SUPERSTRING INSPIRED MODELS

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A B S T R A C T

We discuss the structure of low energy groups arising from compactified models based on the heterotic string. Particular regard is paid to the possibility of intermediate scale breaking which may change the low energy gauge structure and may naturally lead to doublet-triplet splitting and the suppression of proton decay. We present an illustrative example of such a model with a low energy gauge group structure $SU_3^C \times SU_2^L \times SU_2^R \times U_1^{B-L}$ which may be compatible with low energy phenomena including limits on neutrino masses and $\sin^2\theta_W$. Mechanisms leading to the minimal $SU_3^C \times SU_2^L \times U_1^{Y_W}$ low energy gauge group are also presented.

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1. Introduction

Recent work on the field theory limit of the heterotic string[1,2] has opened the possibility of obtaining a realistic model at the weak scale[3,4,5,6]. In particular, the possibility of models with SU_3 holonomy is interesting for it leads to four dimensional models with N=1 supersymmetry, chiral fermions, and an $E_8 x E_6$[7] internal symmetry group. In this paper we will concentrate on such models and on the question whether symmetry breaking can lead to phenomenologically viable low energy models. We will consider only the E_6 structure of the visible sector, assuming that reasonable supersymmetry breaking will be triggered from the (E_8 symmetric) hidden sector. N=1 supersymmetric models with SU_3 holonomy have gauge groups contained in E_6 and light matter consisting of $N_g x$ 27 representations, where $N_g = \frac{1}{2}|\chi|$ and χ is the Euler characteristic of the Calabi-Yau[8] manifold K of the compactified extra six dimensions. In addition there may be further light matter contained in $b_{1,1}(27+\overline{27})$ representations where $b_{1,1}$ is the Betti-Hodge number of K. Existing models of this type use polynominal Calabi-Yau manifolds with $b_{1,1}=1$[7]. Models with $b_{1,1} > 1$ are known to exist[9] and are attractive for they are the only known models with just 3 generations, but, so far, have not been fully exploited for model building.

The gauge symmetry breaking in these models may proceed via some of the scalar fields in these matter supermultiplets acquiring vacuum expectation values (vevs) in the normal manner. However the singlets under the $SU_3^C x SU_2^L x U_1^Y$ subgroup in the 27 and $\overline{27}$ representations are also singlets of

SU_5 and so this mechanism on its own will lead to a low energy group containing SU_5 with unacceptable baryon number violation. Luckily there is another symmetry breaking mechanism "the flux breaking mechanism", possible for non-simply connected Calabi-Yau manifolds[10,11,7,3].

Non-trivial gauge configurations U_g, $g \varepsilon G$ where G is a discrete group, can be trapped in non-simply connected manifolds even though the gauge field strength vanishes. The group elements U_g form a discrete subgroup $\bar{G}$ of E_6 and break E_6 to the subgroup H where

$$[H, U_g] = 0 \quad , \qquad g \varepsilon G \tag{1}$$

Thus once the discrete subgroup of E_6 is specified the pattern of symmetry breaking by the flux mechanism is determined. Together with further symmetry breaking through vevs for components of 27 and $\overline{27}$ representations the low energy subgroup will be determined.

Armed with these symmetry breaking mechanisms, can we arrive at reasonable low energy models? We concentrate here on the possible patterns of gauge symmetry breaking, assuming that acceptable supersymmetry breaking will be fed through from the "hidden" E_8 sector to the visible sector in the form of soft supersymmetry breaking terms[12]. At this stage it is easy to identify several desirable features for a viable model.

(i) Inhibition of baryon and lepton number violation. In supersymmetric models baryon and lepton number may be violated by dimension d=4 terms and dimension d=5 terms as well as the usual dimension 6 terms present also in non-supersymmetric models. In addition to the 15 known fermions per family a 27 representation of E_6 contains 12 new fermions which are dangerous because they can lead to baryon and lepton number violating terms via Yukawa couplings present in the N=1 superpotential (see eq (2)). There are two possible "fixes" for this problem. First one may make the assumption that the Yukawa couplings leading to these terms are absent or small, an approach made less convincing because no set of discrete symmetries seem to emerge forbidding the undesirable couplings[3]. The second possibility is that the new (D) quarks* can be assigned in the part of 27 which transforms as a 10 under $SO_{10} \subset E_6$ (see eq (3)) and are very heavy ($\gtrsim O(10^{14}$ GeV)). In this case if we assign the known fermions in the part of the 27 which transforms as a 16 under $SO_{10} \subset E_6$ the models automatically have a matter parity[5] (see eq (2)) forbidding d=4 operators and the d=5 operators do not give unacceptably fast proton decay. However, for this to happen there must be almost flat directions in the potential allowing large vevs of $O(10^{14}$ GeV). As we will discuss this is actually the case in specific models.

(ii) Light Higgs doublets. All grand unified supersymmetric models have the problem of ensuring that the Higgs doublets, necessary for electroweak breaking, remain light while keeping their colour triplets heavy or decoupled from light fermions to inhibit proton decay. However, in the superstring inspired models, the sliding singlet or missing multiplet mechanisms, proposed to solve this problem by making the colour triplets heavy, seem not to work because of the restricted representation content of the matter fields. Thus we must look for alternatives in the superstring. There are two possibilities.

*We refer to the three quark flavours in each 27 family as the (u,d) electroweak doublet and the new D quark, an electroweak singlet.

The first allows the Higgs doublets and their partners to both be light but assumes that the dangerous baryon number violating couplings of the colour triplets to light fermions are absent, while keeping the Yukawa couplings to Higgs doublets necessary for mass generation. This is technically allowed in the superstring since, after flux breaking, the light doublets and triplets may not be E_6 partners and their Yukawa couplings may not be related by E_6. However, to date no-one has shown that the undesired couplings are absent in a realistic model.

The second possibility is that flux breaking may automatically split the $b_{1,1}(27+\overline{27})$ multiplets leaving the Higgs doublets light while their colour triplet partners acquire mass of order the compactification scale. As stressed in reference [3], this will happen for an unbroken subgroup after flux breaking containing $SU_3^C x SU_2^L x SU_2^R x U_1^{B-L}$. We will discuss whether this symmetry may be further broken at the intermediate scale, and how it is that the light Higgs doublets may not acquire intermediate scale breaking.

(iii) <u>Neutrino masses</u>. The existence of a right-handed neutrino state, ν_R, in the 27 representation gives rise to the possibility of Dirac neutrino masses and the need to explain why the observed neutrino states are nearly massless. One possibility is to make the strong assumption of zero Dirac mass for the neutrino. This leads to complications with cosmology because of the extra massless ν_R states, which are solved if ν_R can be given a mass, either Majorana or Dirac through coupling to new neutral gauge singlet states[13].

A more natural solution would be to allow Dirac neutrino masses but to use the "see-saw" mechanism[14] to give light neutrino states. This requires Majorana masses for the ν_R states $\gtrsim O(10^4 \text{GeV})$. Such masses are not present in zeroth order in these models, but can arise in higher order through, for example, a term $\frac{1}{M_*}\nu_R\nu_R \langle\tilde{\nu}_R\rangle\langle\tilde{\nu}_R\rangle$, where $\langle\tilde{\nu}_R\rangle$ is a vev of the scalar component of a right-handed neutrino and this term is present in the E_6 product $(27)^2(\overline{27})^2$. Whether such a term can give adequate Majorana mass depends on the possible magnitude of $\langle\tilde{\nu}_R\rangle$ and the mass scale M_* associated with this operator.

(iv) <u>M_X and $\sin^2\Theta_W$.</u> Although, after flux breaking, E_6 relations for Yukawa couplings of fields belonging to different H subgroup representations are lost, the same is not true for gauge couplings since the light gauge bosons belong to a single E_6 adjoint representation. At the scale of flux breaking the E_6 relations require $\sin^2\Theta_W = \frac{3}{8}$ and $\alpha_3 = \alpha_2$, where these are the SU_3^C and SU_2^L weak gauge couplings (possibly embedded in some larger symmetry group). Radiative corrections will change these predictions at laboratory energies in the usual way[15] and a viable model should give a reasonable $\sin^2\Theta_W$ and a value for the unification scale M_X compatible with the compactification scale at which the flux breaking mechanism works. These predictions are sensitive to the group structure after flux breaking and to any intermediate scale of symmetry breaking following from vevs of 27 or $\overline{27}$ matter fields. Moreover, as the low energy group may be larger than that of the minimal $SU_3^C \times SU_2^L \times U_1^Y$ electroweak model it is important to reanalyse low energy phenomena, allowing for any additional light gauge bosons, before making a comparison with the predictions for $\sin^2\Theta_W$.

In this paper we will analyse in detail the patterns of symmetry breaking in superstring inspired models, paying particular regard to the intermediate scale breaking possibilities. In section 2 we estimate the natural scale for intermediate breaking and discuss ways in which this scale may be made larger. We show that, in models with suitable discrete symmetries, the intermediate scale may be such that it inhibits baryon number violating processes. Section 3 is devoted to the construction of a complete list of low-energy groups resulting from models with E_6 symmetry broken by the flux mechanism and by intermediate scale breaking. Section 4 discusses the prospects for these models to satisfy the desirable ingredient for a viable model. We point out that, if no assumptions of anomalously small Yukawa couplings are made , the low energy group is greatly restricted, and must contain $SU_3^C \times SU_2^L \times SU_2^R \times U_1^{B-L}$ in models with Betti-Hodge number $b_{1,1}=1$. Remarkably, such models may have acceptable levels of baryon number violation and acceptable neutrino masses. The value of $\sin^2\Theta_W$ in such models is large, but subject to large errors. If the scale of breaking of SU_2^R is close to the electroweak breaking scale, the new neutral currents may increase the experimental determination of $\sin^2\Theta_W$ and may be consistent with the predictions following from the E_6 symmetry. In models with Betti-Hodge number $b_{1,1}$ greater than one the low energy symmetry may be the minimal $SU_3^C \times SU_2^L \times U_1^Y$ and give a three generation model leading to this structure. In section 5 we give a concrete example of a model with a $SU_3^C \times SU_2^L \times SU_2^R \times U_1^{B-L}$ gauge symmetry. We show that the discrete symmetries in this model can give intermediate scale breaking at a scale $O(10^{14}GeV)$ and naturally suppress baryon number violating effects. Remarkably the same discrete symmetries forbid large masses for the Higgs doublets while their colour triplet partners acquire mass after flux breaking. The existence of a low scale of (B-L) violation allows for Majorana neutrino masses of $O(10^4GeV)$, so neutrinos may have masses consistent with laboratory bounds. Section 6 presents a three generation example of a model with $b_{1,1}= 2$ and low energy group $SU_3^C \times SU_2^L \times U_1^Y$. Section 7 presents a summary of our results and gives our conclusions.

2. Intermediate scale breaking

We have considered how each of the desirable features of a viable model is sensitive to intermediate scale breaking via matter vevs. Before listing viable candidate low energy theories let us discuss the possibilities for intermediate scale breaking. In the absence of light components in the $b_{1,1}(27+\overline{27})$ representations, there is no possibility for intermediate scale breaking[4]. A vev, v, for a 27 component, leaving the standard model unbroken, will give rise to a non-vanishing D term and a potential energy of $O(v^4)$. Allowing for supersymmetry breaking contributions to the potential in the gauge-non-singlet sector of $O(\frac{M_W^4}{\alpha^2})$ (to maintain the supersymmetric solution of the hierarchy problem[16]), minimisation of the potential will require $v^4 \lesssim O(\frac{M_W^4}{\alpha^2})$, where α is a gauge fine structure constant.

Once we have light components in the $b_{1,1}(27+\overline{27})$ representations the situation changes because the D term will vanish if identical components of a 27 and $\overline{27}$ have equal vevs. Along these flat directions a large vev may develop, triggered by supersymmetry breaking terms coming from the hidden sector[12], and cut-off at a scale determined by F terms[4] or by renormalisation group effects changing the sign of the soft supersymmetry-breaking mass squared terms[17]. In estimating the effect of F terms in limiting the intermediate scale it is necessary to consider the couplings appearing from the singlet product of three 27 chiral superfields in the superpotential. This has the form[5]

$$
\begin{aligned}
P = d_{ijk}\Big[&(\lambda_i(1:1)\ \lambda_j(10:5)\ \lambda_k(10:\bar{5}) \\
&+ \lambda_i(10:5)\{\lambda_j(10:10)\lambda_k(16:10) + \lambda_j(16:1)\lambda_k(16:\bar{5}) \\
&\qquad + \lambda_j(1:1)\ \lambda_k(10:\bar{5})\} \\
&+ \lambda_i(10:\bar{5})\{\lambda_j(16:10)\lambda_k(16:\bar{5}) + \lambda_j(10:5)\lambda_k(1:1)\} \\
&+ \lambda_i(16:1)\lambda_j(10:5)\lambda_k(16:\bar{5}) \\
&+ \lambda_i(16:\bar{5})\{\lambda_j(10:\bar{5})\lambda_k(16:10) + \lambda_j(10:5)\lambda_k(16:1)\} \\
&+ \lambda_i(16:10)\{\lambda_j(16:10)\lambda_k(10:5) + \lambda_j(10:\bar{5})\lambda_k(16:\bar{5})\}+[j \leftrightarrow k])\Big] \qquad (2)
\end{aligned}
$$

where λ_i, λ_j, λ_k are chiral superfields transforming as the 27 of E_6 and chosen from the $N_g+b_{1,1}$ copies of light 27's subject to the discrete symmetries of the model. The indices in parenthesis indicate the embedding of SO_{10} in E_6 and the embedding of SU_5 in SO_{10}

$$27 = \left[16+10+1\right]_{SO_{10}} = \left[(10+\bar{5}+1) + (5+\bar{5})+1\right]_{SU_5} \tag{3}$$

We must remember that although light fields fill complete 27's, they need not belong to the same E_6 representation and no particular E_6 relations hold amongst the d_{ijk} except those following for the unbroken group after flux breaking . Thus the coefficients d_{ijk} may be different for different components of the λ_i, λ_j and λ_k; ie the notation of eq (2) is schematic.

From eq (2) we see that the $|F|^2$ terms $\sum_{i,a,b} \left| \frac{\partial P\ (\lambda)}{\partial\lambda_i(a:b)}\right|^2$, also possess a flat direction, for example there is no term in $|F|^2$ of the form $|\lambda(1:1)|^4$. For vevs along these directions we must look for additional terms in P, which can generate a non vanishing $|F|^2$. For example a term in P with E_6 content

$$P = \frac{1}{M}\,(27)^2(\overline{27}^2) \tag{4}$$

will give a contribution to the potential V of the form

$$V = \frac{4}{M^2}\,\left\{\lambda^2(1:1)\ \bar{\lambda}^4(1:1) + \lambda^4(1:1)\ \bar{\lambda}^2(1:1)\right\} \tag{5}$$

Given a supersymmetry breaking soft mass term of the form

$$V_s = -m_W^2 |\lambda(1:1)|^2 \qquad (6)$$

the full potential will have a minimum at

$$< \lambda(1:1)> = <\bar{\lambda}(1:1)> \approx [m_W M]^{1/2} \qquad (7)$$

The non-renormalisable term in eq (4) may be expected in general as a result of the exchange massive string excitations, and the scale M will be the compactification scale, expected close to the Planck scale[18]. Inserting this in eq (7) gives an estimate of $O(10^{10}\text{GeV})$ for the possible intermediate scale $<\lambda(1:1)>$. Our choice of the $\lambda(1:1)$ was purely illustrative, the potential following from eq (2) has many other flat directions that may acquire this intermediate scale breaking.

Our discussion in section 1.1 suggests that an intermediate scale of $O(10^{10}\text{GeV})$ is insufficient adequately to suppress d=5 contributions to nucleon decay. Is this the maximum possible? In ref [4], it was argued that it is, because of the large number of massive string modes capable of generating eq (4). However, it may be that the full string theory possesses (discrete) symmetries forbidding a term of the form eq (4) and in this case larger intermediate scales are possible. Whether this happens depends on the specific model. In section 5 we present a realistic model in which there are such discrete symmetries which prevent the terms of eq (4). In that case the first non-zero contributions to V come from a superpotential term (allowed by the discrete symmetries) of the form

$$P = \frac{1}{M} (27)^3(\overline{27})^3 \quad (8)$$

This would give an intermediate scale of the form

$$\langle\lambda(1,1)\rangle = \langle\bar{\lambda}(1,1)\rangle \approx \left[m_W M^3\right]^{1/4}$$

$$\approx O(10^{14}\text{GeV}) \quad (9)$$

Thus specific models may have much higher intermediate scales than the one given in eq (7) and large enough to suppress nucleon decay (cf section 1(i)). Of course, in any complete model one must check whether the necessary soft supersymmetry breaking terms of eq (6) are present with a negative coefficient even at the intermediate scale of breaking. Moreover it is necessary to check that, after flux breaking, there do indeed remain the light components from the $b_{1,1}(27+\overline{27})$ supermultiplets required for intermediate scale breaking. In the next section we consider this in more detail and discuss the possible structure of low-energy groups after flux and intermediate scale breaking.

3.1 Symmetry breaking at the compactification scale

As discussed in section 1, the low energy group after flux breaking is determined once the discrete group elements U_g are given. It is convenient to write the U_g in the basis of the maximal $SU_3xSU_3xSU_3$ subgroup of E_6^*, with the convention that SU_3^C and SU_2^L factors of the standard model are given

*Throughout this paper we use the representation (3,3,3) of $SU_3xSU_3xSU_3$ when dealing with U_g elements.

by the first SU_3 factor and by the first two rows and columns of the second SU_3 factor respectively. We consider first the case that U_g represents an abelian discrete subgroup of E_6, leaving the rank of the unbroken subgroup unchanged - ie rank six.

In this case we may write U_g in the general diagonal form

$$U_g = \begin{pmatrix} 1 & & \\ & 1 & \\ & & 1 \end{pmatrix} \times \begin{pmatrix} \alpha & & \\ & \alpha & \\ & & \alpha^{-2} \end{pmatrix} \times \begin{pmatrix} \beta & & \\ & \gamma & \\ & & \delta \end{pmatrix} \tag{10}$$

$$\beta\gamma\delta = 1$$

where we have required that the unbroken subgroup contains $SU_3^C x SU_2^L$. It is now straightforward to enumerate all the possibilities for unbroken subgroups of E_6 following from all possible choices for the U_g. These are given in Table I with the corresponding U_g given in Table III. In Table I we also list the decomposition of the 27 matter representations under the unbroken subgroup, and H^C, the maximal commuting subgroup, in which U_g must be embedded. A comparison of H^C with the specific forms of U_g given in Table III shows that this is indeed the case.

In writing Table I we have included all possible E_6 subgroups, even though they are not all candidates for a low energy gauge theory because they may possess baryon number violating gauge bosons. However, as discussed above, there may be a further stage of symmetry breaking at an intermediate scale which may give the dangerous baryon number violating gauge bosons a large mass, and so we must, at this stage keep all the candidate groups in

Table I.

We turn now to the case the U_g may represent a non-abelian discrete subgroup of E_6, with the result that the rank of the unbroken subgroup is lowered. As discussed by Witten[3] the rank of the unbroken subgroup will be at least five if the standard model is to be included. To see this we use the convention that the charge operator (in the vector representation) be given in the $SU_3 x SU_3 x SU_3$ basis by

$$Q \equiv Q^L + Q^R = (T_3^L + \frac{1}{\sqrt{3}} Y^L) + (T_3^R + \frac{1}{\sqrt{3}} Y^R) \tag{11}$$

where

$$\begin{aligned}
T_3^R &= \frac{1}{2} \left[\begin{pmatrix} 0 & & \\ & 0 & \\ & & 0 \end{pmatrix} + \begin{pmatrix} 0 & & \\ & 0 & \\ & & 0 \end{pmatrix} + \begin{pmatrix} 1 & & \\ & -1 & \\ & & 0 \end{pmatrix} \right] \\
Y^L &= \frac{1}{2\sqrt{3}} \left[\begin{pmatrix} 0 & & \\ & 0 & \\ & & 0 \end{pmatrix} + \begin{pmatrix} 1 & & \\ & 1 & \\ & & -2 \end{pmatrix} + \begin{pmatrix} 0 & & \\ & 0 & \\ & & 0 \end{pmatrix} \right] \\
Y^R &= \frac{1}{2\sqrt{3}} \left[\begin{pmatrix} 0 & & \\ & 0 & \\ & & 0 \end{pmatrix} + \begin{pmatrix} 0 & & \\ & 0 & \\ & & 0 \end{pmatrix} + \begin{pmatrix} 1 & & \\ & 1 & \\ & & -2 \end{pmatrix} \right]
\end{aligned} \tag{12}$$

With this convention we may write U_g in the most general form, which leaves the charge unbroken, given by

$$U_g = \begin{pmatrix} 1 & & \\ & 1 & \\ & & 1 \end{pmatrix} \times \begin{pmatrix} \alpha & & \\ & \alpha & \\ & & \alpha^{-2} \end{pmatrix} \times \begin{pmatrix} \beta & \\ & V \end{pmatrix} \tag{13}$$

$$\beta \det V = 1$$

where V is a (2x2) matrix corresponding to an element of a discrete subgroup of SU_2. It is clear that U_g given in eq (13) commutes with the colour group and three neutral generators, namely Q, $Q^L - Q^R$ and $(\frac{1}{\sqrt{3}} T_3^L - Y^L)$ leaving, at least, a rank 5 unbroken subgroup.

In Table II we list the unbroken subgroups of E_6 following from the general non-abelian U_g of the form given in eq (13), and in Table IV the corresponding U_g. Eq (13) and Table IV give the most general forms of U_g consistent with our choice of charge operator in eq (11). However the form of eq (10) and Table III allow for other choices of charge operator. Rather than do this we prefer to keep our definition of Q in eq (11) and allow for all possible forms of U_g consistent with this choice. This is done in Table III for each of the cases discussed in Table I. This completes the list of possible subgroups of E_6 after flux breaking. If there is no further intermediate scale breaking, many of the groups listed are unacceptabe as low energy models, for they may give rise to fast baryon number violating processes. Thus the possible low energy groups are given by[4] :

$$\text{Rank 6} \quad [3], [7], [9], [10], [11], [12] \quad \text{in Table I} \qquad (14)$$

$$\text{Rank 5} \quad [3], [4], [5] \quad \text{in Table II} \qquad (15)$$

3.2 Intermediate scale symmetry breaking

As discussed in section 2 (see also [4]), for intermediate scale breaking to proceed we need components of the $b_{1,1}(27+\overline{27})$ representations to remain light after flux breaking. Most work to date has concentrated on models with polynominal Calabi-Yau manifolds with Betti-Hodge number $b_{1,1} = 1$. Recently models with $b_{1,1} > 1$ have been discovered[9] and may be promising for model building. As we will discuss, the possible low energy group structure depends sensitively on the Betti-Hodge number and for this reason it is convenient to discuss the possibilities for $b_{1,1} = 1$ and $b_{1,1} > 1$ separately.

(i) $\underline{b_{1,1} = 1}$

Before flux breaking there are $N_g 27 + (27+\overline{27})$ E_6 representations of matter chiral multiplets. After flux breaking the only components of the $\overline{27}$ to remain light are those which remain invariant under $G + \bar{G}$ (cf section 1). Since the $\overline{27}$ is a singlet under G, the light components of the $\overline{27}$ are the ones that are singlets under $\bar{G}$. The index theorem then ensures that for each light components of the $\overline{27}$ there are (N_g+1) copies of the corresponding light component from the 27's. For other components of the 27, N_g copies survive since none comes from the $\overline{27}$. Thus before supersymmetry breaking there remain massless N_g complete 27's plus the components of a $(27+\overline{27})$ invariant under $\bar{G}$. The light components of the conjugate pair $(27+\overline{27})$ are very important for the intermediate scale scenario. As it is only along these light directions that an intermediate scale vev may develop we see that the possibilities for further breaking of the groups in Table I are restricted.

To classify the allowed low-energy groups after intermediate scale breaking we must consider each entry in Table I and II in turn. Consider first the form of U_g given in eq (10). The $SU_3^C \times SU_2^L \times U_1^Y$ singlet components of a 27 representation lie in the $(1,\bar{3}_3,3_2)$ and $(1,\bar{3}_3,3_3)$ directions and, for viable models, it is only along these directions that an intermediate scale vev may develop. These components remain light after the breaking parameterised by eq (10) provided $\alpha^2\gamma = 1$ or $\alpha^2\delta = 1$ respectively. Thus it is straightforward to determine whether intermediate scale breaking is possible and, if so, use the decomposition of the 27 detailed in Table I to determine the resultant low energy group. In Table V we list these viable low energy groups, together with the representation of the light states in the $(27+\overline{27})$ responsible for intermediate scale breaking (if any). We also give the structure of the light neutral currents. Note that in this table we do not include groups which contain light, baryon-number violating generators, for we have now exhausted all possible mechanisms for large scale breaking ($\gg O(1\ \text{TeV})$).

If we consider now the case of non-abelian flux breaking parametrised by the form of eq (13) we see that the mass of the states $(1,\bar{3}_3,3_2)$ and $(1,\bar{3}_3,3_3)$ cannot be zero for they transform non-trivially under $\bar{G}$. Consequently for this case there is no possibility of intermediate scale breaking and the possible low energy groups are those given in eq (15).

(ii) $b_{1,1} > 1$

If there are several copies of $(27+\overline{27})$ representations there are new states which may generate intermediate scale breaking. Which components of the additional $(27+\overline{27})$ representations remain light depends on their transformation properties under G.

If they are G singlets then the light states must be $\bar{G}$ singlets just as for the case $b_{1,1} = 1$. Thus the light states have the same transformation properties as the light states in the single $(27+\overline{27})$ for the case $b_{1,1} = 1$ discussed above. In table V, column B, we list the minimal low energy group which may result from such additional $(27+\overline{27})$ light components when it differs from the minimal case. Note that this only happens when there is an SU_2^N symmetry after Hosotani breaking, so that both the $(1,\bar{3}_3,3_2)$ and $(1,\bar{3}_3,3_3)$ directions may remain light. In this case, with $b_{1,1} = 1$, we may use the SU_2^N to rotate the vev to the $(1,\bar{3}_3,3_3)$ direction giving the pattern shown in Table V, column A. However with $b_{1,1} > 1$ it is not always possible to rotate the vevs to the $(1,\bar{3}_3,3_3)$ direction giving the additional breaking of Table V, column B. In this way it is possible to break just to the standard $SU_3^C x SU_2^L x U_1^Y$ model with no additional light neutral gauge bosons.

The last possibility is that the additional $(27+\overline{27})$ representations are not G singlets. In this case the condition that they contain light components after flux breaking, namely that they be $G+\bar{G}$ singlets, can be satisfied for all components. Thus there are the light colour singlet neutral components needed for intermediate scale breaking and these models have the freedom to break to the minimal $SU_3^C x SU_2^L x U_1^Y$ standard model. It would be very interesting to have examples of manifolds with such non-trivial G representations.

3.3 Multiplet splitting and the suppression of baryon number violation

To conclude our discussion of the structure of low energy models we consider here which groups may give the possibility for an intermediate scale suppression of baryon number violating processes generated via Yukawa couplings. As discussed in section (1(i)), the suppression of such processes may result from anomalously small Yukawa couplings or from the generation of large D quark masses. At present we neither have an exhaustive list of possible models nor the computation of the Yukawa couplings, so it is not yet known if models exist which selectively suppress the unwanted baryon number violating Yukawa couplings. Here we concentrate on the second possibility which is answerable once the pattern of symmetry breaking is specified. From eq (2) we see that D quarks will acquire a mass if the (16:1) or (1:1) directions (in the notation of eq (2)) acquire vevs. These correspond to the $SU_3^C \times SU_3^L \times SU_3^R$ directions $(1,\bar{3}_3,3_2)$ and $(1,\bar{3}_3,3_3)$ respectively. In table V we list which models have this intermediate scale breaking. However, the fact that the D quarks may be made heavy gives rise to a new problem for, from eq (2), we see that some coupling of quarks and leptons to $(10:5+\bar{5})$ is needed to give them masses ie the Higgs light bosons of the standard model must belong to $(10:5+\bar{5})$ representation. Our mechanism for giving an intermediate scale mass to the D quarks would seem to give all components of $(10:5+\bar{5})$ such masses leaving no light Higgs doublets.

There are just two resolutions to this problem. We could appeal to our ignorance of Yukawa couplings and assume that the terms in eq (2), which would give the Higgs doublets a mass when (16:1) or (1:1) acquire large vevs, are absent, while the terms giving their colour triplet "partners" a large mass are present. Again this is technically possible since the Yukawa couplings need not obey E_6 relations after flux breaking, but again we have no idea whether such models exist.

The second possibility, which we prefer because we can show it applies to a non-empty set, is that the Higgs doublets belong to the $b_{1,1}(27+\overline{27})$ multiplets and their colour triplet partners acquire a large mass after flux breaking. We still require that these Higgs doublets do not have Yukawa couplings to the (16:1) or (1:1) directions with large vevs but this is actually the case in specific models as a result of discrete symmetries which follow from the original string theory. We will give a specific example in section 5 of this mechanism.

Having argued for the worth of split multiplets through flux breaking, let us list the possible low energy models with this feature for the case $b_{1,1}=1$. Of the set of models in Table V, column A, with acceptable intermediate scale breaking only those groups with an SU_2^R unbroken subgroup can have the necessary light doublets. This follows immediately from the form of U_g given in eq (10) and the condition that doublets remain light, namely $\alpha^{-1}\beta=1$ $\alpha^{-1}\gamma=1$. Thus only the groups in Table V, column C, have split multiplets and the possibility for natural suppression of baryon number violating processes while leaving Higgs doublets light.

4. Prospects for viable low energy models

We are now in a position to discuss whether the low energy models listed in the last section can satisfy the requirements for a viable model discussed in section 1. In section 3 we excluded models with unacceptable low energy baryon number violation mediated by gauge boson exchange; thus only baryon number violation from Yukawa couplings need be considered here. As discussed in section 2 Yukawa couplings need not obey E_6 relations, although, of course, they will obey the symmetries of the low energy group. For this reason it is possible to assume that the dangerous baryon-number-violating couplings of D quarks to light fermions are absent. In this case the models of section 3 will not have baryon number violation at unacceptable rates. It would be desirable to show that the unwanted Yukawa couplings are indeed absent and, in principle, all Yukawa couplings are predictable, being related to the underlying gauge couplings in the ten dimensional theory. In practice, we are not yet able to predict such couplings in a general model, although it is known how residual discrete symmetries restrict the allowed form of these couplings. Unfortunately, no model with discrete symmetries forbidding the unwanted baryon-number-violating Yukawa couplings is known.

The only alternative to this assumed suppression of Yukawa couplings is to have the D quarks acquire large ($\gtrsim O(10^{14}\text{GeV})$) masses as a result of intermediate scale breaking. From eq (12) we see this will happen if the components (16:1) or (1:1) acquire large vevs provided the relevant d_{ijk} are non-zero. However it is important, if this scheme is to work, for the known quarks and leptons to belong to the (conventional) $(16:10+\bar{5})$ representation, and their mixing with the $(10:5+\bar{5})$ containing the D quarks to be small.

Only if this is the case will the low energy superpotential following from eq (2) have the matter parity needed to forbid dimension 4 baryon-number-violating operators (see section 1(i)). From eq (2) we see that the 16-10 mixing is proportional to the vev of the (16:1) component, so if we do not assume the absence of 16-10 mixing Yukawa couplings this method for suppressing baryon-number-violation requires models with only intermediate scale vevs principally along the (1:1) components. Typically we require $\frac{\langle(16:1)\rangle}{\langle(1:1)\rangle} < 10^{-10}$. In models with $b_{1,1} = 1$ this is automatic for only one direction may aquire a large vev. In Table V we list the models which can have large intermediate scale-breaking along the (1:1) direction.

Our second criterion was that the Higgs doublet remain light. In models with intermediate scale breaking this may be difficult due to the Yukawa couplings of eq (2). Again one can assume the unwanted couplings are zero; in some cases this will result from the known discrete symmetries. However, if we insist on the natural baryon-number-suppression mechanism discussed above we should try to keep the naturalness in the Higgs sector too. From eq (2) we see that if the (10:5) and $(10:\bar{5})$ matter representations are given intermediate scale mass via a large (1:1) vev then the light fermion fields belong to (16:10) + $(16:\bar{5})$ + (16:1) representations. Again from eq (2) we see that masses for these fields require that the Higgs doublets lie in a (10:5) or $(10:\bar{5})$ representation, apparently in contradiction with our criterion for natural baryon-number suppression. As discussed above the only natural way out in this case is to arrange that the colour triplet partners of the Higgs be heavy after flux breaking. From Table V we see that this requires the low energy group to contain $SU_3^C x SU_2^L x SU_2^R x U_1^{B-L}$ if $b_{1,1} = 1$. There remains the need to forbid intermediate scale masses for these light Higgs but, as we will show in section 5, this can result from a discrete symmetry automatically present in the model.

The third criterion is that neutrinos should have an acceptable mass matrix. In general, one may assume the Yukawa couplings responsible for neutrino masses are absent, although again no models have been shown to have this property. (In models with a low energy symmetry, such as SU_2^R this way out may not be possible since there may be relations between neutrino Dirac masses and charged lepton masses). A less contrived possibility for a reasonable neutrino mass matrix is that the right handed neutrino acquires a large Majorana mass giving rise to a see-saw mechanism[14]. To achieve this we must generate a term for example of the form $\frac{1}{M_*} \nu_R \nu_R \langle\tilde{\nu}_R\rangle\langle\tilde{\nu}_R\rangle$ as discussed in section 1(iii). Such a term will be generated at tree level via gaugino-neutrino mixing but will give mass to only one ν_R generation[13]. In radiative order other generations may acquire mass via the D term $\frac{1}{M_*^3} [\nu_{Ri}\ \nu_{Ri}\ \bar{\nu}_{Rj}\ \bar{\nu}_{Rj}\ \eta^+]_D$, where ν_{Ri} denote chiral superfields and η is a supersymmetry breaking mass insertion $\eta = \Theta\Theta m_s^2$. The mass term m_s^2 is the supersymmetry breaking scale in the gauge non singlet sector ie $m_s \lesssim O(\frac{M_W}{\alpha})$[16]. To avoid large suppressions, since $M_* > O(\langle\nu_R\rangle)$, we require $\langle\nu_R\rangle \approx M_* \approx m_s$ ie the intermediate states giving rise to the desired couplings must be relatively light. In this case Majorana neutrino masses of $O(10^4$ GeV) may be radiatively generated.

To generate higher scales of neutrino mass requires that there may be additional terms in the superpotential of the form $\frac{1}{M_*}(27)^2(\overline{27})^2$ arising from the exchange of massive string modes. In this case the neutrino masses will not be suppressed by the supersymmetry breaking scale, m_s. However the scale M_* will now be the compactification scale, $M_* \approx O(10^{18}\ \mathrm{GeV})$[18]. This, in turn, requires $\langle\nu_R\rangle \gtrsim 10^{11}$ GeV in conflict with our expectations of section 4 unless the couplings of eq (2), giving rise to (16-10) mass mixing via $\langle\nu_R\rangle$, be small.

The final criterion for acceptable models is that $\sin^2\Theta_W$ should be compatible with low energy phenomenology. The predicitions for $\sin^2\Theta_W$ in leading order have been worked out for a wide range of models[4]. In models with an SU_2^R factor, for example, the value of $\sin^2\Theta_W$ is much larger than the quoted experimental value of 0.22. However there are two possible sources of this discrepancy. The renormalisation group predictions are sensitive to the positions of supersymmetry and intermediate scale thresholds, to higher order corrections (particularly as the value of the unified coupling is large) and to the non-renormalisable couplings of the superstring giving an uncertainty in the prediction for $\sin^2\Theta_W$ very difficult to estimate. The second source of discrepancy comes from the additional low energy gauge bosons which may change the neutral current structure. Their effects should be included before analysing the experimental measurements and extracting the experimental value of $\sin^2\Theta_W$. Thus, before ruling out models with a non-standard low energy gauge groups on the basis of $\sin^2\Theta_W$, it is necessary to reanalyse the neutral current data[21].

To summarise this section, the models of Table V may all be compatible with low energy phenomenology provided we choose the Yukawa couplings in eq (2) at will. The strongest constraint is likely to be the value of $\sin^2\Theta_W$ but, until a complete neutral current analysis of these models is performed, we cannot rule out any of the models in Table V.

If we try to build a model without choosing Yukawa couplings at will, there is a much restricted set of possible models, all containing the low-energy gauge group $SU_3^C \times SU_2^L \times SU_2^R \times U_1$. In the next section we will analyse the simplest of such models in detail, to show how the discrete symmetries present in this model allow for an $O(10^{14}\text{GeV})$ intermediate scale, and protect light Higgs doublets from this scale.

Finally, we note that models with $b_{1,1} > 1$, may lead to the minimal low energy group $SU_3^C \times SU_2^L \times U_1^Y$. The extra freedom to introduce intermediate scale breaking may allow for the generation of large Majorana neutrino masses and the suppression of proton decay. In section 6 we discuss possible symmetry breaking patterns in specific three generation models of this type that have been constructed by Yau[9].

5. A model with flat directions

We have already emphasized that flat directions do exist in models derived from superstring. They drastically improve phenomenological prospects. The D-flat directions require light components in $27+\overline{27}$ which are singlets under $SU_3^C \times SU_2^L \times U_1^Y$. For these directions to be also F-flat discrete symmetries or pseudosymmetries are required.

We now analyse a model with $SU_3^C x SU_2^L x SU_2^R x U_1 x U_1$ as gauge group after compactification and a flat direction allowing to break it down to $SU_3^C x SU_2^L x SU_2^R x U_1^{B-L}$ at 10^{15}GeV. This analysis illustrates which properties are relevant, in particular the role played by pseudosymmetries.

The most detailed model existing in the literature is that due to Witten[3] with $SU_3^C x SU_2^L SU_2^R x U_1 x U_1$ as (grand) unified group. It is based on a Calabi-Yau manifold K/G of SU_3 holonomy. K is defined by the zeros of the polynomial

$$P(z_i) = \sum_{i=1}^{5} z_i^5 + C z_1 z_2 z_3 z_4 z_5 , \tag{16}$$

where z_i are five complex variables not all zero such that $(z_1, z_2, z_3, z_4, z_5) \sim \lambda(z_1, z_2, z_3, z_4, z_5)$ for any non-zero complex number λ. G is a $Z_5 x Z_5$ group acting freely on K and generated by

$$S : z_i \to z_{i+1} ,$$

$$T : z_i \to \alpha^i z_i \quad , \alpha = \exp\left(\frac{2\pi i}{5} \right) \tag{17}$$

This manifold has a $Z_4 x Z_5$ symmetry group F_o generated by

$$Y : z_i \rightarrow z_{2i}$$

$$B : z_i \rightarrow \alpha^{3i^2} z_i \ , \qquad (18)$$

and a large group of pseudosymmetries (symmetries of K that do not give symmetries on K/G) E including the z_i permutations and the transformations

$$z_i \rightarrow \alpha^{n_i} z_i \ , \ n_i \ \varepsilon \ Z \ , \ \sum n_i = 0 \ . \qquad (19)$$

The model is further specified by requiring that the fields ψ obey the boundary conditions

$$\psi(g(x)) = U_g \psi(x) \ , \qquad (20)$$

$x \varepsilon K$, $g \varepsilon G$, $U_g \varepsilon E_6$, with

$$U_S = (1) \ x \begin{pmatrix} \alpha & & \\ & \alpha & \\ & & \alpha^{-2} \end{pmatrix} \ x \ \begin{pmatrix} \alpha & & \\ & \alpha & \\ & & \alpha^{-2} \end{pmatrix} \qquad (21)$$

$$U_T = 1$$

The choice (21) breaks E_6 via the flux mechanism down to H = $SU_3^C x SU_2^L x SU_2^R x U_1 x U_1$ and a discrete symmetry group $Z_2 x Z_5$ corresponding to $\tau = Y^2$ and B. The light matter content is four complete 27 representations plus the components of $27+\overline{27}$

$$\left[(1,2,2;-\frac{1}{2\sqrt{3}},\ \frac{1}{2\sqrt{3}})+(1,1,1;\ \frac{1}{\sqrt{3}},\ -\ \frac{1}{\sqrt{3}})\right]+$$

$$\left[(1,2,2;\ \frac{1}{2\sqrt{3}},\ -\ \frac{1}{2\sqrt{3}})+(1,1,1;\ -\frac{1}{\sqrt{3}},\ \frac{1}{\sqrt{3}})\right] \tag{22}$$

corresponding to case $[9]_2$ (a=b) in Table V.

All this concerns the visible sector. The extra E_8, singlet under E_6, is probably responsible for the necessary supersymmetry breaking[12].

Witten[3] has already obtained the allowed Yukawa couplings in such a model. The cubic part of the superpotential has the generic form

$$P_3 = a_1\bar{\lambda}_o^3 + a_2\lambda_o^3 + a_3\lambda_o\lambda_2\lambda_{-2} + a_4(\lambda_2\lambda_{-1}\lambda_{-1} + \lambda_{-2}\lambda_1\lambda_1) \tag{23}$$

$$+ a_5(\lambda_2\lambda_2\lambda_1 + \lambda_{-2}\lambda_{-2}\lambda_{-1})$$

where $\lambda_2, \lambda_{-2}, \lambda_1, \lambda_{-1}$ stand for the H multiplets in the four (2,-2,1-1) complete 27 representations and λ_o and $\bar{\lambda}_o$ for H multiplets in the $27+\overline{27}$ subset in (22). P_3 is constrained by the symmetries and pseudosymmetries of K/G. Thus the symmetries in (18) require invariance under $\lambda_i \to \lambda_{-i}$ and that i+j+k=0 (mod 5) in non-zero $\lambda_i\lambda_j\lambda_k$ couplings; whereas in addition the pseudosymmetries in (19) forbid the $\lambda_o\lambda_1\lambda_{-1}$ term. Furthermore, only couplings appearing in the singlet of the product 27^3 are allowed [19]*.

A priori it could seem that the model is ruled out because of too fast proton decay. Known quarks must lie in the four complete 27's and no symmetry excludes all dangerous $Q_i Q_j Q_k$ terms $(i,j,k \epsilon \{2,-2,1,-1\})$ in (23). However, as we will see, a flat direction exists in this model along the singlet direction $(1,1,-\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}})$ in (22), allowing for the breaking of H down to $SU_3^C x SU_2^L x SU_2^R x U_1^{B-L}$ at 10^{15}GeV. The unbroken gauge group is contained in SO_{10} in the standard way [20]. The flat direction will be defined by

$$<\bar{\lambda}_o^2> = < \lambda_1^2 + \lambda_{-1}^2 > , \qquad (24)$$

with non-zero vev along $(1,1,1; \frac{1}{\sqrt{3}}, - \frac{1}{\sqrt{3}})$ for λ_1, and λ_{-1} and $(1,1,1;- \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}})$ along $\bar{\lambda}_o$. Then looking at (23) and the first term in eq (2) we see that the quarks and leptons in the 10 of SO_{10} inside the 27's will get tree level masses ~ 10^{15} GeV $(<\lambda_1>\lambda_{-2}\lambda_1, <\lambda_{-1}>\lambda_2\lambda_{-1}, <\lambda_1>\lambda_2\lambda_2, <\lambda_{-1}>\lambda_{-2}\lambda_{-2})$, as well as the $\bar{\lambda}_o$ doublets $(<\bar{\lambda}_o> \bar{\lambda}_o\bar{\lambda}_o)$. The singlets not eaten in the 27's and 27 will get radiative masses at one loop. Then there remain four 16's of SO_{10} plus the doublets of λ_o. This realizes the mechanism discussed in section 4 in which the known quarks and lepton belong to the 16's of SO_{10}, leading to a matter parity in the superpotential eq (23) which removes

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*We must remember that although light fields fill complete 27's, they do not come from the same E_6 representation and no particular E_6 relations hold.

$d = 4$ baryon and lepton number violation. The D quarks acquire a 10^{15} GeV mass and do not give unacceptably fast proton decay.

Let us see that the flat direction in (24) does exist. The direction in (24) is D-flat because 27 and $\overline{27}$ have opposite charges. Now we will see that (24) is also a flat direction for F-terms involving the superpotential up to terms with five fields. P to that order reads

$$P = P_3 + P_4 + P_5 + P_6 + \dots$$

where P_3 is given by (23), whereas

$$P_4 = b_1\bar{\lambda}_o^2\lambda_o^2 + b_2\bar{\lambda}_o^2\lambda_2\lambda_{-2} \tag{26}$$

and

$$\begin{aligned} P_5 = {} & c_1\bar{\lambda}_o^4\lambda_o + c_2\bar{\lambda}_o\lambda_o^4 + c_3\bar{\lambda}_o\lambda_o^2\lambda_2\lambda_{-2} + c_4\bar{\lambda}_o\lambda_o(\lambda_1\lambda_1\lambda_{-2}+\lambda_{-1}\lambda_{-1}\lambda_2) \\ & + c_5\bar{\lambda}_o\lambda_o(\lambda_1\lambda_2\lambda_2 + \lambda_{-1}\lambda_{-2}\lambda_{-2}) + c_6\bar{\lambda}_o(\lambda_1\lambda_1\lambda_1\lambda_2 + \lambda_{-1}\lambda_{-1}\lambda_{-1}\lambda_{-2}) \\ & + c_7\bar{\lambda}_o(\lambda_1\lambda_{-2}\lambda_{-2}\lambda_{-2} + \bar{\lambda}_{-1}\lambda_2\lambda_2\lambda_2) + c_8\bar{\lambda}_o\lambda_1\lambda_{-1}\lambda_1\lambda_{-1} \\ & + c_9\bar{\lambda}_o\lambda_1\lambda_{-1}\lambda_2\lambda_{-2} + c_{10}\bar{\lambda}_o\lambda_2\lambda_{-2}\lambda_2\lambda_{-2} \end{aligned} \tag{27}$$

Eqs (26) and (27) display the generation structure but, of course, all independent E_6 invariant products must be included in each of the terms in these equations.

P_4, P_5, ... in (25) are subjected to the same constraints as P_3. P_4 contains terms appearing in the singlet parts of the product $27^2\overline{27}^2$. There are three E_6 singlets in that product and then in principle three sets of terms similar to (26). However, only two will remain when the condition that the $\overline{27}^2$ is symmetric is imposed, due to the fact that the only $\overline{27}$ is $\bar{\lambda}_o$ and this enters as $\bar{\lambda}_o^2$. Besides the symmetries and pseudosymmetries of K/G only allow for the two terms in (26) (see [3] and above). P_5 contains the terms in the products $\overline{27}^4 27$ and $27^4\overline{27}$; there are six E_6 singlets in each of such products, but not all of these appear in the final expression due to symmetry considerations as in the case of P_4, whereas the ten different terms are those allowed by symmetries and pseudosymmetries. P_6 which will lift the flat direction has many terms, for example

$$d_1\bar{\lambda}_o^3(\lambda_1\lambda_1\lambda_{-2} + \lambda_{-1}\lambda_{-1}\lambda_2) \tag{28}$$

Let us see that (24) is a flat direction of $P_3 + P_4 + P_5$. F terms have $\frac{\partial P}{\partial \lambda}$ as factors . $\frac{\partial P_3}{\partial \lambda}$ will give zero in that direction because the singlet in $27^3(\overline{27}^3)$ does not have any term with the flat direction $(1,1,1; \frac{1}{\sqrt{3}} - \frac{1}{\sqrt{3}})$ repeated (see (2)), which is the only direction getting a vev. $\frac{\partial P_4}{\partial \lambda}$ gives zero because all the terms in (26) contain terms with zero vev's $(\lambda_o, \lambda_2, \lambda_{-2})$

more than once. Also $\frac{\partial P_5}{\partial \lambda}$ does not contribute by the same reason. Finally the flat direction is removed by terms in $|\frac{\partial P_6}{\partial \lambda}|^2$, for instance by $|\frac{\partial P_6}{\partial \lambda_2}|^2$ in (28). Then, as stated in Section 2, eqns (8) and (9), the corresponding vev in the flat direction is $\sim 10^{14-15}$ Gev.

Obviously the appropriate supersymmetry breaking has to be assumed. As it stands the model has no problem with proton decay. What about other predictions. The model predicts a fourth family and all the superpartners at the Fermi scale. Experiment will tell. The problem seems to be in the neutrino masses and in the Weinberg angle. As stated above the intermediate group is $SU_3^C x SU_2^L x SU_2^R x U_1^{B-L}$ and the light matter content four 16's plus two SU_2^L doublets in the 10 of SU_{10}, all from 27's of E_6. There is no light singlet under the standard model in 27 for breaking $SU_2^R x U_1^{B-L} \to U_1^Y$, implying that this breaking must occur at most at 10^4 GeV, because non-zero D terms imply supersymmetry breaking which cannot be at higher energies. Assuming that this breaking occurs at that energy neutrino masses can be acceptable (see Section 2). The Weinberg angle appears at first sight to be too large, essentially because SU_2^R survives down to low energies. However, the calculation is subjected to many ambiguities. Besides a proper calculation of weak current data taking into account the possibility of new weak interactions could allow for a larger value of $\sin^2\Theta_W$ than presently quoted[21]. Finally, the light fermion mass structure in this model is interesting since only the λ_2, λ_{-2} generations get masses at tree level once the Higgs scalars in λ_o acquire vevs breaking the electroweak group. The other fermion generations in λ_1, λ_{-1} will acquire mass only in radiative order.

6. Models with $b_{1,1} > 1$.

As discussed in section 4, models with $b_{1,1} > 1$ may break E_6 via flux and intermediate scale breaking to just the standard model. To illustrate this let us consider the three generation models devised by Yau[9] which have $b_{1,1} = 2$. These models are based on manifolds $K = K_o/G$ where K_o is a constrained $CP^n xCP^n$ space and G, the discrete group, is Z_3 and Z_3xZ_3 for the two models proposed.

If we are to break to the standard model then, after flux breaking, there must remain at least the group $SU_3^C xSU_2^L xSU_2^N xU_1 xU_1$ [cf. our discussion of section 4]. There is only one possible choice for the U_g which does this leaving the directions $(1,\bar{3}_3, 3_2)$ and $(1,\bar{3}_3, 3_3)$ massless with no light baryon-number violating gauge bosons. This is

$$U_g = \begin{pmatrix} 1 & & \\ & 1 & \\ & & 1 \end{pmatrix} x \begin{pmatrix} \alpha & & \\ & \alpha & \\ & & \alpha \end{pmatrix} x \begin{pmatrix} \alpha & & \\ & \alpha & \\ & & \alpha \end{pmatrix}$$

$$E_6 \rightarrow SU_3^C xSU_3^L xSU_3^R$$

Here $\alpha^3 = 1$, $\alpha \neq 1$ and U_g are the elements of $\bar{G}$, a Z_3 discrete subgroup of

E_6. Since the $(1,\bar{3}_3, 3_2)$ and $(1,\bar{3}_3, 3_3)$ are light (both copies of $\overline{27}$ are G singlets and hence their light components are $\vec{G}$ singlets) they may both acquire large vevs along D flat directions as discussed in section 5.

Because there are two copies of $\overline{27}$ representations it is not possible, in general, to use the residual SU_2^N symmetry to rotate their vevs into the $(1,\bar{3}_3, 3_3)$ direction. Hence, in all the above cases, the low energy group after this intermediate scale breaking will be the standard model $SU_3^C x SU_2^L x U_1^Y$. In addition there will be residual light Higgs components from the $(1,\bar{3}, 3)$ part of the $2(27+\overline{27})$ representations. These will be sufficient to break the electroweak group. Their colour triplet partners will be heavy. Thus these models automatically have split multiplets after flux breaking. Whether the Higgs doublets needed for electroweak breaking remain light after intermediate scale breaking depends on the Yukawa couplings. We have not yet investigated the discrete symmetries in these models to see if they will guarantee light Higgs in a similar manner to our example of section 5. It will also be important to investigate the discrete symmetries to determine the limits on the intermediate breaking scale. If it can be as large as 10^{14} GeV, we expect that baryon and lepton number violation may be naturally suppressed as in the example of section 5. The value for $\sin^2\Theta_W$ in these models will be close to the experimental value if the scale of intermediate breaking is large.

7. Summary and Conclusions

We have investigated which low energy groups may result from the underlying

E_6 structure in superstring models with SU_3 holomony. With flux breaking alone, the low energy group must be at least rank five and, in this case, the new neutral current is uniquely specified. Once we include the possible effects of intermediate scale breaking via vevs for fields transforming as the 27 under E_6 there are more possibilities for the low energy structure. Models with Betti-Hodge number $b_{1,1} = 1$ still must have a low energy group of at least rank five, but in this case there are other possibilities for the new neutral current. Models with $b_{1,1} > 1$ can have a low energy group of rank four ie the standard model.

The major difficulty in building a viable theory based on these superstring inspired models is the need to inhibit baryon and lepton number violating processes. These may be suppressed if the relevant Yukawa couplings are small enough, but at present it is not known if manifolds exist with this property. Another possibility arises in models with large intermediate scale breaking giving the D quarks and their SO_{10} partners large mass ($\gtrsim 10^{14}$ GeV). This suppresses d = 4 and d = 5 baryon and lepton number violating processes below the current limits of observation. Although such large scales of intermediate scale breaking are not to be expected in general, we showed that they may arise as a result of discrete symmetries, and presented an example in which this happens. Remarkably in this example, the discrete symmetries also protect the Higgs doublets, left light after flux breaking, from acquiring intermediate scale masses. These doublets may then break the low energy group down to $SU_3^C x U_1^{em}$ at the electroweak scale.

A viable theory must also give an acceptable pattern for neutrino masses. This may be arranged by careful choice of Yukawa couplings but again it is not known if models have this property. Another possibility is that non-gravitational radiative corrections generate large Majorana masses for right handed neutrinos. This can work only if the scale of vevs for right handed sneutrinos is comparable to the supersymmetry breaking scale in the gauge non-singlet sector. It is possible to evade this bound if heavy string modes generate higher order terms in the superpotential but in this case sneutrino vevs of $O(10^{14}$ GeV) are necessary which may give problems with baryon number violation by inducing mixing of D quarks with d quarks.

Perhaps the most promising candidates for a viable model are the three generation models of Yau[9]. These may break down to the standard model and we constructed the flux breaking patterns leading to this structure. The models may have a large scale of intermediate scale breaking, although it is not yet known whether the scale of this breaking and the Yukawa couplings are such as to naturally suppress baryon and lepton number violation and to leave light the Higgs doublets necessary for electroweak breaking.

Our discussion has centred on the group structure possible for low energy models based on the superstring. In this we have required only that the models have SU(3) holonomy, leaving an N = 1 supersymmetry in 4 dimensions unbroken to low scales. Our discussion does not depend therefore on uncertain details of the compactification. However we do assume that soft supersymmetry breaking terms will trigger gauge symmetry breaking along directions neutral under the standard model and it will be important to determine, in a specific model, whether this does indeed happen. For this we will need the detailed structure of the low energy model. It will also be important to calculate, if possible, the Yukawa couplings. Clearly much remains to be done but we find it remarkable that the preliminary analysis shows the low energy structure of superstring inspired models can so closely resemble the real world.

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Table Captions

Table I : Column(A) contains all possible E_6 subgroups,H, containing $SU_3^C xSU_2^L xU_1^Y xU_1' xU_1''$. This column contains also the decomposition of a 27 multiplet of E_6 with respect to the corresponding subgroup H. Column(B) contains the maximal subgroup H^c which commutes with H. The discrete subgroup { U_g } is embedded in H^c.

Table II : This Table contains information analogous to that of Table I for the case of rank 5 E_6 subgroup containing $SU_3^C xSU_2^L xU_1^Y xU_1'$.

Table III : Column(A) contains the form of U_g corresponding to the entries in Table I. Column(B) contains the inequivalent ways of embedding { U_g } without breaking SU_3^C , SU_2^L and charge Q . Column(C) enumerates the set of models which can break down to the standard model through 27 Higgs breaking (if any).

Table IV : This Table contains information analogous to that of Table III for the case of non-abelian flux breaking mechanism.

Table V : Column(A) contains acceptable models with $b_{1,1}= 1$ (or models with $b_{1,1}> 1$, but with light $27+\overline{27}$ components being replicas of the $b_{1,1}= 1$ case), which admit an intermediate scale. Column(B) contains further possible breaking for models with $b_{1,1}> 1$. Column(C) enumerates the light doublets for the models of column(A).

Table 1

(A)	(B)
H: E_6 subgroup containing $SU_3^C \times SU_2^L \times U_1^Y \times U_1' \times U_1''$ and the corresponding 27 decomposition	H^C: Commuting subgroups. All contain the centre of E_6, Z_3, and are contained in $U_1^Y \times U_1' \times U_1''$
[1] $SU_6 \times SU_2$ $27=(\bar{6},2)+(15,1)$	$Z_2(\times Z_3)$
[2] $SO_{10} \times U_1$ $27=(16;\ \frac{1}{2\sqrt{6}})+(10;-\frac{1}{\sqrt{6}})+(1;\sqrt{\frac{2}{3}})$	U_1
[3] $SU_3 \times SU_3 \times SU_3$ $27=(3,3,1)+(\bar{3},1,\bar{3})+(1,\bar{3},3)$	$Z_3(\times Z_3)$
[4] $SU_6 \times U_1$ $27=(\bar{6};\ \frac{1}{2})+(\bar{6};-\frac{1}{2})+(15;0)$	$U_1(\times Z_3)$
[5] $SU_5 \times SU_2 \times U_1$ $27=(\bar{5},2;-\frac{1}{2\sqrt{15}})+(1,2;\ \frac{1}{2}\sqrt{\frac{5}{3}})+(10,1;\ \frac{1}{\sqrt{15}})+(5,1;-\frac{2}{\sqrt{15}})$	U_1
[6] $SU_4 \times SU_2 \times SU_2 \times U_1$ $27=(\bar{4},1,2;\ \frac{1}{2\sqrt{6}})+(4,2,1;\ \frac{1}{2\sqrt{6}})+(6,1,1-\frac{1}{\sqrt{6}})+(1,2,2;-\frac{1}{\sqrt{6}})+(1,1,1;\ \sqrt{\frac{2}{3}})$	$Z_2 \times U_1$

[7] $SU_3 x SU_3 x SU_2 x U_1$ $U_1(xZ_3)$

$27=(\bar{3},1,2:-\frac{1}{2\sqrt{3}})+(1,\bar{3},2;\frac{1}{2\sqrt{3}})+(3,3,1;0)+(\bar{3},1,1;\frac{1}{\sqrt{3}})+(1,\bar{3},1;-\frac{1}{\sqrt{3}})$

[8] $SU_5 x U_1 x U_1$ $U_1 x U_1$

$27=(10;\frac{1}{2\sqrt{10}},\frac{1}{2\sqrt{6}})+(\bar{5};-\frac{3}{2\sqrt{10}},\frac{1}{2\sqrt{6}})+(1;\frac{1}{2}\sqrt{\frac{5}{2}},\frac{1}{2\sqrt{6}})+(5;-\frac{1}{\sqrt{10}},-\frac{1}{\sqrt{6}})$
$+(\bar{5};\frac{1}{\sqrt{10}},-\frac{1}{\sqrt{6}})+(1;0,\sqrt{\frac{2}{3}})$

[9] $SU_3 x SU_2 x SU_2 x U_1 x U_1$ $U_1 x U_1$

$27=(\bar{3},1,2:0,-\frac{1}{2\sqrt{3}})+(\bar{3},1,1;0,\frac{1}{\sqrt{3}})+(3,2,1;\frac{1}{2\sqrt{3}},0)+(3,1,1;-\frac{1}{\sqrt{3}},0)$
$+(1,2,2;-\frac{1}{2\sqrt{3}},\frac{1}{2\sqrt{3}})+(1,1,2;\frac{1}{\sqrt{3}},\frac{1}{2\sqrt{3}})+(1,2,1;-\frac{1}{2\sqrt{3}},-\frac{1}{\sqrt{3}})+(1,1,1;\frac{1}{\sqrt{3}},-\frac{1}{\sqrt{3}})$

[10] $SU_4 x SU_2 x U_1 x U_1$ $U_1 x U_1$

$27=(\bar{4},1;\frac{1}{2},\frac{1}{2\sqrt{6}})+(\bar{4},1;-\frac{1}{2},\frac{1}{2\sqrt{6}})+(4,2;0,\frac{1}{2\sqrt{6}})+(6,1;0,-\frac{1}{\sqrt{6}})$
$+(1,2;\frac{1}{2}-\frac{1}{\sqrt{6}})+(1,2;-\frac{1}{2},-\frac{1}{\sqrt{6}})+(1,1;0,\sqrt{\frac{2}{3}})$

[11] $SU_3 x SU_3 x U_1 x U_1$ $U_1 x U_1(xZ_3)$

$27=(\bar{3},1;\frac{1}{2},-\frac{1}{2\sqrt{3}})+(\bar{3},1;-\frac{1}{2},-\frac{1}{2\sqrt{3}})+(1,\bar{3};\frac{1}{2},\frac{1}{2\sqrt{3}})+(1,\bar{3};-\frac{1}{2},\frac{1}{2\sqrt{3}})+(3,3;0,0)+(\bar{3},1;0,\frac{1}{\sqrt{3}})$
$+(1,\bar{3};0,-\frac{1}{\sqrt{3}})$

[12] $SU_3^C \times SU_2^L \times U_1^Y \times U_1' \times U_1''$ $U_1^Y \times U_1' \times U_1''$

$$27=(3,2;\ \frac{1}{2\sqrt{15}},\ \frac{1}{\sqrt{15}},0)+(\bar{3},1;-\frac{2}{\sqrt{15}},\ \frac{1}{\sqrt{15}},0)+(1,1;\ \sqrt{\frac{3}{5}},\ \frac{1}{\sqrt{15}},0)+(3,1;-\frac{1}{\sqrt{15}},-\frac{2}{\sqrt{15}},\ 0)$$

$$+(1,2;\ \frac{1}{2}\sqrt{\frac{3}{5}},-\frac{2}{\sqrt{15}},0)+(\bar{3},1;\ \sqrt{\frac{1}{15}},-\frac{1}{2\sqrt{15}},\frac{1}{2})+(\bar{3},1;\ \frac{1}{\sqrt{15}},\ -\frac{1}{2\sqrt{15}}\ -\frac{1}{2})+(1,2;-\frac{1}{2}\sqrt{\frac{3}{5}},\ \frac{1}{2\sqrt{15}},\ \frac{1}{2})$$

$$+(1,2;-\frac{1}{2}\sqrt{\frac{3}{5}},-\frac{1}{2\sqrt{15}},-\frac{1}{2})+(1,1;0,\ \frac{1}{2}\sqrt{\frac{5}{3}},\frac{1}{2})+(1,1;0,\ \frac{1}{2}\sqrt{\frac{5}{3}},-\frac{1}{2})$$

Table II

(A)	(B)
H: E_6 subgroups of rank five containing $SU_3^C \times SU_2^L \times U_1^Y \times U_1'$ and the corresponding 27 decomposition	H^C: Commuting subgroups. All contain the centre of E_6, Z_3, and are contained in $SU_2^N \times U_1^Y \times U_1'$
[1] SU_6 $27=2(\bar{6})+(15)$	$SU_2^N (\times Z_3)$
[2] $SU_5 \times U_1'$ $27=2(\bar{5};-\frac{1}{2\sqrt{15}})+2(1;\frac{1}{2}\sqrt{\frac{5}{3}})+(10;\frac{1}{\sqrt{15}})+(5;-\frac{2}{\sqrt{15}})$	$SU_2^N \times U_1$
[3] $SU_4 \times SU_2 \times U_1$ $27=2(\bar{4},1;\frac{1}{2\sqrt{6}})+(4,2;\frac{1}{2\sqrt{6}})+(6,1;-\frac{1}{\sqrt{6}})+2(1,2;-\frac{1}{\sqrt{6}})+(1,1;\sqrt{\frac{2}{3}})$	$SU_2^N \times U_1$
[4] $SU_3 \times SU_3 \times U_1$ $27=2(\bar{3},1;-\frac{1}{2\sqrt{3}})+2(1,\bar{3};\frac{1}{2\sqrt{3}})+(3,3;0)+(\bar{3},1;\frac{1}{\sqrt{3}})+(1,\bar{3};-\frac{1}{\sqrt{3}})$	$SU_2^N \times U_1 (\times Z_3)$
[5] $SU_3^C \times SU_3^L \times U_1^Y \times U_1'$ $27=(3,2;\frac{1}{2\sqrt{15}},\frac{1}{\sqrt{15}})+(\bar{3},1;-\frac{2}{\sqrt{15}},\frac{1}{\sqrt{15}})+(1,1;\sqrt{\frac{3}{5}},\frac{1}{\sqrt{15}})+(3,1;-\frac{1}{\sqrt{15}},-\frac{2}{\sqrt{15}})$ $+(1,2;\frac{1}{2}\sqrt{\frac{3}{5}},-\frac{2}{\sqrt{15}})+2(\bar{3},1;\frac{1}{\sqrt{15}},-\frac{1}{2\sqrt{15}})+2(1,2;-\frac{1}{2}\sqrt{\frac{3}{5}},-\frac{1}{2\sqrt{15}})+2(1,1;0,\frac{1}{2}\sqrt{\frac{5}{3}})$	$SU_2^N \times U_1^Y \times U_1'$

Table III

	(A)	(B)	(C)
[1]	$(1)(1)\begin{pmatrix}1&&\\&-1&\\&&-1\end{pmatrix}\ (x(1)(e^{i\frac{2\pi}{3}})(e^{-i\frac{2\pi}{3}}))$	3	2
[2]	$(1)\begin{pmatrix}a&&\\&a&\\&&a^{-2}\end{pmatrix}\ \begin{pmatrix}a^{-1}&&\\&a^{-1}&\\&&a^{-2}\end{pmatrix}$	2	1
[3]	$(1)(e^{i\frac{2\pi}{3}})(e^{i\frac{2\pi}{3}})\ (x(1)(e^{i\frac{2\pi}{3}})(e^{-i\frac{2\pi}{3}})$	1	1
[4]	$(1)(1)\begin{pmatrix}1&&\\&a&\\&&a^{-1}\end{pmatrix}\ (x(1)(e^{i\frac{2\pi}{3}})(e^{-i\frac{2\pi}{3}}))$	2	1
[5]	$(1)\begin{pmatrix}a^{2}&&\\&a^{2}&\\&&a^{-4}\end{pmatrix}\ \begin{pmatrix}a^{-2}&&\\&a&\\&&a\end{pmatrix}$	4	3
[6]	$(1)\begin{pmatrix}a&&\\&a&\\&&a^{-2}\end{pmatrix}\ \begin{pmatrix}-a^{-1}&&\\&-a^{-1}&\\&&a^{2}\end{pmatrix}$	2	2
[7]	$(1)(1)\begin{pmatrix}a^{-2}&&\\&a&\\&&a\end{pmatrix}\ (x(1)(e^{i\frac{2\pi}{3}})(e^{-i\frac{2\pi}{3}}))$	3	3
[8]	$(1)\times\begin{pmatrix}ab&&\\&ab&\\&&a^{-2}b^{-2}\end{pmatrix}\times\begin{pmatrix}a^{-1}b^{-1}&&\\&a^{-1}b^{-2}&\\&&a^{2}b^{3}\end{pmatrix}$	2	1
[9]	$(1)\begin{pmatrix}b&&\\&b&\\&&b^{-2}\end{pmatrix}\ \begin{pmatrix}a^{-2}&&\\&a&\\&&a\end{pmatrix}$	2	2
[10]	$(1)\times\begin{pmatrix}a&&\\&a&\\&&a^{-2}\end{pmatrix}\times\begin{pmatrix}a^{-1}b&&\\&a^{-1}b^{-1}&\\&&a^{2}\end{pmatrix}$	2	2

[11] $(1)(1)\begin{pmatrix} a^{-2} & & \\ & ab & \\ & & ab^{-1} \end{pmatrix}$ $(x(1)(e^{i\frac{2\pi}{3}})(e^{-i\frac{2\pi}{3}}))$ 1 1

[12] $(1) \times \begin{pmatrix} b^{2}c & & \\ & b^{2}c & \\ & & b^{-4}c^{-2} \end{pmatrix} \times \begin{pmatrix} b^{-2}c^{4} & & \\ & abc^{-2} & \\ & & a^{-1}bc^{-2} \end{pmatrix}$ 1 1

<u>Table IV</u>

	(A)	(B)	(C)
[1]	$(1)(1)\begin{pmatrix} 1 & \\ & V \end{pmatrix} \left(x(1)(e^{i\frac{2\pi}{3}})(e^{-i\frac{2\pi}{3}})\right)$	1	–
[2]	$(1)\begin{pmatrix} a^2 & & \\ & a^2 & \\ & & a^{-4} \end{pmatrix} \begin{pmatrix} a^{-2} & \\ & aV \end{pmatrix}$	1	–
[3]	$(1)\begin{pmatrix} a^2 & & \\ & a^2 & \\ & & a^{-4} \end{pmatrix} \begin{pmatrix} -a^{-2} & \\ & iaV \end{pmatrix}$	1	1
[4]	$(1)(1)\begin{pmatrix} a^{-2} & \\ & aV \end{pmatrix} \left(x(1)(e^{i\frac{2\pi}{3}})(e^{-i\frac{2\pi}{3}})\right)$	1	1
[5]	$(1) \times \begin{pmatrix} b^2c & & \\ & b^2c & \\ & & b^{-4}c^{-2} \end{pmatrix} \times \begin{pmatrix} b^{-2}c^4 & \\ & bc^{-2}V \end{pmatrix}$	1	1

Table V

	(A)	(B)	(C)
	Light components (+ cc)		
$[1]_1$	$SU_6 x SU_2^L \underset{\text{2 neutrals}}{\overset{(\overline{15},1)}{\rightarrow}} SU_4 x SU_2^L x SU_2^R$	Standard Model	None
$[1]_2$	$SU_6 x SU_2^R \underset{\text{1 neutral}}{\overset{(15,1)}{\rightarrow}} SU_4 x SU_2^L x SU_2^R$	$SU_4 x SU_2^L x SU_2^R$	One (4,2,1)
$[3]$	$SU_3 x SU_3 x SU_3 \underset{\text{2 neutrals}}{\overset{(1,\bar{3},3)}{\rightarrow}} SU_3 x SU_2^L x SU_2^R x U_1$	Standard Model	Three $(1,2,2;0) + (1,2,1;-\frac{1}{2}\sqrt{\frac{3}{2}})$
$[4]$	$SU_6 x U_1^R \underset{\text{1 neutral}}{\overset{(15;0)}{\rightarrow}} SU_4 x SU_2^L x U_1^R$	$SU_4 x SU_2^L x U_1^R$	One (4,2;0)
$[5]_1$	$SU_5 x SU_2^L x U_1$ (a=±i) $\underset{\text{2 neutrals}}{\overset{(\bar{5},1;\frac{2}{\sqrt{15}})}{\rightarrow}} SU_4 x SU_2^L x U_1^R$	Standard Model	None
$[5]_2$	$SU_5 x SU_2^L x U_1$ (a=±i) $\underset{\text{1 neutral}}{\overset{(\bar{5},1;\frac{2}{\sqrt{15}})}{\rightarrow}} SU_4 x SU_2^L x U_1^R$	$SU_4 x SU_2^L x U_1^R$	None

[6]	$SU_4 \times SU_2^L \times SU_2^R \times U_1 \xrightarrow[1\ \text{neutral}]{(1,2,2;-\frac{1}{\sqrt{6}})+(1,1,1;\sqrt{\frac{2}{3}})} SU_4 \times SU_2^L \times SU_2^R$ $(a=\pm i)$	$SU_4 \times SU_2^L \times SU_2^R$	Two (1,2,2)
$[9]_1$	$SU_3 \times SU_2^L \times SU_2^N \times U_1 \times U_1 \xrightarrow[2\ \text{neutrals}]{(1,1,2;\frac{1}{\sqrt{3}},\frac{1}{2\sqrt{3}})} SU_3 \times SU_2^L \times U_1 \times U_1$ $(ab^2=1)$	Standard Model	None
$[9]_2$	$SU_3 \times SU_2^L \times SU_2^R \times U_1 \times U_1 \xrightarrow[1\ \text{neutral}]{(1,1,2;\frac{1}{\sqrt{3}},\frac{1}{2\sqrt{3}})} SU_3 \times SU_2^L \times U_1 \times U_1$ $(ab^2=1)$	$SU_3 \times SU_2^L \times U_1 \times U_1$	None
	$(a=b) \xrightarrow[1\ \text{neutral}]{(1,2,2;-\frac{1}{2\sqrt{3}},\frac{1}{2\sqrt{3}})+(1,1,1;\frac{1}{\sqrt{3}},-\frac{1}{\sqrt{3}})} SU_3 \times SU_2^L \times SU_2^R \times U_1$	$SU_3 \times SU_2^L \times SU_2^R \times U_1$	Two (1,2,2;0)
	$(a=-b) \xrightarrow[1\ \text{neutral}]{(1,1,1;\frac{1}{\sqrt{3}},-\frac{1}{\sqrt{3}})} SU_3 \times SU_2^L \times SU_2^R \times U_1$	$SU_3 \times SU_2^L \times SU_2^R \times U_1$	None
$[10]_1$	$SU_4 \times SU_2^L \times U_1^N \times U_1 \xrightarrow[1\ \text{neutral}]{(\bar{4},1;\frac{1}{2},\frac{1}{2\sqrt{6}})} SU_3 \times SU_2^L \times U_1 \times U_1$ $(a=b)$	$SU_3 \times SU_2^L \times U_1 \times U_1$	None

[10]$_2$	$SU_4 x SU_2^L x U_1^R x U_1$ (a=b) $\xrightarrow[1\ \text{neutral}]{(\bar{4},1;\ \frac{1}{2},\ \frac{1}{2\sqrt{6}})}$	$SU_3 x SU_2^L x U_1 x U_1$	$SU_3 x SU_2^L x U_1 x U_1$	None
	$(6,1;0,-\frac{1}{\sqrt{6}})+(1,1;0,\sqrt{\frac{2}{3}})$ (a = − 1) $\xrightarrow[1\ \text{neutral}]{}$	$SU_4 x SU_2^L x U_1^R$	$SU_4 x SU_2^L x U_1^R$	None
	(a = ±i) $\xrightarrow[1\ \text{neutral}]{(1,1;0,\ \sqrt{\frac{2}{3}})}$	$SU_4 x SU_2^L x U_1^R$	$SU_4 x SU_2^L x U_1^R$	None
[12]	$SU_3 x SU_2^L x U_1^Y x U_1 x U_1$ (ab^5=1) $\xrightarrow[1\ \text{neutral}]{(1,1;0,\ \frac{1}{2}\sqrt{\frac{5}{3}},\ \frac{1}{2})}$	$SU_3 x SU_2^L x U_1^Y x U_1$	$SU_3 x SU_2^L x U_1^Y x U_1$	None