its outputs to become efficient. The inverse of θ_{j0} is the efficiency score of DMU_0. The increase is employed concurrently to all outputs and results in a radial movement towards the envelopment surface [13].

The *CCR* models above are based on constant returns to scale, however, economies of scale are not taken into account. Banker et al. [3] developed another version of *DEA* model that considered variable returns to scale, called the *BCC* model which is different from the *CCR* model in that the former has an additional convexity constraint of all λ_j restricted to sum to 1 in the dual case. Note, the *CCR* and *BCC* models only provide the radial measure of technical efficiency. Further literature by [33] and [34] have shown how DEA formulations could be used to measure Pareto–Koopmans efficiency that allows the presence of nonzero slacks.

3. Equivalence between DEA and MOLP

In a DEA model, an efficiency score is generated for a *DMU* by maximizing outputs with limited inputs, or minimizing inputs with desired or fixed outputs, or simultaneously maximizing outputs and minimizing inputs. Either way, this can be regarded as a kind of multiple objective optimization problem. In this section, the theoretical considerations of combining MOLP and DEA are presented, and the equivalence between the output-orientated DEA dual formulation and the minimax formulation in MOLP will be shown.

A MOLP problem can be represented in a general form as follows:

$$\begin{aligned} \max \quad & [f_1(\lambda), f_2(\lambda), \ldots, f_r(\lambda), \ldots, f_s(\lambda)] \\ \text{s.t.} \quad & \lambda \in \Omega_{j_0}, \end{aligned}$$

where

$$\Omega_{j_0} = \left\{ \lambda \left| \sum_{j=1}^{n} \lambda_j x_{ij} \leqslant x_{ij_0}; i = 1, \ldots, m; \lambda_j \geqslant 0 \ , j = 1, \ldots, n \right. \right\}. \tag{3}$$

The MOLP formulation can then be written in a weighted minimax approach [23,35] as follows:

$$\begin{aligned} \min_{\lambda} \max_{1 \leqslant r \leqslant s} \quad & \{ w_r (f_r^* - f_r(\lambda)) \} \\ \text{s.t.} \quad & \lambda \in \Omega_{j_0}. \end{aligned} \tag{4}$$

The weighted minimax formulation is a special case of the reference point approach with f_r^* as the ideal point, and calculates the minimum of the maximum distance between f_r^*, the maximum value of objective r, and $f_r(\lambda)$, the observed value of objective r.

The weighted minimax MOLP formulation can then be written as follows by introducing an auxiliary variable θ [35,36] as follows:

$$\begin{aligned} \min \quad & \theta \\ \text{s.t.} \quad & w_r(f_r^* - f_r(\lambda)) \leqslant \theta, \quad r = 1, \ldots, s, \\ & \lambda \in \Omega_{j_0}. \end{aligned} \tag{5}$$

From formulation (2), the output-orientated CCR dual DEA model can be equivalently rewritten as follows:

$$\begin{aligned} \min \quad & \theta_{j_0} \\ \text{s.t.} \quad & \theta_{j_0} y_{rj_0} - \sum_{j=1}^{n} \lambda_j y_{rj} \leqslant 0, \quad r = 1, \ldots, s, \\ & \lambda \in \Omega_{j_0}. \end{aligned} \tag{6}$$

In order to prove that the CCR dual formulation (6) is equivalent to the minimax formulation in (5), certain conditions have to be applied. The purpose for establishing the equivalence conditions is to use the interactive techniques in MOLP to locate the MPS on the efficient frontier for target setting and resource allocation. Note that in formulation (5) the weighting parameter w_r is subject to change during the interactive process of locating the MPS. Suppose in formulation (6), the rth composite output is denoted by $f_r(\lambda)$, defined as follows:

$$f_r(\lambda) = \sum_{j=1}^{n} \lambda_j y_{rj}, \quad r = 1, \ldots, s \ \text{ and } \ \lambda = (\lambda_1, \ldots, \lambda_n)^{\mathrm{T}}. \tag{7}$$

In this equivalence analysis, the rth composite output is taken as an objective for maximization, so there are s objectives in total. The maximum feasible value of the rth composite output is denoted by $\overline{f}_{rj_0} = f_r(\lambda^r)$ where λ^r can be found by solving the following single objective optimization problem:

$$\begin{aligned} \max \quad & f_r(\lambda) = \sum_{j=1}^{n} \lambda_j y_{rj} \\ \text{s.t.} \quad & \lambda \in \Omega_{j_0}. \end{aligned} \tag{8}$$

Suppose the feasible decision space Ω_{j_0} in formulations (3) and (5) is the same as defined for formulation (6). The equivalence relationship between the output-oriented CCR dual model (6) and the minimax formulation (5) can be established by the following theorem.

Theorem 1. *Suppose $y_{rj_0} > 0$ for any $r = 1, \ldots, s$ and $j_0 = 1, \ldots, n$. The output-oriented CCR dual model* (6) *can be equivalently transformed to the minimax formulation* (5) *using Eqs.* (7) *and* (8) *and the following equations*:

$$w_r = \frac{1}{y_{rj_0}}, \tag{9}$$

$$f_r^* = \frac{F^{\max}}{w_r} = y_{rj_0} F^{\max}, \tag{10}$$

$$\theta = F^{\max} - \theta_{j_0}, \tag{11}$$

with

$$F^{\max} = \max_{1 \leqslant r \leqslant s} \{w_r \overline{f}_{rj_0}\} = \max_{1 \leqslant r \leqslant s} \left\{ \frac{\overline{f}_{rj_0}}{y_{rj_0}} \right\}. \tag{12}$$

Proof. Using Eqs. (7) and (9), the output-orientated CCR dual model can be equivalently rewritten as follows:

$$\begin{aligned} \min \quad & \theta_{j_0} \\ \text{s.t.} \quad & \theta_{j_0} \frac{1}{w_r} - f_r(\lambda) \leqslant 0, \quad r = 1, \ldots, s, \\ & \lambda \in \Omega_{j_0}. \end{aligned} \tag{13}$$

The s constraints in (13) except for those included in Ω_{j_0} can be equivalently transformed as follows, where "$\Leftrightarrow$" means "is equivalent to". For any $r = 1, \ldots, s$, we have

$$\begin{aligned} & \theta_{j_0} \frac{1}{w_r} - f_r(\lambda) \leqslant 0 \\ & \Leftrightarrow -w_r f_r(\lambda) \leqslant -\theta_{j_0} \\ & \Leftrightarrow F^{\max} - w_r f_r(\lambda) \leqslant F^{\max} - \theta_{j_0} \\ & \Leftrightarrow w_r \left(\frac{F^{\max}}{w_r} - f_r(\lambda) \right) \leqslant \theta_{j_0} \\ & \Leftrightarrow w_r (f_r^* - f_r(\lambda)) \leqslant \theta_{j_0}. \end{aligned} \tag{14}$$

Moreover, the objective function of (13) becomes

$$\begin{aligned} \max \ \theta_{j_0} &= \min(-\theta_{j_0}) \\ &= \min(F^{\max} - \theta_{j_0}) \\ &= \min \theta. \end{aligned} \tag{15}$$

Note that for any $\lambda \in \Omega_{j_0}$,

$$\begin{aligned} \theta &= F^{\max} - \theta_{j_0} \\ &\geqslant w_r \overline{f}_{rj_0} - \theta_{j_0} \quad \text{for any } r = 1, \ldots, s \\ &\geqslant w_r f_r(\lambda) - \theta_{j_0} \quad \text{for any } r = 1, \ldots, s \\ &\geqslant 0. \end{aligned} \tag{16}$$

Also,

$$f_r^* = \frac{F^{\max}}{w_r} \geqslant \frac{w_r \overline{f}_{rj_0}}{w_r} = \overline{f}_{rj_0} = \max_{\lambda \in \Omega_{j_0}} f_r(\lambda) \quad \text{for any } r = 1, \ldots, s. \tag{17}$$

The equivalence model between the CCR dual DEA of (6) and the minimax MOLP formulation of (5) is established since Eqs. (14) and (15) holds. □

From Theorem 1, the output-oriented CCR dual model can be equivalently rewritten as a minimax formulation of (5) as follows:

$$\begin{aligned} \max \quad & \theta \\ \text{s.t.} \quad & w_r \left(f_r^* - \sum_{j=1}^{n} \lambda_j y_{rj} \right) \leqslant \theta, \quad r = 1, \ldots, s, \\ & \sum_{j=1}^{n} \lambda_j x_{ij} \leqslant x_{ij_0}, \quad i = 1, \ldots, m, \\ & \lambda_j \geqslant 0. \end{aligned} \tag{18}$$

The output-oriented *BCC* dual model can also be transformed to an equivalent minimax formulation similar to the above model, by adding the convexity constraint $\sum_{j=1}^{n} \lambda_j = 1$.

Alternatively, since formulation (5) is equivalent to (3), formulation (18) can be rewritten as follows:

$$\begin{aligned} \max \quad & \left[\sum_{j=1}^{n} \lambda_j y_{1j}, \sum_{j=1}^{n} \lambda_j y_{2j}, \ldots, \sum_{j=1}^{n} \lambda_j y_{rj}, \ldots, \sum_{j=1}^{n} \lambda_j y_{sj} \right] \\ \text{s.t.} \quad & \sum_{j=1}^{n} \lambda_j x_{ij} \leqslant x_{ij_0}, \quad i = 1, \ldots, m, \\ & \lambda_j \geqslant 0. \end{aligned} \tag{19}$$

From Theorem 1, the following remarks can be drawn:

Remark 1. Formulation (18) is equivalent to formulation (6) if w_r in formulation (18) is calculated using Eq. (9) (or $w_r = 1/y_{rj_0}$) and f_r^* in formulation (18) is calculated using Eq. (10) (or $f_r^* = y_{rj_0} F^{\max}$). Likewise, formulation (19) is equivalent to formulation (6). In other words, the efficiency score of the jth *DMU* can be generated by solving formulation (18) or (19). Hence, an interactive MOLP method can be used to solve the DEA problem.

The equivalence model shown above is only valid for output-oriented DEA models (for more details about this equivalence see also [37]). Similarly, the equivalence model has been extended to input-oriented DEA models in [38].

4. Interactive multiobjective programming

Interactive multiobjective programming methods constitute techniques that allow the DM to search for different solutions along the efficient frontier, so that the DM can reach the MPS. At each stage, the current solution is adapted to the structure of preferences of the DM. It can be said that an interactive method is designed to drive the DM towards his MPS, or at least, to a good solution, in the sense that it is acceptable by the DM. For this reason, interactive methods are powerful tools for solving MOLP problems.

The first pioneer interactive multiobjective programming method, known as the step method or STEM, was proposed by [24] that is devoted to solve MOLP problems. Then, two other well-known methods were published, the G-D-F algorithm [22] and Zionts–Wallenius's method [39]. The former uses a widely spread concept in MCDM by applying local weights or tradeoffs on objectives.

There are several interactive methods that are adaptations of a priori information algorithms. For example, in the reference point method proposed by [26], the scalarised achievement function is the technical support of the process, while the interactivity relies on the variation of some parameters of this function, which represent the reference values. The visual interactive approach (VIA) algorithm ([27]) and its Pareto Race software ([40]) works in a similar manner. At the same time, [41] proposed the Tchebychev method whereby the DM has to select the best solution among different possible solutions generated at each iteration.

The earlier methods mentioned above proved to be highly effective for solving multiobjective programming, particularly, MOLP problems and a wider range of interactive methods were developed in the last two decades [42], such as the interactive surrogate worth tradeoff (ISWT) [43], satisfacing tradeoff method (STOM) [28], light beam search [44], NIMBUS [45], GUESS [46], and the gradient projection [25]. Nevertheless, the proliferation of such methods, apart from being a proof of the good health of the interactive scheme, has also brought up one of the main dilemma, that is, which of these methods should be used when it comes to applying an interactive algorithm to solve a real decision problem. It is evident that the election of the interactive method would indeed influence the final solution.

In the next section, an empirical application is solved with several interactive methods. Results are compared and analysed highlighting the quality of the solutions with corresponding comments on advantages and disadvantages found in the solution. The interactive methods that are considered in this paper are namely: G-D-F, Wierzbicki, STEM, Tchebychev and STOM. To explain these interactive methods, let us consider the general MOLP formulation in (3).

4.1. G-D-F method

The G-D-F method was proposed by [22]. It is applicable to problems in (3) where the objective functions are concave and Ω is a convex set. This method assumed that DM's preferences could be described by means of a utility function that is continuously differentiable, concave and increasing utility. It is based on the use of a procedure of optimization, known as Frank–Wolfe algorithm [47], which searches an ascending direction in the DM's utility function.

This procedure moves from a solution to other solution by means of a gradient direction with ascending step length. At each iteration, a feasible solution is obtained through maximizing the direction of the gradient of the utility function that was evaluated in the previous solution where the DM provides local tradeoffs information on objectives or local weights. At iteration h, the local weights on objectives in the solution of the previous iteration x^{h-1} are determined, $w^h = (w_1^h, w_2^h, \ldots, w_s^h)$, by means of the interaction with the DM. With these values, an optimal solution y^h to the problem search of ascending direction of the DM's utility is obtained solving

$$\begin{aligned} \max_{y} \quad & \left(\sum_{i=1}^{s} w_i^h \nabla_x f_i(x^{h-1})\right)^t \cdot y \\ \text{s.t.} \quad & y \in \Omega. \end{aligned} \tag{20}$$

Given y^h, some solutions that lie in the segment $\overline{x^{h-1}, y^h}$ are presented to the DM, who must choose one of the solutions (x^h, f^h) where $f^h = f(x^h)$ denotes the objective vector.

4.2. Wierzbicki method

The Wierzbicki method [26] is based on the achievement scalarising function, created to obtain efficient solutions in a multiobjective problem. The basis of the method is at each iteration, several efficient solutions are obtained by means of minimizing an achievement function through the use of a reference point that is provided by the DM. A range of reference points can be determined where each point only differs from the original reference point by a single component. This allows the DM to choose one of the solutions on the efficient frontier. The efficient solutions are generated by minimizing an achievement function which is representative of the aspiration levels provided by the DM. It is noted the achievement function weights are fixed and they are not changed during the whole solution process, and it is the reference points that can be changed. The weights attached to the reference points will be provided by the DM or set arbitrarily with normalizing character. The achievement function used in this method is

$$s(\overline{q}, f(x), w) = \max_{i=1,\ldots,s} \{w_i(\bar{q}_i - f_i(x))\} + \rho \sum_{i=1}^{s} (\bar{q}_i - f_i(x)), \tag{21}$$

where $w = (w_1, \ldots, w_s)$ is the weights vector, $\overline{q} = (\bar{q}_1, \ldots, \bar{q}_s)$ is the reference point and ρ is a small positive value.

Given the solution (x^{h-1}, f^{h-1}), the DM must provide the new reference levels on the objectives, namely, the new reference point $\overline{q}^h$. With this point, we obtain the objective vector $f^{h,0}$ that minimizes the achievement function. The following problem is as follows:

$$\min_{x \in \Omega} s(\overline{q}^h, f(x), w) \tag{22}$$

and is equivalent to

$$\begin{aligned} \min_{x,\alpha} \quad & \left\{ \alpha + \rho \sum_{i=1}^{s} (\bar{q}_i^h - f_i(x)) \right\} \\ \text{s.t.} \quad & w_i(\bar{q}_i^h - f_i(x)) \leqslant \alpha, \quad i = 1, \ldots, s, \\ & x \in \Omega. \end{aligned} \tag{23}$$

The two vectors $\overline{q}^h$ and $f^{h,0}$, can be used to obtain k reference points:

$$\overline{q}^{h,j} = \overline{q}^h - (0, \ldots, 0, \overset{j)}{\delta^h}, 0, \ldots, 0), \quad \delta^h = \|\overline{q}^h - f^{h,0}\|_2, \quad j = 1, \ldots, s. \tag{24}$$

And its corresponding objective vectors can be calculated as

$$\min_{x \in \Omega} s(\overline{q}^{h,j}, f(x), w) \Rightarrow f^{h,j}, \quad j = 1, 2, \ldots, s. \tag{25}$$

From these $s + 1$ objective vectors, $\{f^{h,0}, f^{h,1}, \ldots, f^{h,s}\}$, the DM must choose one of the solutions (x^h, f^h).

4.3. STEM method

The STEM method [24] is based on minimizing the Tchebychev distance from the ideal point to the criterion space. The parameters of the distance formula and the feasible space can be changed by a normalized weighting method based on the DM's preferences in the previous solution. The procedure of STEM allows the DM to recognize good solutions and the relative importance of the objectives.

At each iteration, the DM is able to improve some objectives, by sacrificing others. In addition, the DM must provide the maximum amount by which the objective functions can be sacrificed, although it is not necessary to provide tradeoffs on objectives. To carry out an iteration in the STEM method, given a solution x^{h-1}, the DM must provide their preferences for objective functions to be improved $\{f_i, i \in \{1, \ldots, s\} - J^h\}$, as well as the objective functions to be relaxed $\{f_i, i \in J^h\}$ with corresponding maximal amounts to relax $\{\Delta f_i^h, i \in J^h\}$.

The following problem can be solved using the above preferences:

$$\begin{aligned} \min_{x,\alpha} \quad & \alpha \\ \text{s.t.} \quad & w_i(f_i^* - f_i(x)) \leqslant \alpha, \quad i \in \{1, \ldots, s\} - J^h, \\ & f_j(x) \geqslant f_j(x^{h-1}) - \Delta f_j^h, \quad j \in J^h, \\ & f_j(x) \geqslant f_j(x^{h-1}), \quad j \in \{1, \ldots, s\} - J^h, \\ & x \in \Omega, \quad \alpha \geqslant 0, \end{aligned} \tag{26}$$

where $f_i^* = \max_{x \in \Omega} f_i(x)$, $i = 1, \ldots, s$ are the ideal values, resulting from maximizing the objective functions individually.

4.4. Tchebychev method

The Tchebychev method was proposed by [23,41]. At each iteration, given the current solution, weights are calculated which projects the ideal vector to the current objective vector. A new weight interval is obtained and the DM must choose one solution among different solutions. If the DM is not satisfied with the current solution, a further new set of solutions can be generated in the next iteration. It is noted as the number of iterations increases, the size of the weight interval and range of the solutions will become smaller.

The problem can then be solved by minimizing the augmented Tchebychev metric based on each weight vector:

$$\begin{aligned} \min_{x,\alpha} \quad & \left\{ \alpha + \rho \sum_{i=1}^{s} (f_i^* - f_i(x)) \right\} \\ \text{s.t.} \quad & w_i(f_i^* - f_i(x)) \leqslant \alpha, \quad i = 1, \ldots, s, \\ & x \in \Omega, \quad \alpha \geqslant 0. \end{aligned} \tag{27}$$

4.5. STOM method

The STOM method [28] is another method that uses the achievement function like the Wierzbicki method. An important aspect of the method is the fact that in its original version, the reference point of the achievement function does not change within the whole solution process. The reference levels of the objectives are provided by the DM at each iteration and incorporated in the weights of the achievement function.

Given a solution (x^{h-1}, f^{h-1}), the DM must provide the objective functions that are to be improved, and those that has to be maintained or relaxed. $\{f_i, i \in I_I^h\}$ represents the objective functions to improve where $\{\Delta f_i^h, i \in I_I^h\}$ are the amounts to be improved, $\{f_i, i \in I_M^h\}$ are objective functions that are maintained and $\{f_i, i \in I_R^h\}$ are objective functions to relax $\{f_i, i \in I_R^h\}$ with the amounts to relax as $\{\Delta f_i^h, i \in I_R^h\}$.

Hence, the new reference point $q^h = (q_1^h, \ldots, q_s^h)$ can be shown as

$$\begin{aligned} q_i^h &= f_i^{h-1} + \Delta f_i^h \quad \forall i \in I_I^h, \\ q_i^h &= f_i^{h-1} \quad \forall i \in I_M^h, \\ q_i^h &= f_i^{h-1} - \Delta f_i^h \quad \forall i \in I_R^h. \end{aligned} \tag{28}$$

And the following problem can be solved:

$$\min_{x\in\Omega} s\left(f^*, f(x), \frac{1}{f^* - q^h}\right) \tag{29}$$

which is equivalent to

$$\begin{aligned} \min_{x,\alpha} \quad & \alpha \\ \text{s.t.} \quad & w_i^h(f_i^* - f_i(x)) \leqslant \alpha, \quad i = 1, \ldots, s, \\ & x \in \Omega, \quad \alpha \geqslant 0, \end{aligned} \tag{30}$$

where

$$w_i^h = \frac{1}{f_i^* - q_i^h} \quad \forall i = 1, \ldots, s.$$

The solution process of the above methods can be carried out using a decision system developed by [42] known as *PROMOIN*©(interactive multiobjective programming). The windows-based software has a user-friendly interface and allows for solving problems in multiobjective programming interactively. *PROMOIN* has a choice of 10 well-known interactive methods [32,48].

5. Empirical application: assessment of retail banks in the UK

The UK retail bank industry, specifically seven major retail banks, is examined to demonstrate the interactive approach to search for the MPS on the efficient frontier. The data set is obtained from [49] through a study on using DEA and the ER approach for performance measurement of UK retail banks (Table 1).

For the DEA formulation, the reference set consists of seven *DMU*s, and three inputs and three outputs are considered. The *DMU*s are homogenous comparable major banks in the UK including Abbey National, Barclays, Halifax, HSBC, Lloyds TSB, NatWest and RBS. The three inputs are namely number of branches, number of ATMs and number of staffs, while the three outputs are total revenue, corporate image and customer satisfaction. The first author of this paper is the DM for the assessment of retail banks and has consulted with senior bank managers with regards to the choice of variables used in the DEA model.

The results of BCC DEA dual formulation is shown in Table 2, which is maximizing the amount by which outputs must be proportionally increased for the observed *DMU* to be efficient. The efficiency score for NatWest is 73%, implying that it is operating as an inefficient bank with respect to all seven banks in the survey.

Further analysis on the areas of improvement NatWest needs to focus upon and the amount of improvement needed for each input and output are shown in Table 3. For instance, if the amounts of inputs of NatWest are maintained, all

Table 1
Data set of the UK retail banks

DMU	Bank	Inputs			Outputs		
		No. of branches ('000)	No. of ATMs ('000)	No. of staff ('0,000)	Total revenue (£m)	Corporate image [a]	Customer satisfaction [a]
1	Abbey Nat.	2.00	2.18	2.35	10.57	3.40	6.79
2	Barclays	1.95	3.19	8.43	13.35	6.66	2.55
3	Halifax	0.80	2.10	3.21	8.14	1.92	9.17
4	HSBC	1.75	4.00	13.30	23.67	8.47	5.82
5	Lloyds TSB	2.50	4.30	9.27	14.01	3.44	6.57
6	NatWest	1.73	3.30	7.70	12.04	2.53	4.86
7	RBS	0.65	1.53	2.67	7.36	1.26	7.28

[a]Corporate image and customer satisfaction values are converted scores based on the average expected utility of survey respondents.

Table 2
DEA efficiency results

Observed DMU's composite unit									
DMU	Bank	Efficiency	1	2	3	4	5	6	7
1	Abbey Nat.	1.00	1.00						
2	Barclays	1.00		1.00					
3	Halifax	1.00			1.00				
4	HSBC	1.00				1.00			
5	Lloyds TSB	0.88			0.50	0.50			
6	NatWest	0.73	0.38		0.14	0.48			
7	RBS	1.00							1.00

Table 3
DEA efficiency results of NatWest

Performance	Inputs			Outputs		
	No. of branches ('000)	No. of ATMs ('000)	No. of staff ('0000)	Total revenue (£m)	Corporate image [a]	Customer satisfaction[a]
Evaluated unit (current)	1.73	3.30	7.70	12.04	2.53	4.86
Composite unit	1.71	3.04	7.70	16.49	5.62	6.66
Improvement	−0.02	−0.26	0.00	4.45	3.09	1.80
%	99	92	100	137	222	137
Efficiency score	0.73					

[a]Corporate image and customer satisfaction values are converted scores based on the average expected utility of survey respondents.

outputs can at least be optimized further by 37%. In actual fact, inputs such as the number of ATMs should be reduced from 3300 to 3040, a decrease of 8%, while the number of branches should be reduced by 20 from 1730 to 1710 in order for NatWest to be efficient. Conversely, the total revenue, corporate image and customer satisfaction can be increased by 37%, 122% and 37% respectively. So, instead of a current total revenue of £12.04 m, the target total revenue that can be achieved is £16.49 m.

However, the DEA efficiency results generated do not consider the value judgements of the DM. Hence, interactive MOLP methods will be used to search for the MPS along the efficient frontier for NatWest. The DEA problem will be transformed into an MOLP formulation using (18) and solve with the aid of the multiobjective programming software, *PROMOIN*.

Table 4
G-D-F interactive results

Iteration 1	Iteration 2	Iteration 3
Solution		
Ab_Nat = 0.4044	Ab_Nat = 0.3235	Ab_Nat = 0.0647
Barclays = 0.0000	Barclays = 0.0000	Barclays = 0.0000
Halifax = 0.0000	Halifax = 0.2000	Halifax = 0.4839
HSBC = 0.4855	HSBC = 0.3884	HSBC = 0.4338
Lloyds = 0.0000	Lloyds = 0.0000	Lloyds = 0.0000
NatWest = 0.0000	NatWest = 0.0000	NatWest = 0.0000
RBS = 0.1101	RBS = 0.0881	RBS = 0.0176
Objective functions values		
Tot_rev = 16.5789	Tot_rev = 14.8907	Tot_rev = 15.0209
Corp_Im = 5.6255	Corp_Im = 4.8844	Corp_Im = 4.8461
Cus_Sat = 6.3725	Cus_Sat = 6.9320	Cus_Sat = 7.5295

Table 5
Comparison table of final solutions

Interactive methods	Final solutions				Observed DMU's composite unit						
	No. of iterations	Total revenue	Corporate image	Customer satisfaction	1	2	3	4	5	6	7
G-D-F	3	15.02	4.85	7.53	0.065		0.484	0.434			0.018
STEM	2	15.18	4.91	7.59	0.034		0.518	0.449			
Tchebychev	7	15.00	4.81	7.69			0.558	0.442			
Wierzbicki	7	14.97	4.80	7.70			0.560	0.440			
STOM	2	15.07	4.85	7.67			0.550	0.446			

In order to compare the different interactive models, a set of priority goal rules will be introduced. The first rule is revenue will be targeted at £15 m, the second rule is to improve the level of customer satisfaction to above a value of 7.5 and the final rule is to maintain the level of corporate image at its current levels. Interactive multiobjective methods are examined to allow us to incorporate the DM's preferences into our model and generate solutions according to the above preferences, through the use of reference levels, local weights or local tradeoffs methods. The five interactive methods which will be compared and analysed are namely: G-D-F, Wierzbicki, STEM, Tchebychev and STOM.

The full solution process of NatWest using the G-D-F method is shown in Table 4. Initially, this method generates an initial solution considering all local weights equally. In our case, the achieved output values of revenue, corporate image and customer satisfaction are 16.58, 5.63, 6.37 respectively. However, the DM is not satisfied with the current level of outputs as he wants to improve customer satisfaction, a new set of local weights of 1.0, 1.0, 7.0 are provided where the weight of the third output (the local weight of customer satisfaction) is greater than the other two. With these values, a new set of solutions is generated with the output values of 14.89, 4.88, 6.93. In iteration 2, both the output levels of revenue and corporate image are sacrificed for a higher output level of customer satisfaction. By adhering to the DM's preferences that target revenue should be set at £15 m objective functions. Again, the DM is still not satisfied with the new solutions and carry on interactively to search for the MPS that satisfied all his preferences. Since some intermediate solutions in this method can be not efficient and the DM is not satisfied with the current values, a set of new solutions is generated in iteration 3 by introducing similar local weights in revenue and customer satisfaction outputs, and a smaller local weight for corporate image output, and choosing the last solution, that is efficient. The achieved values are 15.02, 4.85, 7.53. Once all the conditions are met, and the DM is fully satisfied with the indifference tradeoffs between the objectives, the interactive process will terminate and the MPS is found. Here, the local weights values allow us to generate solutions according to DM's preferences.

The comparison Table (Table 5) shows the MPS generated for each interactive method used. One of the methods preferred by the DM was the Tchebychev method because it fully satisfied the first goal by achieving an exact targeted revenue of £15 m. The Tchebychev, Wierzbicki and STOM methods have rather similar final solutions and it can be

Table 6
Assessment of interactive methods

Interactive methods	Criteria					
	DM's confidence	Ease of understanding	Ease of elicitation	Computational charge	No. of iterations	Total score
G-D-F	3	3	2	4	4	16
STEM	2	2	2	5	5	16
Tchebychev	5	5	5	2	3	20
Wierzbicki	5	4	3	3	3	18
STOM	3	3	3	5	5	19

seen that NatWest's MPS are the convex combination of Halifax and HSBC. For the second goal criteria, Wierzbicki provides the highest value for customer satisfaction with 7.70, although the shortfall is that revenue is slightly lower than its target level. For this case study, STEM and STOM methods require the least number of iterations in order to reach its MPS. It is important to point out that these results do not necessarily recommend any one interactive method. The DM's preference on the interactive methods in multiobjective programming depends on the data set under consideration. This issue was also highlighted by [30] who suggested the need to combine different interactive methods under a common methodology to obtain better results.

The G-D-F method utilizes weights as local tradeoffs and would not necessarily provide the "best" MPS as envisaged by the DM. Its MPS is a convex combination of four banks: Abbey National, Halifax, HSBC and RBS. The STEM method shows an improvement in subsequent iterations. However, the main problem is than it depends on strictly extreme values where the interactive process requires the DM to specify which objective functions that will be allowed to relax with corresponding maximal amounts. The MPS obtained seems to be relatively the worst off, particularly, in achieving set target levels such as the revenue figure of £15.18 m, a deviation of 0.18 from the target level.

The Tchebychev method is preferred by the DM because of its user-friendly and easy-to-understand solution process. It is based on the minimum distance from the ideal levels, where the weights interval and range of solution provided will become smaller and is able to fine-tune the search process until the MPS is found. Although the number of iterations may be higher than other interactive methods, but the accuracy of the results far outweighs the lengthy solution process. The Wierzbicki method could also require a high number of iterations in order to find the MPS. The search process is a tedious step-by-step progression because the differences between generated solutions can be very close between them depending on the form of the efficient frontier. Hence, the small tradeoffs are being made at each iteration. Nevertheless, the accuracy of the results are much higher than some of the other interactive methods and provides highest value of customer satisfaction, yet maintaining the output levels of the other two objectives. The STOM method shows a relatively good result and the references in the achievement function are always ideal and are introduced in the aspiration levels that are incorporated in the weighting factor. Hence, when the values are very near to its ideal levels, the inverse value of the weights will affect the solution.

Finally, following [50–52] who consider comparative studies of interactive methods in multiobjective programming, the five interactive methods will be assessed by the DM and the analyst according to several criteria, in order to carry out a more rigorous comparative study on the obtained solutions. The five criteria that are considered includes the DM's confidence in the final solution; the ease of understanding of the interactive method; ease of eliciting information from the DM; computational charge, mainly, number of intermediate problems that are solved at each iteration; and the number of iterations.

The scores for each criteria are based on a scale from 1 to 5, where 1 is deemed the least favourable and 5 is deemed the most favourable. It is necessary to point out that the first three criteria are evaluated by the DM, the fourth criteria by the analyst and the fifth corresponds to the number of iterations carried out during the solution process.

It can be seen from Table 6 that the Tchebychev method has the highest total score of 20, followed by STOM method, Wierzbicki's method and then G-D-F and STEM method.

Although the attempt to rank the interactive methods was shown, however, the total scores generated between the methods were very close. Hence, there may not be one best interactive method to obtain the MPS, and the variety of methods available would only further confuse the DM on which method would be best suited for the problem. *PROMOIN*, the multiobjective programming software, has the "change-of-method" option that allows the DM the choice to switch to another interactive method during the solution process. The programme was developed on a theoretical basis that has the ability to transfer and minimising the loss of information between methods. The change of method does not necessarily mean a restart to the interactive procedure but retaining all prior information obtained so far in the previous iterations carried out. It is particularly useful for circumstances when the DM found the current interactive method difficult to make a significant progress towards the MPS, and the option to switch to another method allows to overcome this problem. Moreover, the DM can backtrack and change previous inconsistent decision if desired.

6. Conclusion

The paper establishes the equivalence relationship between the output-orientated DEA dual models and minimax reference point approach of MOLP, showing how a DEA problem can be solved interactively without any prior judgements by transforming it into an MOLP formulation. This provides the basis to apply interactive techniques in MOLP to solve DEA problems and further locate the MPS along the efficient frontier for each *DMU*. Moreover, it effectively allows evaluating past performances and planning future targets by taking into account of the DM's preferences in an interactive and progressive fashion where the DM explores what could be achieved technically. The MPS generated using interactive MOLP methods provides a rich insight into the performance assessment and the efficiency analysis of each *DMU* with realistic and technically feasible target values that incorporate DM's value judgements. An example on the assessment of UK retail banks was shown to illustrate the equivalence model and its potential to be applied to other performance assessment and target setting applications. This equivalence model has been solved satisfactory by using five well-known interactive methods in multiobjective programming. In addition, an analysis of results comparing the various interactive MOLP methods was examined.

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