Universal Temporal Concurrent Constraint Programming.

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Motivation

Concurrent Systems are everywhere:

- Engineering : Security protocols, service oriented computing, mobile computing, synchronous systems.
- Science : Biological and chemical systems.
- Arts : Multimedia Interaction.

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Formal Models to describe and analyze concurrent systems. They must be:

- Simple.
- Expressive.
- Formal.
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Some Examples : CCS [Mil89], the π-calculus [MPW92], CSP [Hoa85], ACP [BK85], **CCP** [Sar93].

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- Concurrent Constraint Programming (CCP) [Sar93] is a declarative model for Concurrency where agents interact by telling and asking information represented as constraints in a global store .
- The type of constraints and the entailment relation is given by a Constraint System (e.g. $x > 42 \models x > 0$).

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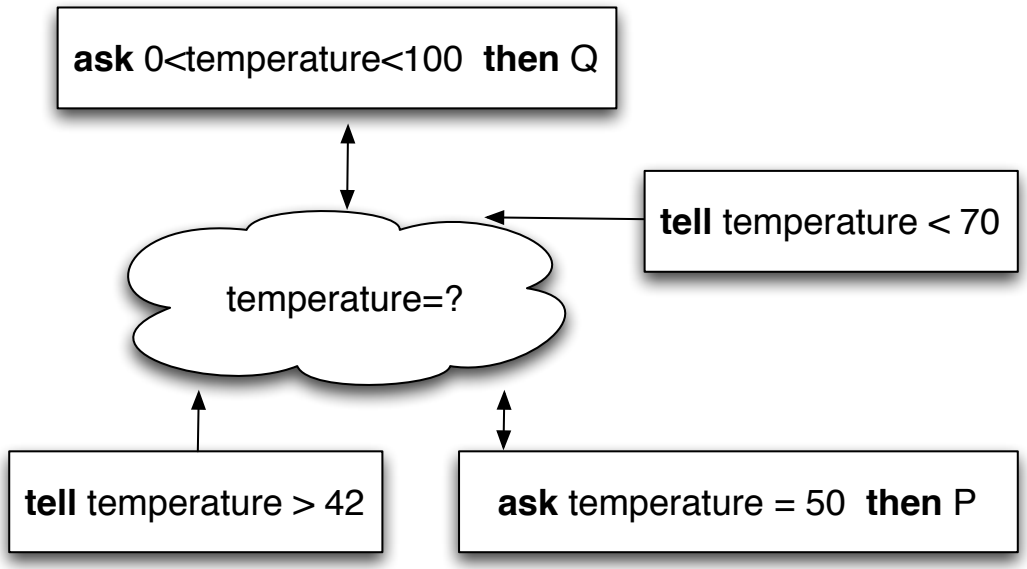

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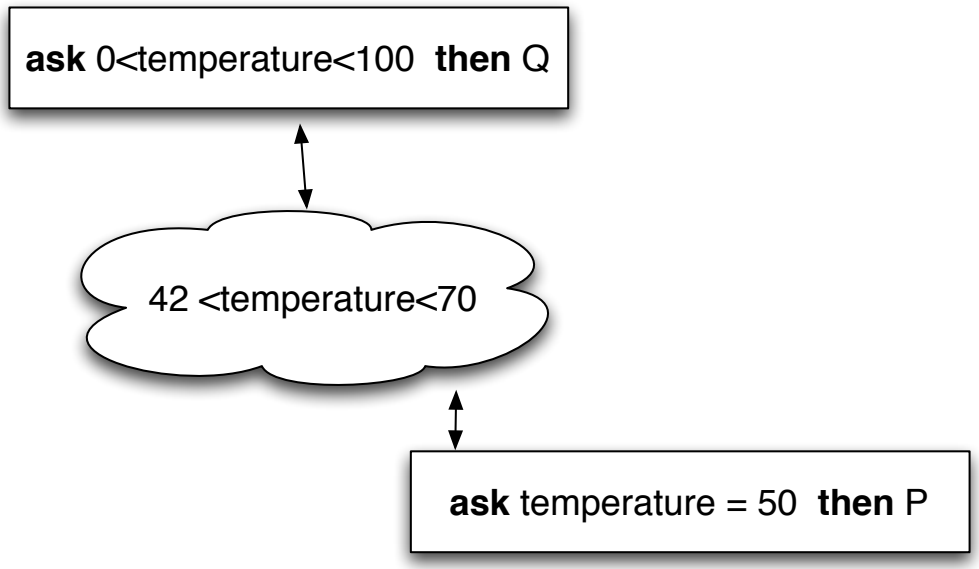

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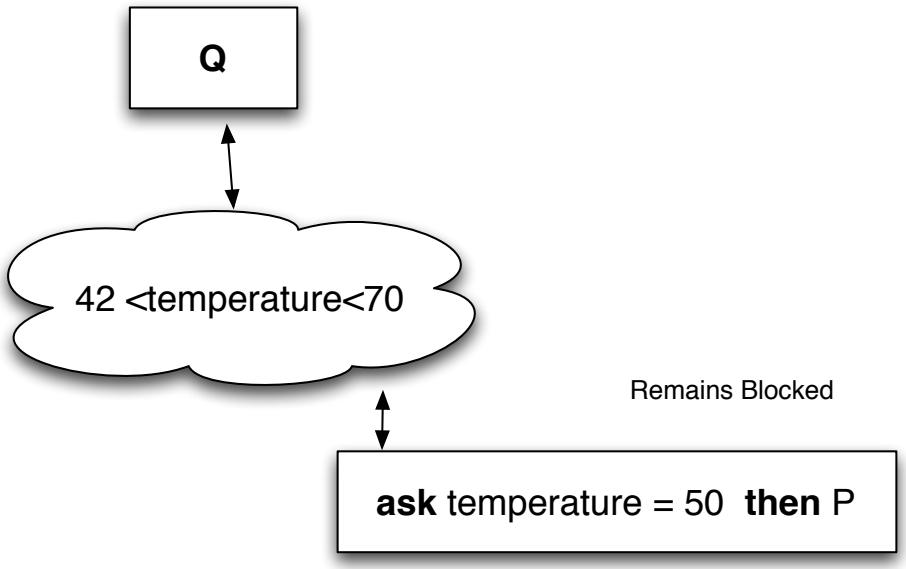

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- We aim at developing a theory for a CCP-based model for the specification of mobile reactive systems where logic and behavioral approaches coexist coherently.

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Criteria:

- Declarative : allowing for rechability analysis using deduction in logic.
- Determinism : which is the source of CCP's elegant and simple semantic characterizations.
- Applications in emergent application areas.

Our Approach

Mobility in CCP

Local variables define boundaries of interaction . How can we change the communication structure of the processes?

- Variables as communication channels [Sar93].
  – channels can be used only once.
- Atomic CCP with pattern matching [LM92].
  – Atomic CCP is non-deterministic.
- Adding linear parametric asks (LCC [FRS01, SL92]).
  – Non-determinism is introduced.
- Adding persistent parametric asks.
  –Not all the inputs must be persistent.

Our approach : A CCP-based calculus with temporary parametric asks : The Universal Timed CCP calculus (`utcc`).

Our Contributions

Reasoning Techniques for `utcc`:

- A novel symbolic semantics based on temporal formulae.
- Interpretation of `utcc` processes as formulae in Pnueli's FLTL .
- A denotational semantics based on closure operators .
- Abstract semantics as basis for the static analysis of `utcc` programs.

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- We prove the undecidability of the Monadic fragment of Pnueli's First-Order LTL.

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- We prove the undecidability of the Monadic fragment of Pnueli's First-Order LTL.

Applications:

- Closure operator semantics for languages for security .
- Declarative interpretation and temporal extensions for sessions .
- Modeling of multimedia interactive systems.

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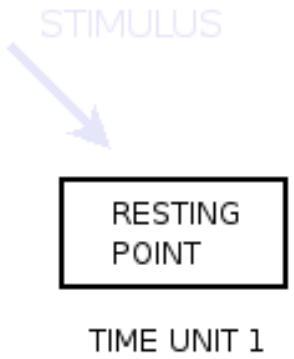

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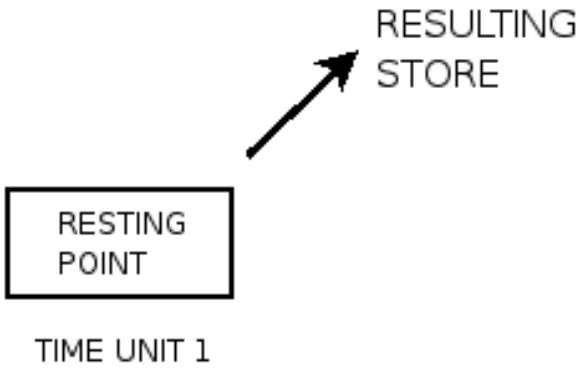

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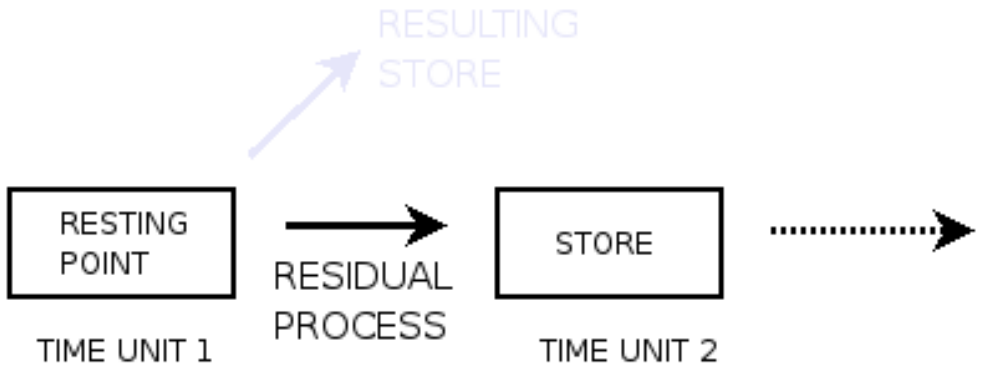

1. Receives a stimulus (i.e a constraint) from the environment.
2. Computes a CCP process in the current time-unit and wait for stability.
3. Responds with the resulting store.
4. Executes the Residual process in the *next* time-unit.

* **Note**: Stores are not automatically transferred from a time unit to the next one.

The tcc calculus [SJG94]

Syntax

$$P, Q := \textbf{skip} \mid \textbf{tell}(c) \mid \textbf{when } c \textbf{ do } P \mid P \parallel Q \mid (\textbf{local } \vec{x}; c)\, P \mid \textbf{next } P \mid \textbf{unless } c \textbf{ next } P \mid !\, P$$

- $\textbf{tell}(c)$: adds c to the store in the current time interval.
- $\textbf{when } c \textbf{ do } P$: executes P if c can be deduced from the current store.
- $(\textbf{local } \vec{x})\, P$: behaves like P but the information about variables in $\vec{x}$ is local to P
- $\textbf{next } P$: executes P in the next time unit.
- $\textbf{unless } c \textbf{ next } P$: executes P in the next time unit if c cannot be entailed now.
- $!\, P$: Unboundedly many copies of P, one at a time.

Abstractions and the utcc Calculus

Example

Let $Q = \textbf{tell}(\mathtt{out}(42))$ and $P = \textbf{when}\ \ \mathtt{out}(x)\ \textbf{do}\ R$.

- x is not a place holder in P
- $\mathtt{out}(42) \not\models \mathtt{out}(x)$. Then $P \parallel Q \not\longrightarrow$.

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The idea of Abstractions in utcc

The abstraction construct $S = (\textbf{abs}\ \vec{x}; c)\, R$:

- S can be seen as a λ-abstraction of R on $\vec{x}$ with guard c.
- S is a parametric ask that executes $R[\vec{x}/\vec{t}]$ for each $\vec{t}$ s.t. $c[\vec{x}/\vec{t}]$ can be deduced from the current store. E.g., $S \parallel Q \longrightarrow R[42/x]$.
- The variables in $\vec{x}$ can be seen as the formal parameters of R.
- Logical point of view: S corresponds to a formula $\forall\vec{x}.(c \Rightarrow F)$.

Operational Semantics

Internal transitions ($\longrightarrow$)

$$\mathrm{R}_{\mathrm{TELL}}\ \frac{}{\langle \mathbf{tell}(c), d\rangle \longrightarrow \langle \mathbf{skip}, d \wedge c\rangle}$$

$$\mathrm{R}_{\mathrm{PAR}}\ \frac{\langle P, c\rangle \longrightarrow \langle P', d\rangle}{\langle P \parallel Q, c\rangle \longrightarrow \langle P' \parallel Q, d\rangle}$$

$$\mathrm{R}_{\mathrm{ABS}}\ \frac{d \models_{\Delta} c[\vec{t}/\vec{x}] \qquad [\vec{t}/\vec{x}] \text{ is admissible.}}{\langle (\mathbf{abs}\ \vec{x}; c)\, P, d\rangle \longrightarrow \left\langle P[\vec{t}/\vec{x}] \parallel (\mathbf{abs}\ \vec{x}; c \wedge \vec{x} \neq \vec{t}\ )\, P, d\right\rangle}$$

Observable transitions ($\Longrightarrow$)

$$\mathrm{R}_{\mathrm{OBS}}\ \frac{\langle P, c\rangle \longrightarrow^{*} \langle Q, d\rangle \not\longrightarrow}{P \overset{(c,d)}{\Longrightarrow} F(Q)}$$

$$\begin{cases} F((\mathbf{abs}\ \vec{x}; c)\, Q) = \mathbf{skip} \\ F(\mathbf{next}\, Q) = Q \\ F(\mathbf{unless}\ c\ \mathbf{next}\, Q) = Q \end{cases}$$

▸ C. System ▸ SOS Details

Input-output Behavior

- $P \xrightarrow{(c,c')} Q$: P under input c outputs c' and executes Q in the next time-unit.
- Similarly, $P \xrightarrow{(\alpha,\alpha')}$, or $(\alpha, \alpha') \in io(P)$, whenever $\alpha = c_1.c_2...$, $\alpha' = c'_1.c'_2....$ and $P = P_1 \xrightarrow{(c_1,c'_1)} P_2 \xrightarrow{(c_2,c'_2)} ...P_i \xrightarrow{(c_i,c'_i)} ....$

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Theorem (Determinism)

Let α, β and β' be sequences of constraints. If both (α, β), $(\alpha, \beta') \in io(P)$ then for all $i > 0$, $\beta(i) \equiv \beta'(i)$.

Some Examples

Communication using global channels

$$Alice \xrightarrow{\{m\}_B} Bob$$

$$\begin{array}{lcl} A & = & (\mathbf{local}\, m)\,(\mathbf{tell}(\mathtt{out}(\{m\}_B))) \\ B & = & (\mathbf{abs}\; x; \mathtt{out}(\{x\}_B))\, B' \end{array}$$

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•	$(\textbf{local}\, m; \mathtt{out}(\{m\}_B))\,(B'[m/x] \parallel B'')$	$\exists m.(\mathtt{out}(\{m\}_B))$

where $B'' = (\textbf{abs}\; x; \mathtt{out}(\{x\}_B \wedge x \neq m))\, B'$.

Some Examples

Communication of Local Names

$$\begin{array}{lcl} P_\pi & = & (\nu b)(\overline{a}b.b(y).R_\pi) \\ Q_\pi & = & a(x).(\nu c)(\overline{x}c) \end{array}$$

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Some Examples

Encoding Recursive Definitions

The variables $\vec{x}$ in $(\textbf{abs}\ \vec{x}; c)\, P$ can be seen as the formal parameters of P.

- Proc. definitions: $\ulcorner p(\vec{y}) \stackrel{\texttt{def}}{=} P \urcorner = !\,(\textbf{abs}\ \vec{y}; \mathit{call}_p(\vec{y}))\, \widehat{P}$
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A simple Example

- Definition:

$$!\,(\textbf{abs}\ N, M, X; \mathit{fact}(N, M, X)) \quad \begin{array}{l} \textbf{when}\ N \leq 1\ \textbf{do}\ \textbf{tell}(X = M) \parallel \\ \textbf{when}\ N > 1\ \textbf{do}\ \textbf{tell}(\mathit{fact}(N - 1, N \times M, X)) \end{array}$$

- Call:

$$\langle \textbf{tell}(\mathit{fact}(3, 1, X)), \texttt{true} \rangle \longrightarrow^* \langle P, X = 6 \rangle$$

Infinite Behavior

No observable transition!!!

The **abs** construct may introduce infinitely many internal reductions:

- Loops : $(\mathbf{abs}\ x; c(x))\,\mathbf{tell}(c(x+1))$
- Infinitely many substitutions : $P = (\mathbf{abs}\ x; x > 1)\,R$.

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Example (Message Composition)

$$P \quad = \quad (\textbf{abs}\ x, y; \mathtt{out}(x) \wedge \mathtt{out}(y))\,\textbf{tell}(\mathtt{out}(\{x, y\}))$$

What do we observe from $P \parallel \textbf{tell}(\mathtt{out}(m))$? :
$\mathtt{out}(\{m, m\}), \mathtt{out}(\{m, \{m, m\}\}), \mathtt{out}(\{m, \{m, \{m, m\}\}\}) \ldots$

Outline

Symbolic Semantics for utcc

$$P = \quad (\textbf{abs}\ x, y; \mathtt{out}(x) \wedge \mathtt{out}(y))\, \textbf{tell}(\mathtt{out}(\{x, y\}))$$

Symbolically

$\langle P, \mathtt{out}(m)\rangle \longrightarrow_s$

$\langle \textbf{skip}, \mathtt{out}(m) \wedge \forall x, y : (\mathtt{out}(x) \wedge \mathtt{out}(y) \Rightarrow \mathtt{out}(\{x, y\}))\rangle$

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Temporal Dependencies

$$P = (\textbf{abs}\ x, y; \mathtt{out}(x) \wedge \mathtt{out}(y))\ \textbf{next}\ \textbf{tell}(\mathtt{out}(\{x, y\}))$$

$\langle P, \mathtt{out}(m)\rangle \not\longrightarrow_s$
$\Rightarrow_s \langle \textbf{skip}, \ominus\, \mathtt{out}(m) \wedge \forall x, y : (\ominus(\mathtt{out}(x) \wedge \mathtt{out}(y)) \Rightarrow \mathtt{out}(\{x, y\}))\rangle$

Symbolic Semantics

Symbolic Reductions

$$\mathrm{R}_{\mathrm{ABS-SYM}}\ \frac{\langle P, \exists \vec{x} d\rangle \longrightarrow_s \langle P', \exists \vec{x} d \wedge d'\rangle}{\langle (\mathbf{abs}\ \vec{x}; c)\, P, d\rangle \longrightarrow_s \langle (\mathbf{abs}\ \vec{x}; c)\, P', d \wedge \forall \vec{x}(c \Rightarrow d')\rangle}$$

$$\mathrm{R}_{\mathrm{OBS-SYM}}\ \frac{\langle P, c\rangle \longrightarrow_s^* \langle Q, d\rangle \not\longrightarrow_s}{P \stackrel{(c,d)}{\Longrightarrow} F_s(Q, d)}$$

Symbolic Future Function : $F_s(P, d) = \mathbf{tell}(\ominus d) \parallel F'_s(P)$

$$F'_s(P) \quad = \quad \begin{cases} (\mathbf{abs}\ \vec{x}; \ominus c)\, F_s(P) & \text{if } P = (\mathbf{abs}\ \vec{x}; c)\, P \\ (\mathbf{local}\ \vec{x}; \ominus c)\, F_s(Q) & \text{if } P = (\mathbf{local}\ \vec{x}; c)\, Q \end{cases}$$

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Theorem (Semantic Correspondence)

Let P be an abstracted-unless free process. The symbolic and the operational outputs of P entail the same basic constraints . ▸ Details

FLTL Correspondence (Declarative view of Processes)

Definition (FLTL Syntax)

$$F, G, \ldots := c \mid F \wedge G \mid \neg F \mid \exists x F \mid \ominus F \mid \circ F \mid \Box F$$

c is a constraint in $\mathcal{L}$. $\Diamond F = \neg\Box\neg F$ (eventually F). ▸ LTL Sem.

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Definition

FLTL Interpretation of `utccProcesses`

$$\begin{array}{lcllcl}
[\![\textbf{skip}]\!] & = & \texttt{true} & [\![\textbf{tell}(c)]\!] & = & c \\
[\![(\textbf{abs}\ \vec{y}; c)\ P]\!] & = & \forall\vec{y}(c \Rightarrow [\![P]\!]) & [\![P \parallel Q]\!] & = & [\![P]\!] \wedge [\![Q]\!] \\
[\![(\textbf{local}\ \vec{x}; c)\ P]\!] & = & \exists\vec{x}(c \wedge [\![P]\!]) & [\![\textbf{next}\ P]\!] & = & \circ[\![P]\!] \\
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Theorem (Logic Correspondence)

If P is a monotonic process, $[\![P]\!] \models_T \Diamond c$ iff $P \Downarrow_s^c$.

Outline

Undecidability of Monadic FLTL

Theorem (utcc is Turing powerful)

The Minsky machine $M(0,0)$ *halts iff* $[\![M(0,0)]\!] \Downarrow^{\texttt{halt}}$ ▸ Encoding

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Monadic FLTL is Incomplete

Let F be the FLTL formulae corresponding to $[\![M(0,0)]\!]$:

- F is a monadic FLTL formula without functions nor equality.
- Let $G = (F \implies \Box\,\texttt{running})$. G is valid iff M never halts .

Undecidability of Monadic FLTL

Theorem (utcc is Turing powerful)

The Minsky machine $M(0,0)$ *halts iff* $[\![M(0,0)]\!] \Downarrow^{\texttt{halt}}$ ▸ Encoding

Monadic FLTL is Incomplete

Let F be the FLTL formulae corresponding to $[\![M(0,0)]\!]$:

- F is a monadic FLTL formula without functions nor equality.
- Let $G = (F \implies \Box\,\texttt{running})$. G is valid iff M never halts .

Theorem (Incompleteness of Pnueli's FLTL)

Monadic FLTL without function symbols nor equality is incomplete .

Undecidability of Monadic FLTL

Our result in Context

1. [Mer92] proved the Monadic fragment of FLTL to be decidable!!.
   - Quantification of flexible variables was not allowed in [Mer92].
2. [Val05] conjectures that the negation-free restriction can be drop and still obtain decidability fragments of FLTL.
   - With negation the FLTL in [Val05] is the same studied here.

Our result shows (1) and (2) to be necessary to obtain decidability.

Outline

Strongest Postcondition

Symbolic Input-output Relation

- Input-output Relation : (w, v) s.t. $P \xrightarrow{(w,v)}_s$
- Strongest Postcondition : $w \in sp_s(P)$ if P cannot add any information to w. ▸ Details.
- if P is monotonic, $io_s(P)$ is a closure operator , i.e, a function satisfying extensiveness , idempotence and monotonicity whose set of fixed points is $sp_s(P)$:

$$(w, w') \in io_s(P) \text{ iff } w' = min(sp_s(P) \cap \{w \mid s \leq w\})$$

Denotational Semantics

Compositional Characterization of the $sp_s(\cdot)$ relation.

$$\begin{array}{lll}
\text{D}_{\text{TELL}} & [\![\textbf{tell}(c)]\!] & = \{e.w \mid e \models_T c\} \\
\text{D}_{\text{PAR}} & [\![P \parallel Q]\!] & = [\![P]\!] \cap [\![Q]\!] \\
\text{D}_{\text{LOC}} & [\![(\textbf{local}\, \vec{x}; c)\, P]\!] & = \{w \mid \text{there exists an } \vec{x}\text{-variant} \\
 & & \quad w' \text{ of } w \text{ s.t } w'(1) \models_T c \text{ and } w' \in [\![P]\!]\} \\
\text{D}_{\text{ABS}} & [\![(\textbf{abs}\ \vec{x}; c)\, P]\!] & = \{w \mid \text{for every } \vec{x}\text{-variant } w' \text{ of } w \text{ if } w'(1) \models_T c \\
 & & \quad \text{and } w' \succeq (\vec{x} = \vec{t})^{\omega} \text{ for some admissible } \vec{t} \text{ then } w' \in [\![P]\!]\}
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\end{array}
$$

Theorem (Full Abstraction)

Let P, Q be monotonic processes. It holds:

- $P \approx_s^{io} Q$ *iff* $[\![P]\!] = [\![Q]\!]$.

Outline

Closure operator semantics for Sec. Languages.

SCCP Syntax

$$
\begin{array}{llll}
\text{Values} & v, v' & ::= & n \mid x \\
\text{Keys} & k & ::= & pub(v) \mid priv(v) \\
\text{Messages} & M, N & ::= & v \mid k \mid X \mid \{M, N\} \mid \{M\}_k \\
\text{Processes} & R & ::= & \mathbf{nil} \\
 & & & \mid \mathbf{new}(x)R \\
 & & & \mid \mathbf{out}(M).R \\
 & & & \mid \mathbf{in}\ (\vec{x})[M].R \\
 & & & \mid !R \\
 & & & \mid R_i \parallel R_j
\end{array}
$$

Closure operator semantics for Sec. Languages.

SCCP Syntax

Values	v, v'	$::=$	$n \mid x$	
Keys	k	$::=$	$pub(v) \mid priv(v)$	
Messages	M, N	$::=$	$v \mid k \mid X \mid \{M, N\} \mid \{M\}_k$	
Processes	R	$::=$	$\mathbf{nil}$	$\Longrightarrow \mathbf{skip}$
			$\mid \mathbf{new}(x)R$	$\Longrightarrow (\mathbf{local}\, x)\, [\![R]\!]$
			$\mid \mathbf{out}(M).R$	$\Longrightarrow ! \, \mathbf{tell}(\mathtt{out}\,(M)) \parallel \mathbf{next}\, [\![R]\!]$
			$\mid \mathbf{in}\ (\vec{x})[M].R$	$\Longrightarrow (\mathbf{abs}\ \vec{x}; \mathtt{out}(M))\, \mathbf{next}\, [\![R]\!]$
			$\mid !R$	$\Longrightarrow ! \, [\![R]\!]$
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 & & & \mid \mathbf{out}(M).R \Longrightarrow !\, \mathbf{tell}(\mathtt{out}\,(M)) \| \mathbf{next}\, [\![R]\!] \\
 & & & \mid \mathbf{in}\ (\vec{x})[M].R \Longrightarrow (\mathbf{abs}\ \vec{x}; \mathtt{out}(M))\, \mathbf{next}\, [\![R]\!] \\
 & & & \mid !R \Longrightarrow !\, [\![R]\!] \\
 & & & \mid R_i \parallel R_j \Longrightarrow [\![R_i]\!] \parallel [\![R_j]\!]
\end{array}
$$

Security Constraint System

$$PRJ\ \frac{F \models \mathtt{out}(\{m_1, m_2\})}{F \models \mathtt{out}(m_{1/2})} \qquad DEC\ \frac{F \models \mathtt{out}(k^{-1}) \quad F \models \mathtt{out}(\{m\}_k)}{F \models \mathtt{out}(m)}$$

$$ENC\ \frac{F \models \mathtt{out}(m_1) \quad F \models \mathtt{out}(m_2)}{F \models \mathtt{out}(\{m_1\}_{m_2})} \qquad PAIR\ \frac{F \models \mathtt{out}(m_1) \quad F \models \mathtt{out}(m_2)}{F \models \mathtt{out}(\{m_1, m_2\})}$$

An Example

Denning-Sacco key distribution protocol:

$$\begin{array}{lll} msg_1 & A \rightarrow B & : \quad \{(A, m)\}_{pub(B)} \\ msg_2 & B \rightarrow A & : \quad \{n\}_{pub(m)} \end{array}$$

$$\begin{array}{lcl} Init(A, B) & = & !\,\mathbf{new}(m)\mathbf{out}(\{(m, A)\}_{pub(B)}).\mathbf{nil} \\ Resp(B) & = & !\,\mathbf{in}\ (x, u)[\{(x, u)\}_{priv(B)}]. \\ & & \quad \mathbf{new}(n)(\mathbf{out}(\{n\}_{pub(u)}).\mathbf{nil}) \parallel !\,\mathbf{in}\ [n].\mathbf{out}(attack).\mathbf{nil} \end{array}$$

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Proposition

Let R be a SCCP process, P the utcc process representing R and $f = [\![P]\!] \cap [\![!\,\mathbf{when}\ \mathtt{out}(attack)\ \mathbf{do}\ !\,\mathbf{tell}(\mathtt{false})]\!]$.

- *Symbolic Output : $P \Downarrow_s^{attack}$ iff*
- *Closure Operator Semantics : All the fixed points of f take the form $w.\mathtt{false}^\omega$ iff*
- *FLTL Characterization : $\mathrm{TL}[\![P]\!] \models_T \Diamond attack$.*

Language for Structured Communication (sessions)

Definition (The HVK language (Honda 98))

P,Q	::=	**request** $a(k)$ **in** P	Session Request
	|	**accept** $a(x)$ **in** P	Session Acceptance
	|	$k![\vec{e}]$; P	Data Sending
	|	$k?(x)$ **in** P	Data Reception
	|	$k \lhd l$;P	Label Selection
	|	$k \rhd \{l_1 : P_1 \parallel \cdots \parallel l_n : P_n\}$	Label Branching

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Further guarantees are needed when dealing with sessions, e.g.:

- Sessions should be of finite time.
- One should be able to cancel a session.

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Further guarantees are needed when dealing with sessions, e.g.:

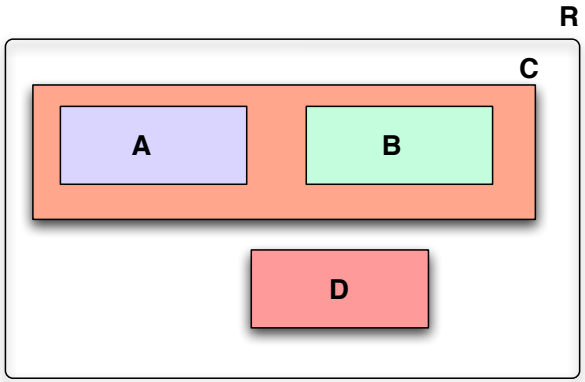
- Sessions should be of finite time.
- One should be able to cancel a session.

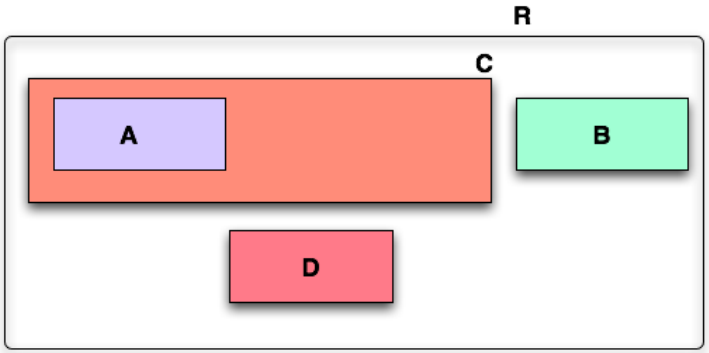
HVK-T

HVK + session cancellation and constraint-guarded **accepts**.

P	::=	**request** $a(k)$ **during** m **in** P	Timed Session Request
	|	**accept** $a(k)$ **given** c **in** P	Declarative Session Acceptance
	|	$\cdots$	
	|	**kill** c_k	Session Abortion

Dynamic Interactive Scores





Dynamic Reconfiguration when Interacting:

- Moving boxes.
- Adding/deleting intervals.

Verification : Minimal conditions to avoid raise conditions.

Outline

Abstract Interpretation of utcc programs

- The behavior of a program P can be approximated by computing $[\![P]\!]^{\tau}_{\alpha}$, a compact representation of $[\![P]\!]$.

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Two steps:

Abstracting the constraint system $(\mathcal{C}, \alpha, \mathcal{A})$

- To reuse previously defined abstract domains for logic programming (e.g. groundness analysis, types, etc).
- To bound the behavior of the operator (**abs** $\vec{x}$; c) P.

Abstract Interpretation of utcc programs

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- To reuse previously defined abstract domains for logic programming (e.g. groundness analysis, types, etc).
- To bound the behavior of the operator (**abs** $\vec{x}$; c) P.

Abstracting the sequences (τ)

- To obtain a finite cut approximating the infinite sequences of the concrete semantics

Applications

- Groundness Analysis of `utcc` programs. We reuse two abstract domains from logic programming:
  - Pos [AMSS98]: Positive propositional formulae to represent groundness dependencies. E.g., $\alpha_g(x = [y|z]) = \text{iff}(x, \{y, z\})$
  - Type dependencies [CSS99]: e.g., $\alpha(x = [a|y]) = \mathit{list}(x, y)$
- Secrecy Analysis : Depth-κ cut to approximate the behavior of the protocol:

$$\mathit{cut}_\kappa(m) = \begin{cases} m & \text{if } \mathit{length}(m) \leq \kappa \\ m_\top & \text{otherwise} \end{cases}$$

Outline

Concluding Remarks

- We proposed `utcc`, a declarative model for the specification of mobile reactive systems.
- Reasoning techniques for `utcc`:
  - Operational and symbolic semantics.
  - Semantics based on closure operators.
  - Declarative interpretation of processes as formulae in FLTL.
  - Abstract semantics for the static analysis of `utcc` programs.
- We showed the applicability of `utcc` in several domains:
  - Decidability of Pnueli's FLTL.
  - Analysis of security protocols.
  - Declarative interpretation of sessions.
  - Modeling of Multimedia Interactive Systems.

Concluding Remarks

Publications from this dissertation

- C. Olarte and F. Valencia. *Universal Concurrent Constraint Programing: Symbolic Semantics and Applications to Security*. In SAC'08. ACM Press, 2008.
- C. Olarte and F. Valencia. *The Expressivity of Universal Timed CCP: Undecidability of Monadic FLTL and Closure Operators for Security*. In PPDP'08. ACM Press. 2008.
- M. Falaschi, C. Olarte, C. Palamidessi and F. Valencia. *Declarative Diagnosis of Temporal Concurrent Constraint Programs*. In ICLP'07. Springer, 2007.
- M. Falaschi, C. Olarte and C. Palamidessi. *A Framework for Abstract Interpretation of Timed Concurrent Constraint Programs*. In PPDP'09. ACM Press. 2009.
- C. Olarte and C. Rueda. *A declarative language for dynamic multimedia interaction systems*. In MCM'09. Springer, 2009.
- H. A. López, C. Olarte, and J. A. Pérez. *Towards a Unified Framework for Declarative Structured Communications*. In Proc. of PLACES'09.
- C. Olarte and F. Valencia. Undecidability of Monadic First-Order Linear-Time Temporal Logic. Submitted to Studia Logica.

Concluding Remarks

Future Work

- Non-deterministic / probabilistic choices for modeling purposes.
- Proof system in the lines of [NPV02].
- Studying the minimal number of global variables required to obtain undecidability in Monadic FLTL [DFL02].
- Abstract debugging of `utcc` programs.
- Analysis of properties related to mobile systems (e.g. [Fer05]).
- Automatic verification of `tcc` and `utcc`.

Thank you!


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Approaches to mobility in CCP

1. [Sar93] Suppose a injective function f. Let $P = \exists_z \textbf{tell}(x = f(z))$ and $Q = \exists_y(\textbf{ask } x = f(y) \textbf{ then tell}(x = f(y)) \parallel R)$.
   Drawbacks: If two names are sent on x those names must be equals (otherwise an inconsistency arises).
2. [LM92] Let
   $P = \exists z \textbf{tell}(x =< 0, z >)$ and
   $Q = \exists_{y,y'}(\textbf{ask } x =< y, y' > \textbf{ then tell}(x =< y, y' >) \rightarrow R)$.
   Here the atomic tell avoids inconsistencies when two messages are sent on x.
   Drawbacks: Atomic tells introduce non-determinism loosing the strong connection CCP has with logic.

Operational Semantics

Constraint System

- A constraint system is a tuple $\langle \sum, \Delta \rangle$ where $\sum$ is a signature and Δ a consistent first-order theory over $\sum$
- Constraints are first-order formulae over $\sum$
- Entailment relation $c \models d$ holds iff $c \Rightarrow d$ is valid on Δ. $c \equiv d$ iff $c \models d$ and $d \models c$
- $\mathcal{C}$ denotes the set of constraints modulo $\equiv$ in $\langle \sum, \Delta \rangle$

Operational Semantics

Structural Congruence

Structural Congruence

Let $\equiv$ be the smallest congruence satisfying:

1. $P \equiv Q$ if they differ only by a renaming of bound variables (alpha-conversion).
2. $P \parallel \mathbf{skip} \equiv P$
3. $P \parallel Q \equiv Q \parallel P$
4. $P \parallel (Q \parallel R) \equiv (P \parallel Q) \parallel R$
5. $P \parallel (\mathbf{local}\, \vec{x}; c)\, Q \equiv (\mathbf{local}\, \vec{x}; c)\, (P \parallel Q)$ if $\vec{x} \notin fv(P)$ (Scope Extrusion)
6. $(\mathbf{local}\, \vec{x}; c)\, (\mathbf{local}\, \vec{y}; d)\, P \equiv (\mathbf{local}\, \vec{x}; \vec{y}\ ; c \wedge d)\, P \quad$ if $\vec{x} \cap \vec{y} = \emptyset$ and $\vec{y} \notin fv(c)$.

Operational Semantics

Internal Reductions

$$\mathrm{R}_{\mathrm{TELL}}\ \frac{}{\langle \mathbf{tell}(c), d\rangle \longrightarrow \langle \mathbf{skip}, d \wedge c\rangle}$$

$$\mathrm{R}_{\mathrm{PAR}}\ \frac{\langle P, c\rangle \longrightarrow \langle P', d\rangle}{\langle P \parallel Q, c\rangle \longrightarrow \langle P' \parallel Q, d\rangle}$$

$$\mathrm{R}_{\mathrm{LOC}}\ \frac{\langle P, c \wedge (\exists \vec{x} d)\rangle \longrightarrow \langle P', c' \wedge (\exists \vec{x} d)\rangle}{\langle (\mathbf{local}\, \vec{x}; c)\, P, d\rangle \longrightarrow \langle (\mathbf{local}\, \vec{x}; c')\, P', d \wedge \exists \vec{x} c'\rangle}$$

$$\mathrm{R}_{\mathrm{UNL}}\ \frac{d \models_\Delta c}{\langle \mathbf{unless}\ c\ \mathbf{next}\ P, d\rangle \longrightarrow \langle \mathbf{skip}, d\rangle}$$

$$\mathrm{R}_{\mathrm{REP}}\ \frac{}{\langle !\, P, d\rangle \longrightarrow \langle P \parallel \mathbf{next}\, !\, P, d\rangle}$$

$$\mathrm{R}_{\mathrm{ABS}}\ \frac{d \models_\Delta c[\vec{t}/\vec{x}] \qquad [\vec{t}/\vec{x}] \text{ is admissible.}}{\langle (\mathbf{abs}\ \vec{x}; c)\, P, d\rangle \longrightarrow \langle P[\vec{t}/\vec{x}] \parallel (\mathbf{abs}\ \vec{x}; c \wedge \vec{x} \neq \vec{t}\ )\, P, d\rangle}$$

$$\mathrm{R}_{\mathrm{STR}}\ \frac{\gamma_1 \longrightarrow \gamma_2}{\gamma_1' \longrightarrow \gamma_2'}\ \text{if } \gamma_1 \equiv \gamma_1' \text{ and } \gamma_2 \equiv \gamma_2'$$

Operational Semantics

Observable Transition

$$\mathrm{R}_{\mathrm{OBS}} \ \frac{\langle P, c\rangle \longrightarrow^* \langle Q, d\rangle \not\longrightarrow}{P \stackrel{(c,d)}{\Longrightarrow} F(Q)}$$

Future Function

Let F be a partial function defined as:

$$F(P) = \begin{cases} \textbf{skip} & \text{if } P = \textbf{skip} \\ \textbf{skip} & \text{if } P = (\textbf{abs}\ \vec{x}; c)\ Q \\ F(P_1) \parallel F(P_2) & \text{if } P = P_1 \parallel P_2 \\ (\textbf{local}\ \vec{x})\ F(Q) & \text{if } P = (\textbf{local}\ \vec{x}; c)\ Q \\ Q & \text{if } P = \textbf{next}\ Q \\ Q & \text{if } P = \textbf{unless}\ c\ \textbf{next}\ Q \end{cases}$$

Theorem (Semantic Correspondence)

Let P be an abstracted-unless free process. Suppose that $P \xrightarrow{(c_1,d_1)} P_1 \xrightarrow{(c_2,d_2)} \ldots \xrightarrow{(c_i,d_i)} P_i$ *and* $P \xrightarrow{(c_1,e_1)}_s P_1{}' \xrightarrow{(c_2,e_2)}_s \ldots \xrightarrow{(c_i,e_i)} P_i'$. *Then for every $c \in \mathcal{C}$ and $j \in \{1,\ldots,i\}$, $d_i \models c$ iff $e_i \models_T c$.*

Expressiveness of utcc

Minsky Machines

Instructions using two counters c_0 and c_1:

$$
\begin{array}{lcl}
L_i & : & \texttt{HALT} \\
L_i & : & c_n := c_n + 1; \texttt{goto } L_j \\
L_i & : & \texttt{if } c_n = 0 \texttt{ then goto } L_j \texttt{ else } c_n := c_n - 1; \texttt{ goto } L_k
\end{array}
$$

The machine M:

1. Starts at instruction L_1 with $c_0 = c_1 = 0$.
2. Halts if the control reaches the location of a halt instruction.

Encoding of Minsky Machines into utcc

$$\texttt{ZERO}_n \stackrel{\text{def}}{=} \textbf{when } \mathit{inc}_n \textbf{ do next } (\textbf{local } a)\,(\texttt{NOT-ZERO}_n(a) \parallel !\,\textbf{when } \texttt{out}(a) \textbf{ do } \texttt{ZERO}_n) \parallel \textbf{when } \mathit{idle}_n \textbf{ do next } \texttt{ZERO}_n \parallel \textbf{tell}(\mathit{isz}_n)$$

$$\texttt{NOT-ZERO}_n(x) \stackrel{\text{def}}{=} \textbf{when } \mathit{inc}_n \textbf{ do next } (\textbf{local } b)\,(\texttt{NOT-ZERO}_n(b) \parallel !\,\textbf{when } \texttt{out}(b) \textbf{ do } \texttt{NOT-ZERO}_n(x)) \parallel \textbf{when } \mathit{dec}_n \textbf{ do next tell}(\texttt{out}(x)) \parallel \textbf{when } \mathit{idle}_n \textbf{ do next } \texttt{NOT-ZERO}_n(x) \parallel \textbf{tell}(\mathit{not\text{-}zero}_n)$$

$$\begin{aligned}
\mathit{ins}(l_i, \texttt{HALT}) &= \textbf{tell}(\texttt{halt}) \parallel \textbf{next tell}(\texttt{out}(l_i)) \parallel \textbf{tell}(\mathit{idle}_0 \wedge \mathit{idle}_1) \\
\mathit{ins}(l_i, \texttt{INC}(c_n, l_j)) &= \textbf{tell}(\mathit{inc}_n) \parallel \textbf{next tell}(\texttt{out}(l_j)) \parallel \textbf{tell}(\mathit{idle}_{1-n}) \\
\mathit{ins}(l_i, \texttt{DECJ}(c_n, l_j, l_k)) &= \textbf{when } \mathit{isz}_n \textbf{ do } (\textbf{next tell}(\texttt{out}(l_j)) \parallel \textbf{tell}(\mathit{idle}_n)) \parallel \\
&\quad \textbf{when } \mathit{not\text{-}zero}_n \textbf{ do } (\textbf{tell}(\mathit{dec}_n) \parallel \textbf{next tell}(\texttt{out}(l_k))) \parallel \\
&\quad \textbf{tell}(\mathit{idle}_{1-n})
\end{aligned}$$

Strongest Postcondition and Fixed Formulae

Strongest Postcondition

The Strongest Postcondition of P, denoted by $sp_s(P)$, is defined as the set $\{w \mid P \xrightarrow{(w,v)}_s \text{ and } w \in Fix(v)\}$

Definition (Fixed Formulae)

Let $n \geq 0$ and $Fix : FF \rightharpoonup \mathcal{P}(FF)$ be defined as

$$
\begin{array}{lcl}
Fix(c) & = & \{F \in FF \mid F \models_T c\} \\
Fix(F_1 \wedge F_2) & = & \{F \in FF \mid F \in Fix(F_1) \text{ and } F \in Fix(F_2)\} \\
Fix(\forall_{\vec{x}} \ominus^n (c) \Rightarrow F_1) & = & \{F \in FF \mid \text{ for all } \vec{x}\text{-variant } F' \text{ of } F, \text{ if } F' \models_T (\ominus^n(c) \wedge \vec{x} = \vec{t}) \\
 & & \quad \text{for some } \vec{t} \in \mathcal{T} \text{ s.t. } adm(\vec{x}, \vec{t}) \text{ then } F' \in Fix(F_1) \} \\
Fix(\exists_{\vec{x}} F_1) & = & \{F \in FF \mid \text{ there exists an } \vec{x}\text{-variant } F' \text{ of } F \text{ s.t. } F' \in Fix(F_1)\} \\
Fix(\ominus F_1) & = & \{F \in FF \mid F = \ominus F' \text{ and } F' \in Fix(F_1)\}
\end{array}
$$

Given the future-free formulae F and G, if $F \in Fix(G)$ we say that F is a fixed formula for G.

Constraint Systems.

A Constraint System defines the basic constraints agents can tell or ask .
A cylindric c.s. is a structure $\mathbf{C} = \langle \mathcal{C}, \leq, \sqcup, \mathtt{true}, \mathtt{false}, \mathit{Var}, \exists, d \rangle$ s.t.:

- $\langle \mathcal{C}, \leq, \sqcup, \mathtt{true}, \mathtt{false} \rangle$ is a lattice.
- Var is a denumerable set of variables.
- $\exists_x : \mathcal{C} \rightarrow \mathcal{C}$ is a cylindrification operator helpful to define the local (hiding) operator.
- For each $x, y \in \mathit{Var}$, $d_{xy} \in \mathcal{C}$ is a diagonal element (e.g. $x = y$) to model parameter passing .

Semantic Equations

$\mathrm{D}_{\mathrm{SKIP}}$	$[\![\textbf{skip}]\!]_I$	$=$	$\mathcal{C}^\omega$
$\mathrm{D}_{\mathrm{TELL}}$	$[\![\textbf{tell}(c)]\!]_I$	$=$	$\{d.s \in \mathcal{C}^\omega \mid d \models c\}$
$\mathrm{D}_{\mathrm{PAR}}$	$[\![P \parallel Q]\!]_I$	$=$	$[\![P]\!]_I \cap [\![Q]\!]_I$
$\mathrm{D}_{\mathrm{NEXT}}$	$[\![\textbf{next}\ P]\!]_I$	$=$	$\{d.s \in \mathcal{C}^\omega \mid s \in [\![P]\!]_I\}$
$\mathrm{D}_{\mathrm{UNL}}$	$[\![\textbf{unless}\ c\ \textbf{next}\ P]\!]_I$	$=$	$\{d.s \in \mathcal{C}^\omega \mid d \not\models c \text{ and } s \in [\![P]\!]_I\}$ $\cup \{d.s \in \mathcal{C}^\omega \mid d \models c\}$
$\mathrm{D}_{\mathrm{REP}}$	$[\![!\ P]\!]_I$	$=$	$\{s \in \mathcal{C}^\omega \mid$ for all w, s' st $s = w.s'$ and $s' \in [\![P]\!]_I\}$
$\mathrm{D}_{\mathrm{LOC}}$	$[\![(\textbf{local}\ \vec{x}; c)\ P]\!]_I$	$=$	$\{s \in \mathcal{C}^\omega \mid$ there exists an $\vec{x}$-variant s' of s st $s'(1) \models c$ and $s' \in [\![P]\!]_I\}$
$\mathrm{D}'_{\mathrm{ABS}}$	$[\![\textbf{when}\ c\ \textbf{do}\ P]\!]_I$	$=$	$\{d.s \in \mathcal{C}^\omega \mid d \models c \text{ and } d.s \in [\![P]\!]_I\}$ $\cup \{d.s \in \mathcal{C}^\omega \mid d \not\models c\}$
$\mathrm{D}_{\mathrm{ABS}}$	$[\![(\textbf{abs}\ \vec{x}; c)\ P]\!]_I$	$=$	$\bigcap_{\vec{t} \in \mathcal{T}^{\lvert\vec{x}\rvert}} [\![(\textbf{when}\ c\ \textbf{do}\ P)[\vec{t}/\vec{x}]]\!]_I$
$\mathrm{D}_{\mathrm{CALL}}$	$[\![p(\vec{x})]\!]_I$	$=$	$I(p(\vec{x}))$

Abs. Semantics Equations

$\mathrm{A}_{\mathrm{SKIP}}$	$[\![\mathbf{skip}]\!]^{\tau}_{X}$	$=$	$\tau(\mathcal{A}^{\omega})$
$\mathrm{A}_{\mathrm{TELL}}$	$[\![\mathbf{tell}(c)]\!]^{\tau}_{X}$	$=$	$\tau(\{d_\kappa.s_\kappa \in \mathcal{A}^{\omega} \mid d_\kappa \models^{\alpha} \alpha(c)\})$
$\mathrm{A}_{\mathrm{PAR}}$	$[\![P \parallel Q]\!]^{\tau}_{X}$	$=$	$[\![P]\!]^{\tau}_{X} \cap [\![Q]\!]^{\tau}_{X}$
$\mathrm{A}_{\mathrm{NEXT}}$	$[\![\mathbf{next}\ P]\!]^{\tau}_{X}$	$=$	$\tau(\{d_\kappa.s_\kappa \in \mathcal{A}^{*} \mid s_\kappa \in [\![P]\!]^{\tau}_{X}\})$
$\mathrm{A}_{\mathrm{UNL}}$	$[\![\mathbf{unless}\ c\ \mathbf{next}\ P]\!]^{\tau}_{X}$	$=$	$\tau(\mathcal{A}^{\omega})$
$\mathrm{A}_{\mathrm{REP}}$	$[\![!\ P]\!]^{\tau}_{X}$	$=$	$\tau(\{s_\kappa \in \mathcal{A}^{*} \mid$ for all s'_κ, w_κ s.t. $s_\kappa = w_\kappa.s'_\kappa, s'_\kappa \in [\![P]\!]^{\tau}_{X}\})$
$\mathrm{A}_{\mathrm{LOC}}$	$[\![(\mathbf{local}\ \vec{x}; c)\ P]\!]^{\tau}_{X}$	$=$	$\tau(\{s_\kappa \in \mathcal{A}^{*} \mid$ there exists a $\vec{x}$-variant s'_κ of s_κ s.t. $s'_\kappa(1) \models^{\alpha} \alpha(c)$ and $s'_\kappa \in [\![P]\!]^{\tau}_{X}\})$
$\mathrm{A}'_{\mathrm{ABS}}$	$[\![\mathbf{when}\ c\ \mathbf{do}\ P]\!]^{\tau}_{X}$	$=$	$\tau(\{d_\kappa.s_\kappa \in \mathcal{A}^{\omega} \mid d_\kappa \not\models_{\mathcal{A}} c\}) \cup \{d_\kappa.s_\kappa \in \mathcal{A}^{*} \mid d_\kappa \models_{\mathcal{A}} c$ and $d_\kappa.s_\kappa \in [\![P]\!]^{\tau}_{X}\}$
$\mathrm{A}_{\mathrm{ABS}}$	$[\![(\mathbf{abs}\ \vec{x}; c)\ P]\!]^{\tau}_{X}$	$=$	$\bigcap_{\vec{t_\kappa} \in \mathcal{T}_\kappa{}^{\lvert\vec{x}\rvert}} [\![(\mathbf{when}\ c\ \mathbf{do}\ P)[\vec{t'}/\vec{x}]]\!]^{\tau}_{X}$ where $\alpha_t(\vec{t'}) = \vec{t_\kappa}$
$\mathrm{A}_{\mathrm{CALL}}$	$[\![p(\vec{x})]\!]^{\tau}_{X}$	$=$	$X(p(\vec{x}))$

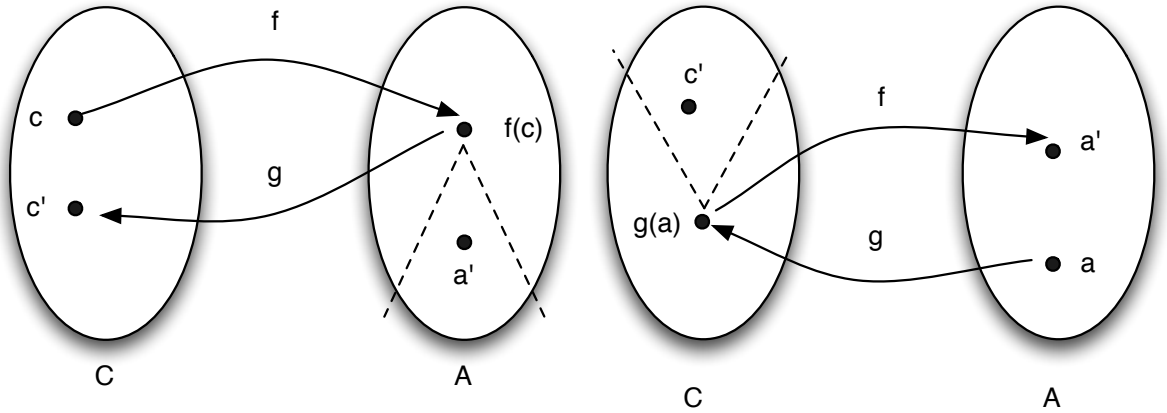
DS Attack

$$
\begin{array}{lcl}
msg_1 & A \rightarrow C & : \quad \{A, m\}_{pub(C)} \\
msg_1' & C \rightarrow B & : \quad \{A, m\}_{pub(B)} \\
msg_2 & B \rightarrow A & : \quad \{n\}_{pub(m)}
\end{array}
$$

Abstract Semantics

Abstract Constraint Systems

Let **C** and **A** be constraint systems. A description $(\mathcal{C}, \alpha, \mathcal{A})$ consists of an abstract domain $(\mathcal{A}, \leq^{\alpha})$ and a monotonic abstraction function $\alpha : \mathcal{C} \rightarrow \mathcal{A}$.



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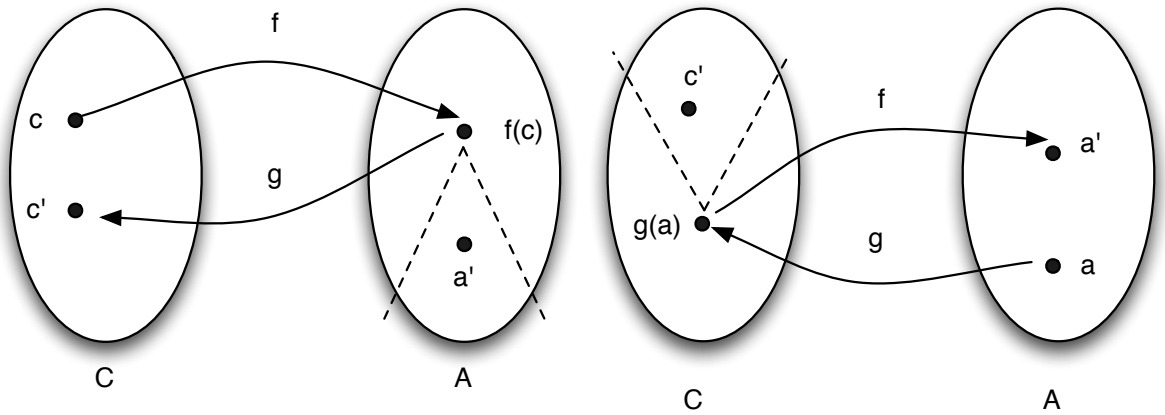

Sequence Abstraction

$\tau : (\mathcal{A}^{\omega} \cup \mathcal{A}^{*}) \rightarrow \mathcal{A}^{*}$ is a reductive operator $(\tau(s_{\kappa}) \leq^{\alpha} s_{\kappa})$.

Abstract Semantics

Semantic Equations

$$
\begin{array}{lll}
\text{A}_{\text{TELL}} & [\![\textbf{tell}(c)]\!]^{\tau}_{X} & = \tau(\{d_\kappa.s_\kappa \in \mathcal{A}^\omega \mid d_\kappa \models^{\alpha} \alpha(c)\}) \\
\text{A}'_{\text{ABS}} & [\![\textbf{when } c \textbf{ do } P]\!]^{\tau}_{X} & = \tau(\{d_\kappa.s_\kappa \in \mathcal{A}^\omega \mid d_\kappa \not\models_{\mathcal{A}} c\}) \\
 & & \cup \{d_\kappa.s_\kappa \in \mathcal{A}^* \mid d_\kappa \models_{\mathcal{A}} c \text{ and } d_\kappa.s_\kappa \in [\![P]\!]^{\tau}_{X}\} \\
\text{A}_{\text{CALL}} & [\![p(\vec{x})]\!]^{\tau}_{X} & = X(p(\vec{x}))
\end{array}
$$

Abstract Semantics

The Abs. semantics is defined as the lfp of

$T^{\alpha}_{\mathcal{D}}(X)(p(\vec{x})) = [\![(\Delta^{\vec{y}}_{\vec{x}} P)]\!]^{\tau}_{X}$ if $p(\vec{y}) \stackrel{\texttt{def}}{=} P \in \mathcal{D}$

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Theorem (Soundness of the approximation)

Given a utcc program $\mathcal{D}.P$, if $s \in [\![P]\!]$ then $\tau(\alpha(s)) \in [\![P]\!]^{\tau}$.

Correctness of the Abstraction

Approximation: Let $d_\kappa = \alpha(d)$. We say that d_κ is the best approximation of d. Furthermore, for all $c_\kappa \leq^\alpha d_\kappa$ we say that c_κ approximates d and we write $c_\kappa \propto d$.

Correctness

Let $\alpha : \mathcal{C} \to \mathcal{A}$ be monotone. We say that **A** is upper correct w.r.t **C** if for all $c \in \mathcal{C}$ and $x, y \in \mathcal{V}$:

1. $\alpha(\exists_x c) = \exists_x^\alpha \alpha(c)$.
2. $\alpha(d_{xy}) = d_{xy}^\alpha$. And
3. $\alpha(c \sqcup d) \models^\alpha \alpha(c) \sqcup^\alpha \alpha(d)$.

Let α_t be the induced term-abstraction by α. Given the sequence of variables $\vec{x}$ and $\vec{t}, \vec{t'} \in \mathcal{T}^{|\vec{x}|}$, 4) if $\alpha_t(\vec{t}) = \alpha_t(\vec{t'})$ then $\alpha(c[\vec{t}/\vec{x}]) = \alpha(c[\vec{t'}/\vec{x}])$

Soundness of the Abstraction

Abstract domain

$\mathbb{A} = (A, \subseteq^{\alpha})$ where $A = \mathcal{P}(\mathcal{A}^*)$ and $\subseteq^{\alpha}$ is defined similarly to $\subseteq^{c}$ (Smyth powerdomain). We require $\mathbb{A}$ to be noetherian

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Galois Connection

$$\begin{aligned} \alpha(E) &:= \tau(\{\alpha'(s) \mid s \in E\}) \\ \gamma(A) &:= \{s \mid \tau(\alpha'(s)) \in A\} \end{aligned}$$

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Let $I : \mathit{ProcHeads} \to E$ (an interpretation), $X : \mathit{ProcHeads} \to A$ (abstract interpretation) and p a procedure name. Then

$$\begin{aligned} \alpha(I)(p) &:= \tau(\{\alpha'(s) \mid s \in I(p)\}) \\ \gamma(X)(p) &:= \{s \mid \tau(\alpha'(s)) \in X(p)\} \end{aligned}$$