

Source-Channel Coding under Energy, Delay and Buffer Constraints

Oner Orhan, *Student Member, IEEE*, Deniz Gündüz, *Senior Member, IEEE*, and Elza Erkip, *Fellow, IEEE*

Abstract—Source and channel coding for an energy-limited wireless sensor node is investigated. The sensor node observes independent Gaussian source samples with variances changing over time slots. The channel is modeled as a flat fading channel, whose gain remains constant during each time slot, and changes from one time slot to the next. The compressed samples are stored in a finite data buffer, and need to be delivered to the destination in at most d time slots. The objective is to minimize the average squared-error distortion between the source samples and their reconstructions. First, a battery operated system, in which the sensor node has a finite amount of energy at the beginning of transmission, is investigated. Then, the impact of energy harvesting, and the energy cost of processing and sampling are considered. The optimal compression and transmission policy is formulated as the solution of a convex optimization problem, and the properties of the optimal policies are identified. For the strict delay case, $d = 1$, a two-dimensional (2D) waterfilling interpretation is provided. Numerical results are presented to illustrate the structure of the optimal policy, and to analyze the effect of the delay constraints, data buffer size, energy harvesting, and processing and sampling costs.

I. INTRODUCTION

Wireless sensor nodes measure physical phenomena, compress their measurements and transmit the compressed data to a destination such that the reconstruction distortion at the destination is minimized subject to delay constraints. Various components of a wireless sensor node consume energy, including sensing, processing and communications modules. The small size and low cost of typical sensors impose restrictions on the available energy, size of the battery and data buffers, and efficiency of sensing and transmission circuitry. When the time variations of the physical environment and the communication channel are also considered, the optimal management of the available energy is essential to ensure minimal reconstruction distortion at the destination with limited resources.

A. Contributions

We consider a wireless sensor node that collects samples of a Gaussian source and delivers them to a destination. To model the time-varying nature of the source and the channel, we consider a time slotted system such that the source variance and the channel power gain remain constant within each time slot, which spans n uses of the channel, and change from one time slot to the next. We assume that the source samples arrive at the beginning of each time slot, and need to be delivered within d time slots. The data buffer, which stores

the compressed samples, has finite capacity. We first assume that the sensor node is run by a battery, and energy is only consumed for data transmission. Our goal is to identify the optimal power and compression rate/distortion allocation over a finite time horizon such that the average distortion at the destination is minimized. This problem is formulated under the offline optimization framework, that is, we assume that the sensor node knows all the source variances and channel gains a priori. We show that this problem can be cast into the convex optimization framework, which allows us to identify the necessary and sufficient conditions for the optimal power and distortion allocation. For the special case of strict delay constraints, i.e., $d = 1$, we show that the optimal strategy has a *two-dimensional (2D) waterfilling* interpretation.

We then extend the above model to study various practical energy constraints on the sensor node. First, we investigate the effect of energy harvesting, and consider a model in which a new energy packet arrives (or becomes available) at the beginning of each time slot. Then, we concentrate on various sources of energy consumption in the sensor such as the operation of transmitter circuitry (digital-to-analog converters, mixers, filters) and the sensing components (source acquisition, sampling, quantization, and compression). We model the former energy cost by the processing cost ϵ_p Joules per channel use, and the latter by the sampling cost ϵ_s Joules per sample. We assume that these energy costs are constant and independent of the transmission power. We show that the offline optimization problem retains its convexity in the presence of energy harvesting, and processing and sampling costs. Accordingly, we identify the properties of the optimal power and distortion allocation under these constraints.

B. Related Work

In recent years optimal energy management policies for joint source-channel coding has received increasing attention. In [1] the fundamental energy-distortion trade-off is studied for an energy-limited joint source-channel coding system. Optimal energy allocation to minimize the sum distortion for uncoded analog transmission of multiple sensors is investigated in [2], [3], where [2] considers a battery operated system, while [3] extends [2] to energy harvesting wireless nodes under finite and infinite energy storage with both causal and non-causal side information about channel gains and energy arrivals. In contrast, we consider coded source-channel transmission strategies. Separate source and channel coding is also considered in [4], [5], [7] for an energy harvesting transmitter, where optimal energy allocation is investigated. In [4], compression

and transmission rates are jointly optimized to guarantee stability of the data queue for stochastic energy arrivals taking into consideration the energy used for source compression. This is extended in [5] to incorporate battery and memory constraints. Our work, on the other hand, considers non-causal knowledge of channel gains, source variances and energy arrivals. Also, note that [2]-[5] do not take into account delay considerations. Another related work is [6], where the problem of sensing and transmission for parallel Gaussian sources for a battery operated transmitter with sensing cost is studied. Our model generalizes that of [6] to arbitrary delay $d > 1$, energy harvesting and processing cost. Our previous work [7] considers delay limited transmission of a time varying Gaussian source over a fading channel with infinite memory size, which is extended in this paper to finite memory size under the energy cost of sampling and processing.

There is also a rich literature on energy harvesting transmission policies for throughput optimization ignoring the source coding aspects. See [8] for an overview of the recent developments. The throughput maximization problem for a fading link is studied in [9]. In [10], an energy harvesting system is studied under battery constraints, such as battery leakage and limited battery size. The effect of processing cost on throughput maximizing policies are studied for battery operated parallel Gaussian channels in [11], for energy harvesting single-link in [12]-[14], and for energy harvesting broadband channel in [15].

The paper is organized as follows. In the next section, we describe the system model. In Section III, we investigate distortion minimization for a battery-run system, and provide the properties of optimal distortion and power allocation. We also propose a $2D$ waterfilling algorithm for $d = 1$. We study distortion minimization with energy constraints in Section IV. We investigate the structure of the optimal distortion and power allocation, and provide $2D$ directional waterfilling algorithm in the presence of energy harvesting, and processing and sampling costs in Sections IV-A, IV-B, IV-C respectively. In Section V, numerical results are presented, and Section VI concludes the paper.

II. SYSTEM MODEL

We consider a wireless sensor node measuring source samples that are independent and identically distributed (i.i.d.) with a given distribution. Due to the potentially time-varying nature of the underlying physical phenomena, we assume that the statistical properties of the source samples change over time. To model this change, we consider a time slotted system with N time slots, with time slot containing n source samples. We denote the samples arriving at time slot i as source i , and assume that the samples of source i come from a zero-mean Gaussian distribution with variance σ_i^2 . The samples are compressed and stored in a data buffer of size B_{max} bits/source sample. In addition, in order to model delay-limited scenarios, e.g., real-time applications, we impose delay constraints on the samples, such that samples arriving in a time slot need to be delivered within at most d time slots. After d time slots, samples become stale, and we set the corresponding distortion to its maximum value, σ_i^2 .

We consider that the collected samples are delivered over a fading channel having an additive white Gaussian noise (AWGN) with zero mean and unit variance. We assume that the real valued channel power gain remains constant within each time slot, and its value for time slot i is denoted by h_i . Assuming that the time slot durations in terms of channel uses are large enough to invoke Shannon capacity arguments, the maximum transmission rate in time slot i is given by the Shannon capacity $\frac{1}{2} \log(1 + h_i p_i)$, where p_i indicates the average transmission power in time slot i . Since the source statistics do not change within a time slot, constant power transmission within each time slot can be shown to be optimal. This follows from the concavity and the monotonically increasing property of the Shannon capacity. We also assume that the number of source samples collected in each time slot is equal to the number of channel uses. However, the results in this paper can be easily extended to bandwidth expansion/compression.

Since the samples are continuous valued, lossy reconstruction at the destination is unavoidable. We consider mean squared error distortion criterion on the samples at the destination. Denoting the average distortion of the source i by D_i , the objective is to minimize $D \triangleq \frac{1}{N} \sum_{i=1}^N D_i$. We are interested in *offline optimization*, that is, we assume that the transmitter knows all the sample variances and the channel gains for time slots $i = 1, \dots, N$ in advance. A *transmission policy* refers to the average transmission power p_i and average distortion D_i allocated to channel i and source samples collected in time slot i , respectively, for $i = 1, \dots, N$. We study the optimal transmission policy under different energy constraints. First, we consider a battery operated system in which the sensor node has a total of E Joules of energy available at the beginning of transmission. Then, we take over other energy constraints into account, including energy harvesting, and energy cost of processing and sampling. For the energy harvesting system, we assume that the sensor harvests energy packets of size E_i Joules at the beginning of time slot i , $i = 1, \dots, N$. The processing cost is modelled as constant ϵ_p Joules per transmitted symbol, and it is assumed to be independent of the transmission power. The sampling cost is also assumed to be constant, and considered as ϵ_s Joules per source sample and independent of the sampling rate [4].

This formulation considers separate source and channel coding. We can equivalently model this point-to-point communication problem as multiterminal source-channel communication under orthogonal multiple access as shown in Figure 1. In this correspondence, Encoder i corresponds to the encoder at time slot i which observes source samples over the last d time slots, and transmits over the channel within time slot i . Similarly, we can consider a separate decoder for each time slot i , $i = d, d+1, \dots, N$, such that Decoder i observes channel outputs $i-d+1, \dots, i$, and reconstructs the source samples that have been accumulated within time slot $i - (d-1)$. Note that this is equivalent to decoding the source samples just before their deadline expires, since decoding them earlier does not gain anything in terms of the average distortion. Using [16] we can argue the optimality of source-channel separation in this setting; hence the above formulation gives us the optimal average distortion.

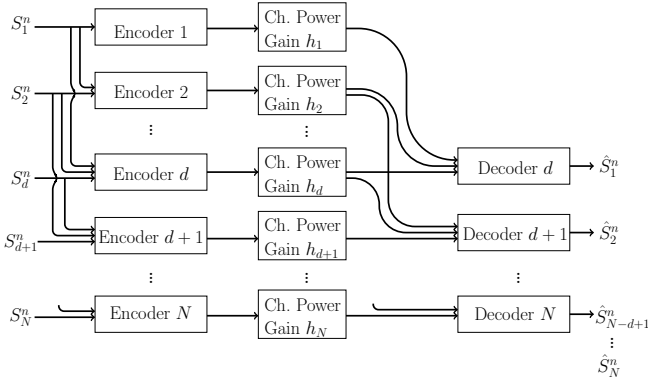


Figure 1. Equivalent multiterminal source-channel communication scenario under orthogonal multiple access. S_i^n denotes source samples in time slot i , $\hat{S}_i^n$ denotes their reconstruction at the receiver.

III. DISTORTION MINIMIZATION FOR A BATTERY-RUN SYSTEM

We assume that the sensor node has E Joules of energy at the beginning of transmission. We focus only on the energy consumption of the power amplifier, and ignore any energy cost due to processing and sampling. We denote the rate allocated to source i in time slot j , $j \leq N$, as $R_{i,j}$. Note that $R_{i,j} = 0$ for $i + d \leq j$, or $j < i$. In a feasible transmission policy, the transmission power in time slot j limits the maximum rate that can be transmitted over that time slot. Therefore, any feasible transmission policy should satisfy the following constraints:

$$\sum_{i=j-d+1}^j R_{i,j} \leq \frac{1}{2} \log(1 + h_j p_j), \quad j = 1, \dots, N, \quad (1)$$

where $R_{i,j} = 0$ for $i < 1$. The rate-distortion theorem in [18] states that the average distortion of the samples taken at time slot i , D_i , should satisfy the following inequalities:

$$\frac{1}{2} \log\left(\frac{\sigma_i^2}{D_i}\right) \leq \sum_{j=i}^{i+d-1} R_{i,j}, \quad i = 1, \dots, N. \quad (2)$$

In addition, the limited data buffer size imposes the following constraints:

$$\sum_{j=k}^{k+d-1} \sum_{i=j-d+1}^k R_{i,j} \leq B_{max}, \quad k = 1, \dots, N. \quad (3)$$

Remark 1: Note that the buffer size constraint is in terms of the total bits per sample for those sources that have not yet expired. This would mean that the buffer size is infinite since the above assumptions of capacity and rate-distortion achieving codes stipulate $n \rightarrow \infty$.

The goal is to identify $R_{i,j}$ and D_i values that minimize $D = \frac{1}{N} \sum_{i=1}^N D_i$ under constraints (1)-(3). It can be shown using Fourier-Motzkin elimination [17] that (1)-(3) are equivalent to the following causality, delay and rate constraints, respectively. The proof of Fourier-Motzkin elimination for the case of three time slots with delay constraint $d = 2$ is given in Appendix.

$$\sum_{j=i}^N r_j \leq \sum_{j=i}^N c_j, \quad i = 1, \dots, N, \quad (4)$$

$$\sum_{j=k}^i r_j \leq \sum_{j=k}^{i+d-1} c_j, \quad i = k, \dots, N-d, k = 1, \dots, N-d, \quad (5)$$

$$\sum_{j=k}^{i+1} r_j \leq \sum_{j=k}^i c_j + B_{max}, \quad i = k, \dots, N-1, \quad k = 1, \dots, N-1 \quad (6)$$

$$r_i \leq B_{max}, \quad i = 1, \dots, N, \quad (7)$$

where $r_i \triangleq \frac{1}{2} \log\left(\frac{\sigma_i^2}{D_i}\right)$ and $c_i \triangleq \frac{1}{2} \log(1 + h_i p_i)$. Notice that r_i corresponds to the source coding rate for the samples collected in time slot i , and c_i is the channel capacity for time slot i for power p_i and channel gain h_i . The causality constraints in (4) suggest that the samples can only be transmitted after they have arrived. The delay constraints in (5) stipulate that the samples collected in time slot i need to be delivered to the destination until the end of time slot $i + d - 1$. The data buffer constraints in (6)-(7) impose restrictions on the amount of bits per sample. The goal of the transmitter is to allocate its transmission power p_i within each time slot and choose distortion level D_i for each source, $i = 1, \dots, N$, such that the causality, delay, and data buffer constraints are satisfied, while the average distortion D at the destination is minimized. Then, the optimization problem can be formulated as follows.

$$\min_{r_i, c_i} \quad \frac{1}{N} \sum_{i=1}^N \sigma_i^2 2^{-2r_i} \quad (8a)$$

$$\text{s.t.} \quad \sum_{i=1}^N \frac{2^{2c_i} - 1}{h_i} \leq E, \quad (8b)$$

$$\sum_{j=i}^N r_j \leq \sum_{j=i}^N c_j, \quad i = 1, \dots, N, \quad (8c)$$

$$\sum_{j=k}^i r_j \leq \sum_{j=k}^{i+d-1} c_j, \quad i = k, \dots, N-d, \quad k = 1, \dots, N-d, \quad (8d)$$

$$\sum_{j=k}^{i+1} r_j \leq \sum_{j=k}^i c_j + B_{max}, \quad i = k, \dots, N-1, \quad k = 1, \dots, N-1, \quad (8e)$$

$$0 \leq r_i \leq B_{max} \text{ and } 0 \leq c_i, \quad i = 1, \dots, N. \quad (8f)$$

where the constraint in (8b) ensures that the total consumed energy is less than the energy available in the battery at $t = 0$. The constraints in (8c), (8d), and (8e) are the causality, delay and data buffer size constraints from (4), (5), and (6), respectively. Since the optimization problem in (8) is convex, we can compute the optimal solution by efficient numerical methods [19]. In the following, we investigate the properties of the optimal solution using the Karush-Kuhn-Tucker (KKT) optimality conditions. The Lagrangian of (8) is defined as follows:

$$\begin{aligned}
\mathcal{L} = & \frac{1}{N} \sum_{i=1}^N \sigma_i^2 2^{-2r_i} + \lambda \left(\sum_{i=1}^N \frac{2^{2c_i} - 1}{h_i} - E \right) \\
& + \sum_{i=1}^N \gamma_i \left(\sum_{j=i}^N r_j - \sum_{j=i}^N c_j \right) \\
& + \sum_{k=1}^{N-d} \sum_{i=k}^{N-d} \delta_{i,k} \left(\sum_{j=k}^i r_j - \sum_{j=k}^{i+d-1} c_j \right) \\
& + \sum_{k=1}^{N-1} \sum_{i=k}^{N-1} \zeta_{i,k} \left(\sum_{j=k}^{i+1} r_j - \sum_{j=k}^i c_j - B_{max} \right) \\
& - \sum_{i=1}^N \beta_i r_i + \sum_{i=1}^N \rho_i (r_i - B_{max}) - \sum_{i=1}^N \mu_i c_i, \quad (9)
\end{aligned}$$

where $\lambda \geq 0$, $\gamma_i \geq 0$, $\delta_{i,k} \geq 0$, $\zeta_{i,k} \geq 0$, $\beta_i \geq 0$, $\rho_i \geq 0$ and $\mu_i \geq 0$ are KKT multipliers corresponding to (8b)-(8f).

Taking the derivative of the Lagrangian with respect to r_i and c_i and setting it to zero, we get

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial r_i^*} = & -\frac{2(\ln 2)}{N} \sigma_i^2 2^{-2r_i^*} + \sum_{j=1}^i \gamma_j + \sum_{k=1}^i \sum_{j=i}^{N-d} \delta_{j,k} \\
& + \sum_{k=1}^i \sum_{j=i-1}^{N-1} \zeta_{j,k} - \beta_i + \rho_i = 0, \quad \forall i, \quad (10)
\end{aligned}$$

where $\zeta_{i-1,i} = 0$ for $\forall i$, and

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial c_i^*} = & \lambda \frac{2(\ln 2) 2^{2c_i^*}}{h_i} - \sum_{j=1}^i \gamma_j - \sum_{k=1}^i \sum_{j=i-d+1}^{N-d} \delta_{j,k} \\
& - \sum_{k=1}^i \sum_{j=i}^{N-1} \zeta_{j,k} - \mu_i = 0, \quad \forall i, \quad (11)
\end{aligned}$$

where $\delta_{j,k} = 0$ for $j < k$.

A. Optimal Distortion Allocation

From (10), replacing r_i^* with $\frac{1}{2} \log \left(\frac{\sigma_i^2}{D_i^*} \right)$, we obtain

$$D_i^* = \frac{N}{2 \ln 2} \left[\sum_{j=1}^i \gamma_j + \sum_{k=1}^i \sum_{j=i}^{N-d} \delta_{j,k} + \sum_{k=1}^i \sum_{j=i-1}^{N-1} \zeta_{j,k} - \beta_i + \rho_i \right] \quad (12)$$

The complementary slackness conditions require that, whenever $\beta_i > 0$, we have $D_i^* = \sigma_i^2$, and whenever $\rho_i > 0$, we have $D_i^* = \sigma_i^2 2^{-2B_{max}}$. Therefore, the optimal distortion D_i^* can be further simplified as

$$D_i^* = \begin{cases} \sigma_i^2 2^{-2B_{max}}, & \text{if } \xi_i \leq \sigma_i^2 2^{-2B_{max}}, \\ \xi_i, & \text{if } \sigma_i^2 2^{-2B_{max}} < \xi_i < \sigma_i^2, \\ \sigma_i^2, & \text{if } \xi_i \geq \sigma_i^2, \end{cases} \quad (13)$$

where ξ_i is defined as:

$$\xi_i \triangleq \frac{N}{2 \ln 2} \left(\sum_{j=1}^i \gamma_j + \sum_{k=1}^i \sum_{j=i}^{N-d} \delta_{j,k} + \sum_{k=1}^i \sum_{j=i-1}^{N-1} \zeta_{j,k} \right). \quad (14)$$

Note that ξ_i is similar to the *reverse water level* in the classical solution of the optimal distortion levels for parallel

Gaussian sources [18, Chapter 10, Section 3]. In that classical problem, we are concerned with allocating the available fixed rate among independent Gaussian sources to minimize the average distortion. The optimal solution of this problem is of the “reverse waterfilling type,” that is, there is a fixed reverse water level ξ which determines the optimal distortion of source i as $D_i^* = \{\xi, \sigma_i^2\}$. While the classical solution has a fixed reverse water level, independent of i , in our formulation, due to the causality, delay and data buffer size constraints, the reverse water level depends on the source index i . Note that the optimal distortion D_i^* is confined to the interval $[\sigma_i^2 2^{-2B_{max}}, \sigma_i^2]$ for time slot i . Next, we identify some properties of the optimal distortion allocation.

Lemma 1: Whenever the reverse water level ξ_i in (14) increases from time slot i to time slot $i+1$, all samples collected until time slot i must be transmitted by the end of time slot i , and whenever ξ_i decreases from time slot i to time slot $i+1$, either the data buffer is full at the beginning of time slot i and/or delivery of the samples collected at time slot k , $k \in i+1, \dots, i+d-2$, is postponed by $i-k+d$ time slots.

Proof: From (14), we have

$$\begin{aligned}
\xi_{i+1} - \xi_i = & \frac{1}{2 \ln 2} \left(\gamma_{i+1} + \sum_{j=i+1}^{N-d} \delta_{j,i+1} + \sum_{j=i+1}^{N-1} \zeta_{j,i+1} \right. \\
& \left. - \sum_{k=1}^{i-1} \zeta_{i-1,k} - \sum_{k=1}^i \delta_{i,k} \right), \quad i = 1, \dots, N-1. \quad (15)
\end{aligned}$$

Therefore, when $\xi_{i+1} - \xi_i > 0$, either γ_{i+1} or, for some $j > i$, $\delta_{j,i+1}$ or $\zeta_{j,i+1}$, must be positive. From the complementary slackness conditions, we know that whenever $\gamma_{i+1} > 0$, the constraint in (8c) is satisfied with equality, i.e., $\sum_{j=i+1}^N r_j = \sum_{j=i+1}^N c_j$. This means that all samples collected until time slot i must be transmitted by the end of time slot i since the later time slots can only support the source rates r_j , $j \geq i+1$. In addition, from the complementary slackness conditions and the constraint in (8d), we can conclude that when $\delta_{j,i+1} > 0$, $\sum_{k=i+1}^j r_k = \sum_{k=i+1}^{j+d-1} c_k$ for $j \geq i+1$ must be satisfied. Since only samples collected at time slots $i+1, \dots, j$ are delivered in time slots $i+1, \dots, j+d-1$, and each group of source samples has a delay constraint of d time slots, the samples collected until time slot i should be delivered by the end of time slot i . Similarly, from the complementary slackness conditions and the constraint in (8e), we can argue that if $\zeta_{j,i+1} > 0$ then $\sum_{k=i+1}^{j+1} r_k - \sum_{k=i+1}^j c_k = B_{max}$ for $j \geq i+1$ must be satisfied. This means that the data arriving between time slots $i+1$ and $j+1$ leads to a full data buffer at time slot $j+1$ for $j \geq i+1$, so all the samples collected until time slot i must be transmitted by the end of time slot i . Therefore, whenever ξ_i in (14) increases from time slot i to time slot $i+1$, all samples collected by time slot i must be transmitted until the end of time slot i . Note that this leads to an empty data buffer at the end of time slot i which follows from the positivity of γ_{i+1} , $\delta_{j,i+1}$, $\zeta_{j,i+1}$ for some $j \geq i+1$.

On the other hand, from the complementary slackness conditions and the constraint in (8d), we can conclude that when $\delta_{i,k} > 0$, $\sum_{j=k}^i r_j = \sum_{j=k}^{i+d-1} c_j$ for $k \leq i$ should be satisfied. Therefore, samples collected at time slot $i+1$ should

be delayed d time slots since time slots $i + 1, \dots, i + d - 1$ are allocated for the delivery of samples that have arrived at time slots $k \leq i$. Similarly, from the complementary slackness conditions and the constraint in (8e), we can argue that if $\zeta_{i-1,k} > 0$ then $\sum_{j=k}^i r_j - \sum_{j=k}^{i-1} c_j = B_{max}$ for $k \leq i - 1$ must be satisfied. This means that the data buffer must be full at the beginning of time slot i . Whenever ξ_i decreases from time slot i to time slot $i + 1$, $\delta_{i,k} > 0$ for some $k \leq i$, or $\zeta_{i-1,k} > 0$ for some $k \leq i - 1$. We can conclude that whenever ξ_i decreases from time slot i to time slot $i + 1$, either the data buffer is full at the beginning of time slot i and/or the delivery of the samples collected at time slot k , $k \in i + 1, \dots, i + d - 2$, is postponed by at least $i - k + d$ time slots. ■

B. Optimal Power Allocation

We can identify the optimal power allocation by replacing c_i^* with $\frac{1}{2} \log(1 + h_i p_i^*)$ in (11). The optimal power allocation is given as follows.

$$p_i^* = \left[\frac{1}{2(\ln 2)\lambda} \left(\sum_{j=1}^i \gamma_j + \sum_{k=1}^i \sum_{j=i-d+1}^{N-d} \delta_{j,k} + \sum_{k=1}^i \sum_{j=i}^{N-1} \zeta_{j,k} \right) - \frac{1}{h_i} \right]^+, \quad (16)$$

where $\delta_{j,k} = 0$ for $j < k$. We define $\nu_i \triangleq \frac{\sum_{j=1}^i \gamma_j + \sum_{k=1}^i \sum_{j=i-d+1}^{N-d} \delta_{j,k} + \sum_{k=1}^i \sum_{j=i}^{N-1} \zeta_{j,k}}{2(\ln 2)\lambda}$, which can be interpreted similarly to the classical waterfilling solution obtained for power allocation over parallel channels with *water level* being equal to ν_i . Similarly to (13), ν_i depends on i due to causality, delay and data buffer size constraints. Next, we provide some properties of the optimal power allocation.

Lemma 2: Whenever the water level ν_i in (14) increases from time slot i to time slot $i + 1$, all the samples collected until time slot i must be transmitted by the end of time slot i , and whenever ν_i decreases from time slot i to time slot $i + 1$, either the data buffer is full at the beginning of time slot $i + 1$ and/or the delivery of the samples collected at time slot k , $k \in i - d + 2, \dots, i$, is postponed by at least $i - k + 1$ time slots.

Proof: We can show that $\nu_{i+1} - \nu_i = \frac{\gamma_{i+1} + \sum_{j=i+1}^{N-d} \delta_{j,i+1} + \sum_{j=i+1}^{N-1} \zeta_{j,i+1} - \sum_{k=1}^{i-d+1} \delta_{i-d+1,k} - \sum_{k=1}^i \zeta_{i,k}}{2(\ln 2)\lambda}$. Using arguments similar to the proof of Lemma 1, the proof can be completed. ■

Remark 2: When there is no delay constraint, i.e., $d = N$, the constraint in (8d) is no longer necessary and $\delta_{i,k} = 0$, $\forall i, k$. Therefore, from Lemma 1 (Lemma 2), we can argue that full data buffer at the beginning of time slot i ($i + 1$) is the only reason of a decrease in the reverse water level ξ_i (the water level ν_i) from time slot i to time slot $i + 1$.

Remark 3: When the data buffer size is infinite, i.e., $B_{max} = \infty$, we have $\zeta_{i,k} = 0$, $\forall i, k$. Following the arguments in Lemma 1 (Lemma 2), we can conclude that whenever the reverse water level ξ_i (the water level ν_i) decreases from time slot i to time slot $i + 1$, delivery of the samples collected at time slot k , $k \in i + 1, \dots, i + d - 2$ ($k \in i - d + 2, \dots, i$) must be postponed by $i - k + d$ ($i - k + 1$) time slots.

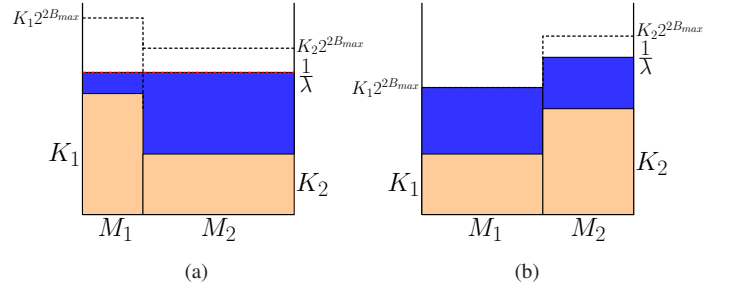


Figure 2. 2D waterfilling algorithm, (a) data buffer constraint is not active, (b) data buffer constraint is active.

C. Strict delay constraint ($d = 1$)

In this section, we investigate the case in which the source samples collected in time slot i need to be transmitted within time slot i , i.e., $d = 1$. Note that this is equivalent to the problem investigated in [6] with zero sensing cost, in which the minimization of total distortion of parallel Gaussian sources for a battery operated transmitter with sensing cost is studied. Here we provide a 2D waterfilling interpretation for the solution. The optimization problem in (8) can be formulated as follows for $d = 1$:

$$\min_{c_i} \quad \frac{1}{N} \sum_{i=1}^N \sigma_i^2 2^{-2c_i} \quad (17a)$$

$$\text{s.t.} \quad \sum_{i=1}^N \frac{2^{2c_i} - 1}{h_i} \leq E, \quad (17b)$$

$$0 \leq c_i \leq B_{max}, \quad i = 1, \dots, N, \quad (17c)$$

where $c_i = \frac{1}{2} \log(1 + h_i p_i) = \frac{1}{2} \log\left(\frac{\sigma_i^2}{D_i}\right)$.

Solving the above optimization problem we find

$$p_i^* = \frac{\sigma_i}{\sqrt{h_i}} \left[\min \left\{ \frac{2^{2B_{max}}}{\sigma_i \sqrt{h_i}}, \frac{1}{\lambda} \right\} - \frac{1}{\sigma_i \sqrt{h_i}} \right]^+. \quad (18)$$

Defining $M_i \triangleq \frac{\sigma_i}{\sqrt{h_i}}$ and $K_i \triangleq \frac{1}{\sigma_i \sqrt{h_i}}$, the optimal power in (18) can be written as

$$p_i^* = M_i \left[\min \left\{ K_i 2^{2B_{max}}, \frac{1}{\lambda} \right\} - K_i \right]^+. \quad (19)$$

Since $\frac{1}{2} \log\left(\frac{\sigma_i^2}{D_i}\right) \leq \frac{1}{2} \log(1 + h_i p_i)$ is satisfied with equality for $d = 1$, from (19) the optimal distortion D_i^* is given by

$$D_i^* = \begin{cases} \sigma_i^2 2^{-2B_{max}}, & \text{if } M_i \lambda \leq \sigma_i^2 2^{-2B_{max}}, \\ M_i \lambda, & \text{if } \sigma_i^2 2^{-2B_{max}} < M_i \lambda < \sigma_i^2, \\ \sigma_i^2, & \text{if } M_i \lambda \geq \sigma_i^2. \end{cases} \quad (20)$$

The above solution is illustrated in Fig. 2 for $N = 2$. For each time slot, we have rectangles of width M_i and height K_i . The total energy is poured above the level K_i for each time slot up to the water level $\frac{1}{\lambda}$. The power allocated to time slot i is given by the shaded area below the water level and above K_i . Note that the water level is bounded by the data buffer size, i.e., $K_i 2^{2B_{max}}$, as argued in (19). If $p_i^* > 0$, the distortion for source i is given by the width M_i times the reciprocal of the water level, and if $p_i^* = 0$, the distortion for source i is $\sigma_i^2 = \frac{M_i}{K_i}$. As seen in Fig. 2(a) the water level is constant over the two time slots, therefore, the optimal allocated power in time slot i is given by $M_i \left(\frac{1}{\lambda} - K_i \right)$ for $i = 1, 2$, and the

optimal distortion is given by $M_i \lambda$. However, in Fig 2(b) the water level in the first time slot is limited by $K_1 2^{2B_{max}}$ due to the data buffer constraint. Therefore, as argued in Lemma 2, the increase in the water level from the first time slot to the second is due to a full data buffer at the first time slot. The optimal power levels for the first and second time slots are given by $M_i K_i (2^{2B_{max}} - 1)$ and $M_i (\frac{1}{\lambda} - K_i)$, respectively. The optimal average distortion values are $\frac{M_1}{K_1 2^{2B_{max}}}$ and $M_2 \lambda$ for source one and two, respectively.

IV. DISTORTION MINIMIZATION UNDER VARIOUS ENERGY CONSTRAINTS

In this section, we consider additional energy constraints on the system including energy harvesting, and the energy cost of processing and sensing. We study the constraints separately to clearly illustrate their impact on the performance. In Section IV-A we identify the effect of energy harvesting on the optimal power and distortion allocation. Then, in Section IV-B we consider the energy cost of processing circuitry together with the transmission energy, and show that the optimal power allocation is bursty in this case. Finally, in Section IV-C we investigate the effect of sampling cost on the optimal power and distortion allocation.

A. Distortion Minimization with Energy Harvesting

In this section, we consider energy harvesting at the sensor node. We consider that the sensor node harvests energy packet of size E_i at the beginning of time slot i , $i = 1, \dots, N$. We consider only the transmission cost and ignore the energy cost of processing and sampling. Due to energy arrivals over time, a feasible transmission policy must satisfy the following energy causality constraints:

$$\sum_{j=1}^i \frac{2^{2c_j} - 1}{h_j} \leq \sum_{j=1}^i E_j, \quad i = 1, \dots, N. \quad (21)$$

Consequently, the optimization problem in (8) remains the same except that the constraint (8b) is replaced by (21). Then the Lagrangian of (8) with energy harvesting becomes:

$$\begin{aligned} \mathcal{L} = & \frac{1}{N} \sum_{i=1}^N \sigma_i^2 2^{-2r_i} + \sum_{i=1}^N \lambda_i \left(\sum_{j=1}^i \frac{2^{2c_j} - 1}{h_j} - \sum_{j=1}^i E_j \right) \\ & + \sum_{i=1}^N \gamma_i \left(\sum_{j=i}^N r_j - \sum_{j=i}^N c_j \right) \\ & + \sum_{k=1}^{N-d} \sum_{i=k}^{N-d} \delta_{i,k} \left(\sum_{j=k}^i r_j - \sum_{j=k}^{i+d-1} c_j \right) \\ & + \sum_{k=1}^{N-1} \sum_{i=k}^{N-1} \zeta_{i,k} \left(\sum_{j=k}^{i+1} r_j - \sum_{j=k}^i c_j - B_{max} \right) \\ & - \sum_{i=1}^N \beta_i r_i + \sum_{i=1}^N \rho_i (r_i - B_{max}) - \sum_{i=1}^N \mu_i c_i, \end{aligned} \quad (22)$$

with $\lambda_i \geq 0$, $\gamma_i \geq 0$, $\delta_{i,k} \geq 0$, $\zeta_{i,k} \geq 0$, $\beta_i \geq 0$, $\rho_i \geq 0$ and $\mu_i \geq 0$ as the KKT multipliers.

The derivative of the Lagrangian with respect to r_i is the same as in (10); hence, the structure of the optimal distortion allocation is the same as in Section III. Therefore, the properties of the optimal distortion stated in Lemma 1 still hold.

Differentiating the Lagrangian with respect to c_i and setting it to zero, we can argue that the optimal channel rate c_i^* of time slot i must satisfy

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial c_i^*} = & \frac{2(\ln 2) 2^{2c_i^*}}{h_i} \sum_{j=i}^N \lambda_j - \sum_{j=1}^i \gamma_j \\ & - \sum_{k=1}^i \sum_{j=i-d+1}^{N-d} \delta_{j,k} - \sum_{k=1}^i \sum_{j=i}^{N-1} \zeta_{j,k} - \mu_i = 0, \end{aligned} \quad (23)$$

for $i = 1, \dots, N$ where $\delta_{j,k} = 0$ for $j < k$. This leads to the optimal power level p_i^* as follows.

$$p_i^* = \left[\frac{1}{2 \ln 2 \sum_{j=i}^N \lambda_j} \left(\sum_{j=1}^i \gamma_j + \sum_{k=1}^i \sum_{j=i-d+1}^{N-d} \delta_{j,k} + \sum_{k=1}^i \sum_{j=i}^{N-1} \zeta_{j,k} \right) - \frac{1}{h_i} \right]^+, \quad \forall i. \quad (24)$$

Defining $\pi_i \triangleq \frac{\sum_{j=1}^i \gamma_j + \sum_{k=1}^i \sum_{j=i-d+1}^{N-d} \delta_{j,k} + \sum_{k=1}^i \sum_{j=i}^{N-1} \zeta_{j,k}}{2 \ln 2 \sum_{j=i}^N \lambda_j}$, we can interpret (24) similarly to the directional waterfilling solution of [9] with water level equal to π_i . Accordingly, Lemma 2 is updated as follows for an energy harvesting sensor node.

Lemma 3: Whenever the water level π_i in (14) increases from time slot i to time slot $i+1$, all the samples collected until time slot i are transmitted by the end of time slot i , and/or the battery is empty at the end of time slot i . Similarly if π_i decreases from time slot i to time slot $i+1$, the data buffer is full at beginning of time slot $i+1$, and/or the transmission of the samples collected within time slot k , $k \in i-d+2, \dots, i$, is postponed by at least $i-k+1$ time slots.

Proof: From complementary slackness conditions, we know that when $\lambda_i > 0$, the constraint in (21) is satisfied with equality, hence, the battery must be empty at the end of time slot i . Therefore, following the arguments in the proofs of Lemma 1 and 2, the proof can be completed. ■

For the case of strict delay constraint, $d = 1$, we can reformulate the optimization problem in (17) by replacing the constraint (17b) with (21). Solving the optimization problem, we obtain the optimal transmission power and distortion in terms of M_i and K_i as follows.

$$p_i^* = M_i \left[\min \left\{ K_i 2^{2B_{max}}, \frac{1}{\sqrt{\sum_{i=i}^N \lambda_i}} \right\} - K_i \right]^+. \quad (25)$$

Similarly, the optimal distortion D_i^* is given by

$$D_i^* = \begin{cases} \sigma_i^2 2^{-2B_{max}}, & \text{if } M_i \sqrt{\sum_{i=i}^N \lambda_i} < \sigma_i^2 2^{-2B_{max}}, \\ M_i \sqrt{\sum_{i=i}^N \lambda_i}, & \text{if } \sigma_i^2 2^{-2B_{max}} < M_i \sqrt{\sum_{i=i}^N \lambda_i} < \sigma_i^2, \\ \sigma_i^2, & \text{if } M_i \sqrt{\sum_{i=i}^N \lambda_i} \geq \sigma_i^2. \end{cases} \quad (26)$$

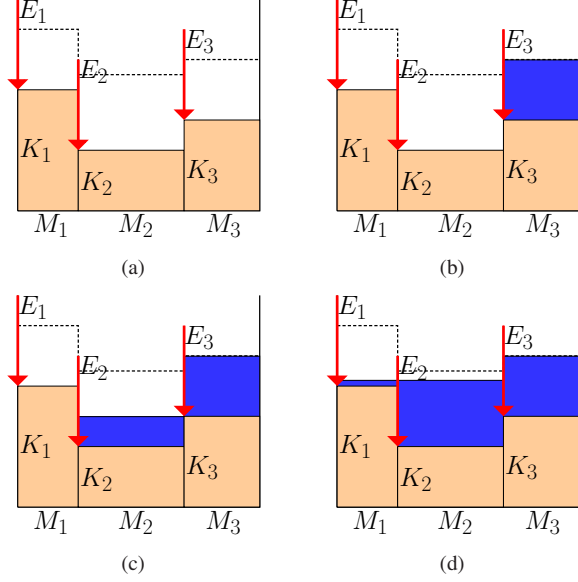


Figure 3. 2D directional water-filling algorithm. Dashed line represents the buffer constraints (a) three time slots with energy arrivals E_i , $i = 1, 2, 3$, (b) E_3 allocated to the third time slot, (c) E_2 allocated to the second time slot, (d) E_1 allocated to time slots 1 and 2.

Extending Section III-C, we can interpret the energy harvesting solution for $d = 1$ as *directional 2D water-filling* such that the harvested energy E_i can only be allocated to time slots $j \geq i$. Accordingly, we allocate energy to the following time slots starting from the last arriving energy and continuing backwards to the first such that the energy causality constraint is satisfied. In addition, allocated power to time slot i is limited by the data buffer size and channel gain, i.e., $p_i^* \leq M_i K_i (2^{2B_{max}} - 1) = \frac{1}{h_i} (2^{2B_{max}} - 1)$.

Consider the illustration in Fig. 3 with three time slots. Similarly to Fig. 2, we have rectangles of width M_i and height K_i . The horizontal dashed lines above the rectangles correspond to $K_i 2^{2B_{max}}$. The arrival times of the energy packets are represented by downward arrows. As argued above, we first allocate the last energy packet E_3 to the third time slot as shown in Fig. 3(a). Note that due to the data buffer constraint, the compression rate and the optimal power in the third time slot are limited by B_{max} and $\frac{1}{h_i} (2^{2B_{max}} - 1)$, respectively. This leads to an excessive energy in the battery if $E_3 > \frac{1}{h_3} (2^{2B_{max}} - 1)$. Then, as shown in Fig. 3(c) the second energy packet E_2 is considered for time slots two and three. Since the water level of the second time slot is lower than the third time slot, E_2 is allocated only to the second time slot. Finally, we consider the first energy packet E_1 and allocate it to the first and second time slots as shown in Fig. 3(d). As argued before, we can obtain the optimal distortion for source i by multiplying M_i with the reciprocal of the water level above rectangle i in Fig. 3(d).

B. Distortion Minimization with Processing Energy Cost

In this section, we investigate the properties of the optimal distortion and power allocation when, in addition to transmission energy, processing energy cost is also taken into account. For ease of exposure, we consider a battery operated system as

in Section III and ignore the sampling cost. We assume that the sensor node consumes energy for processing only when transmitting as in [12]. We consider that the processing energy cost is ϵ_p Joules per transmitted symbol, and it is independent of the transmission power. Note that this energy is consumed only when the transmitter is “on”. As shown in [11], when the constant processing cost is taken into account, the optimal transmission policy becomes bursty. Therefore, the optimal policy may utilize only a fraction of each time slot. We denote the transmission duration within time slot i by θ_i , $0 \leq \theta_i \leq 1$. We redefine the auxiliary variable c_i , the total delivered data in time slot i , as $c_i \triangleq \frac{\theta_i}{2} \log(1 + h_i p_i)$. Accordingly, the optimization problem in (8) remains the same except that there is an additional constraint $0 \leq \theta_i \leq 1$, and the constraint (8b) is replaced by the following energy constraint.

$$\sum_{i=1}^N \theta_i \left(\frac{2^{\frac{2c_i}{\theta_i}} - 1}{h_i} + \epsilon_p \right) \leq E. \quad (27)$$

Then, the Lagrangian of (8) with processing energy cost is given by the following.

$$\begin{aligned} \mathcal{L} = & \frac{1}{N} \sum_{i=1}^N \sigma_i^2 2^{-2r_i} + \lambda \left(\sum_{i=1}^N \theta_i \left(\frac{2^{\frac{2c_i}{\theta_i}} - 1}{h_i} + \epsilon_p \right) - E \right) \\ & + \sum_{i=1}^N \gamma_i \left(\sum_{j=i}^N r_j - \sum_{j=i}^N c_j \right) \\ & + \sum_{k=1}^{N-d} \sum_{i=k}^{N-d} \delta_{i,k} \left(\sum_{j=k}^i r_j - \sum_{j=k}^{i+d-1} c_j \right) \\ & + \sum_{k=1}^{N-1} \sum_{i=k}^{N-1} \zeta_{i,k} \left(\sum_{j=k}^{i+1} r_j - \sum_{j=k}^i c_j - B_{max} \right) \\ & - \sum_{i=1}^N \beta_i r_i + \sum_{i=1}^N \rho_i (r_i - B_{max}) - \sum_{i=1}^N \mu_i c_i \\ & - \sum_{i=1}^N \nu_i \theta_i + \sum_{i=1}^N \psi_i (\theta_i - 1), \end{aligned} \quad (28)$$

where $\lambda \geq 0$, $\gamma_i \geq 0$, $\delta_{i,k} \geq 0$, $\zeta_{i,k} \geq 0$, $\beta_i \geq 0$, $\rho_i \geq 0$, $\mu_i \geq 0$, $\nu_i \geq 0$, and $\psi_i \geq 0$ are KKT multipliers. When we take the derivative of the Lagrangian with respect to r_i , set it to zero, and replace r_i with $\frac{1}{2} \log \left(\frac{\sigma_i^2}{D_i^*} \right)$, we obtain (12). Therefore the optimal distortion allocation satisfies (13), and the properties given in Lemma 1 are also valid in this case. Differentiating the Lagrangian with respect to c_i and setting it to zero, we obtain

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial c_i^*} = & \lambda \frac{2(\ln 2) 2^{\frac{2c_i^*}{\theta_i}}}{h_i} - \sum_{j=1}^i \gamma_j - \sum_{k=1}^i \sum_{j=i-d+1}^{N-d} \delta_{j,k} \\ & - \sum_{k=1}^i \sum_{j=i}^{N-1} \zeta_{j,k} - \mu_i = 0, \quad \forall i, \end{aligned} \quad (29)$$

where $\delta_{j,k} = 0$ for $j < k$. When we replace c_i^* in the above equation with $\frac{\theta_i^*}{2} \log(1 + h_i p_i^*)$, the optimal power allocation is found as in (16). However, unlike the optimal transmission

policy in Section III, due to the processing cost the optimal transmission power p_i^* needs to be allocated θ_i^* fraction of time slot i . Taking derivative of the Lagrangian with respect to θ_i and setting it to zero, we get

$$\frac{\partial \mathcal{L}}{\partial \theta_i^*} = \lambda \left[\frac{2^{\frac{2c_i^*}{\theta_i^*}} - 1}{h_i} + \epsilon_p - \frac{2(\ln 2)c_i^* 2^{\frac{2c_i^*}{\theta_i^*}}}{h_i \theta_i^*} \right] - \nu_i + \psi_i = 0, \forall i. \quad (30)$$

Using complementary slackness conditions together with (30), we can argue that

- If $\theta_i^* = 0$, then $c_i^* = 0$ and $p_i^* = 0$.
- If $0 < \theta_i^* \leq 1$, i.e., $\nu_i = 0$, then assuming that $\lambda > 0$, i.e., the battery is depleted by the end of time slot N , and replacing c_i^* with $\frac{\theta_i^*}{2} \log(1 + h_i p_i^*)$ in (30), we get

$$\ln 2 \log(1 + h_i p_i^*) \left(\frac{1}{h_i} + p_i^* \right) = (\epsilon_p + p_i^*) + \frac{\psi_i}{\lambda}. \quad (31)$$

When $0 < \theta_i < 1$, i.e., $\psi_i = 0$, we obtain the same results as in [12, Eq. (4)]. Therefore, as argued in [12], Equation (31) has a unique solution which depends only on the channel gain and the processing cost. We denote the solution of (31) by $p_i^* = v_{p,i}$. When $\theta_i = 1$, i.e., $\psi_i \geq 0$, it can be argued from (31) that the optimal transmission power satisfies $p_i^* \geq v_{p,i}$. Note that when $\lambda = 0$, i.e., the battery may not be depleted by the end of time slot N , we can restrict the optimal power allocation to the above solution without loss of optimality.

Next, we study the optimal power and distortion allocation for the strict delay constraint, $d = 1$. The optimization problem can be formulated by replacing the constraint (17b) by (27), and inserting an additional constraint $0 \leq \theta_i \leq 1$. Solving the optimization problem, we obtain the optimal power allocation as follows:

$$p_i^* = \frac{\sigma_i^{\frac{2}{\theta_i^*+1}}}{h_i^{\frac{\theta_i^*}{\theta_i^*+1}}} \left[\min \left\{ \frac{2^{\frac{2B_{max}}{\theta_i^*}}}{(\sigma_i \sqrt{h_i})^{\frac{2}{1+\theta_i^*}}}, \frac{1}{\lambda^{\frac{1}{1+\theta_i^*}}} \right\} - \frac{1}{(\sigma_i \sqrt{h_i})^{\frac{2}{1+\theta_i^*}}} \right]^+ \quad (32)$$

where $p_i^* \geq v_{p,i}$. The optimal transmission duration θ_i^* satisfies the properties obtained for general delay constraint. Therefore, the optimal transmission power can be further simplified as follows:

$$p_i^* = \begin{cases} \frac{\sigma_i}{\sqrt{h_i}} \left[\min \left\{ \frac{2^{2B_{max}}}{\sigma_i \sqrt{h_i}}, \frac{1}{\lambda} \right\} - \frac{1}{\sigma_i \sqrt{h_i}} \right]^+ & \text{if } \theta_i^* = 1, \\ v_{p,i} & \text{if } 0 < \theta_i^* < 1, \\ 0 & \text{if } \theta_i^* = 0. \end{cases} \quad (33)$$

Similarly, we can argue that the optimal distortion is given as follows:

$$D_i^* = \begin{cases} \sigma_i^2 2^{2B_{max}} & \text{if } \eta_i \leq \sigma_i^2 2^{2B_{max}} \text{ and } 0 < \theta_i^*, \\ \eta_i & \text{if } \sigma_i^2 2^{2B_{max}} < \eta_i < \sigma_i^2 \text{ and } 0 < \theta_i^*, \\ \sigma_i^2 & \text{if } \eta_i \geq \sigma_i^2 \text{ or } \theta_i^* = 0, \end{cases} \quad (34)$$

where $\eta_i \triangleq \sigma_i^{\frac{2}{\theta_i^*+1}} \left(\frac{\lambda}{h_i} \right)^{\frac{\theta_i^*}{\theta_i^*+1}}$.

Note that for the strict delay constraint case, i.e., $d = 1$, θ_i^* can be interpreted as the number of channel uses per source sample, or the channel-source bandwidth ratio for the source-channel pair in time slot i .

C. Distortion Minimization with Sampling Cost

We now consider the sampling energy cost in addition to the transmission energy. For ease of exposure, we assume a battery operated system and ignore the processing cost, i.e., $\epsilon_p = 0$. Because of the sampling cost, collecting all source samples may not be optimal. Hence, we assume that the sensor collects ϕ_i fraction of the samples with energy cost of ϵ_s Joules per sample. We also assume that the sampling cost is independent of the sampling rate [4]. The distortion of source i is now given by $D_i = \sigma_i^2(1 - \phi_i) + \sigma_i^2 \phi_i 2^{-\frac{2r_i}{\phi_i}}$, where r_i is the compression rate for the samples collected in time slot i . Therefore, we can obtain the corresponding optimization problem by replacing the objective function in (8) with $\sum_{i=1}^N \sigma_i^2(1 - \phi_i) + \sigma_i^2 \phi_i 2^{-\frac{2r_i}{\phi_i}}$, and the constraint (8b) with the following energy constraint:

$$\sum_{i=1}^N \phi_i \epsilon_s + \frac{2^{2c_i} - 1}{h_i} \leq E, \quad (35)$$

where $0 \leq \phi_i \leq 1$.

Accordingly, the Lagrangian of (8b) with $\lambda \geq 0$, $\gamma_i \geq 0$, $\delta_{i,k} \geq 0$, $\zeta_{i,k} \geq 0$, $\beta_i \geq 0$, $\rho_i \geq 0$, $\mu_i \geq 0$, $\eta_i \geq 0$, and $\omega_i \geq 0$ as KKT multipliers, can be written as follows:

$$\begin{aligned} \mathcal{L} = & \frac{1}{N} \sum_{i=1}^N \sigma_i^2(1 - \phi_i) + \sigma_i^2 \phi_i 2^{-\frac{2r_i}{\phi_i}} \\ & + \lambda \left(\sum_{i=1}^N \phi_i \epsilon_s + \frac{2^{2c_i} - 1}{h_i} - E \right) \\ & + \sum_{i=1}^N \gamma_i \left(\sum_{j=i}^N r_j - \sum_{j=i}^N c_j \right) \\ & + \sum_{k=1}^{N-d} \sum_{i=k}^{N-d} \delta_{i,k} \left(\sum_{j=k}^i r_j - \sum_{j=k}^{i+d-1} c_j \right) \\ & + \sum_{k=1}^{N-1} \sum_{i=k}^{N-1} \zeta_{i,k} \left(\sum_{j=k}^{i+1} r_j - \sum_{j=k}^i c_j - B_{max} \right) \\ & - \sum_{i=1}^N \beta_i r_i + \sum_{i=1}^N \rho_i (r_i - B_{max}) - \sum_{i=1}^N \mu_i c_i \\ & - \sum_{i=1}^N \eta_i \phi_i + \sum_{i=1}^N \omega_i (\phi_i - 1). \end{aligned} \quad (36)$$

When we take the derivative of the Lagrangian with respect to c_i , we obtain the optimal transmission power as in (16). Therefore, the properties provided in Lemma 2 still hold. However, when we differentiate the Lagrangian with respect

to r_i and ϕ_i , and set it to zero, we obtain

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial r_i^*} = & -\frac{2(\ln 2)}{N} \sigma_i^2 2^{-\frac{2r_i^*}{\phi_i^*}} + \sum_{j=1}^i \gamma_j + \sum_{k=1}^i \sum_{j=i}^{N-d} \delta_{j,k} \\ & + \sum_{k=1}^i \sum_{j=i-1}^{N-1} \zeta_{j,k} - \beta_i + \rho_i = 0, \quad \forall i, \end{aligned} \quad (37)$$

where $\zeta_{i-1,i} = 0$ for $\forall i$, and

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial \phi_i^*} = & \frac{\sigma_i^2}{N} (2^{-\frac{2r_i^*}{\phi_i^*}} - 1) + \frac{2(\ln 2) \sigma_i^2 r_i^*}{N \phi_i^*} 2^{-\frac{2r_i^*}{\phi_i^*}} \\ & + \lambda \epsilon_s - \eta_i + \omega_i = 0, \quad \forall i, \end{aligned} \quad (38)$$

respectively. Combining (37) with $D_i^* = \sigma_i^2(1 - \phi_i^*) + \sigma_i^2 \phi_i^* 2^{-\frac{2r_i^*}{\phi_i^*}}$ we obtain the optimal distortion for source i as follows:

$$D_i^* = \begin{cases} \sigma_i^2(1 - \phi_i^* + \phi_i^* 2^{-\frac{2B_{max}}{\phi_i^*}}) & \text{if } \xi_i \leq \sigma_i^2 2^{-\frac{2B_{max}}{\phi_i^*}}, 0 < \phi_i^* \\ \sigma_i^2(1 - \phi_i^*) + \phi_i^* \xi_i & \text{if } \sigma_i^2 2^{-\frac{2B_{max}}{\phi_i^*}} < \xi_i < \sigma_i^2, 0 < \phi_i^* \\ \sigma_i^2 & \text{if } \xi_i \geq \sigma_i^2 \text{ or } \phi_i^* = 0, \end{cases} \quad (39)$$

where ξ_i is equal to (14). Therefore, ξ_i in (39) satisfies the properties given in Lemma 1. From (37) we can argue that $\xi_i = \sigma_i^2 2^{-\frac{2r_i^*}{\phi_i^*}}$, and from (38) we obtain:

$$\frac{\lambda \epsilon_s - \eta_i + \omega_i}{\sigma_i^2} = 1 - 2^{-2k_i^*} - 2(\ln 2) k_i^* 2^{-2k_i^*}, \quad (40)$$

where $k_i \triangleq \frac{r_i}{\phi_i}$. We can interpret k_i as the compression rate for the sampled ϕ_i fraction of source i . Note that the right hand side (RHS) of (40) is a monotonically increasing function of k_i^* . When $0 < \phi_i < 1$, i.e., $\eta_i = 0$ and $\omega_i = 0$, there is a unique solution of (40), which is denoted as $k_i^* = v_{s,i}$, for given λ , ϵ_s , and σ_i^2 . In addition, we can argue that whereas ξ_i decreases as source variance σ_i^2 increases, it increases as the sampling cost increases. When $\phi_i = 1$, i.e., $\omega_i \geq 0$, the solution of (40) must satisfy $k_i^* \geq v_{s,i}$.

Next, we investigate the effect of sampling cost on the optimal power and distortion allocation under strict delay constraint. For $d = 1$, the optimization problem can be formulated by replacing the constraint in (17b) with (35), and inserting an additional constraint $0 \leq \phi_i \leq 1$. With the new objective function $\sum_{i=1}^N \sigma_i^2(1 - \phi_i) + \sigma_i^2 \phi_i 2^{-\frac{2c_i}{\phi_i}}$, the Lagrangian of the optimization problem can be written as

$$\begin{aligned} \mathcal{L} = & \frac{1}{N} \sum_{i=1}^N \sigma_i^2(1 - \phi_i) + \sigma_i^2 \phi_i 2^{-\frac{2c_i}{\phi_i}} + \lambda \sum_{i=1}^N \phi_i \epsilon_s \\ & + \frac{2^{2c_i} - 1}{h_i} - E - \sum_{i=1}^N \beta_i c_i + \sum_{i=1}^N \mu_i (c_i - B_{max}) \\ & - \sum_{i=1}^N \eta_i \phi_i + \sum_{i=1}^N \omega_i (\phi_i - 1), \end{aligned} \quad (41)$$

where $\lambda \geq 0$, $\beta_i \geq 0$, $\mu_i \geq 0$, $\eta_i \geq 0$, and $\omega_i \geq 0$ are KKT multipliers. Differentiating the Lagrangian with respect to c_i

and setting it to zero, we obtain

$$\frac{\partial \mathcal{L}}{\partial c_i^*} = -\frac{2(\ln 2)}{N} \sigma_i^2 2^{-\frac{2c_i^*}{\phi_i^*}} + \frac{2(\ln 2) \lambda}{h_i} 2^{2c_i^*} - \beta_i + \mu_i = 0, \quad \forall i. \quad (42)$$

In addition, when we differentiate the Lagrangian with respect to ϕ_i and set it to zero, we get (38). Replacing c_i^* in (42) with $\frac{1}{2} \log(1 + h_i p_i^*)$, we can argue that the optimal power allocation is given by

$$p_i^* = \frac{\sigma_i^2 2^{\frac{2\phi_i^*}{\phi_i^*+1}}}{h_i^{\frac{1}{1+\phi_i^*}}} \left[\min \left\{ \frac{2^{2B_{max}}}{(\sigma_i \sqrt{h_i})^{\frac{2\phi_i^*}{1+\phi_i^*}}}, \frac{1}{\lambda^{\frac{\phi_i^*}{1+\phi_i^*}}} \right\} - \frac{1}{(\sigma_i \sqrt{h_i})^{\frac{2\phi_i^*}{1+\phi_i^*}}} \right]^+ \quad (43)$$

Combining (42) and (38) such that λ is eliminated, we obtain

$$\begin{aligned} -\sigma_i^2 + \sigma_i^2 2^{-\frac{2c_i^*}{\phi_i^*}} + \frac{2(\ln 2) \sigma_i^2 c_i^*}{\phi_i^*} 2^{-\frac{2c_i^*}{\phi_i^*}} + \epsilon_s h_i \sigma_i^2 2^{-\frac{2c_i^*}{\phi_i^*}} 2^{-2c_i^*} \\ + (\beta_i - \mu_i - \eta_i + \omega_i) N = 0. \end{aligned} \quad (44)$$

We can further simplify (44) as follows.

$$\frac{\epsilon_s}{\frac{1}{h_i} + p_i^*} + (\beta_i - \mu_i - \eta_i + \omega_i) N = 2^{2k_i^*} - 2(\ln 2) k_i^* - 1, \quad (45)$$

where $k_i \triangleq \frac{c_i}{\phi_i}$. Using (45), we can argue the following:

- If $\phi_i^* = 0$ or $c_i^* = 0$, then $p_i^* = 0$ and $D_i^* = \sigma_i^2$.
- If $0 < \phi_i^* < 1$ and $0 < c_i^* < B_{max}$, then RHS of (45) is monotonically increasing function of k_i^* , therefore Equation (45) has a unique solution $k_i^* = v_{s,i}$ for a given ϵ_s , h_i , and p_i^* . When h_i and p_i^* are given, $c_i^* = \frac{1}{2} \log(1 + h_i p_i^*)$ is known as well; and hence, we can compute the optimal sampling fraction ϕ_i^* . Then the optimal distortion D_i^* is given by $D_i^* = \sigma_i^2(1 - \phi_i^*) + \sigma_i^2 \phi_i^* 2^{-2v_{s,i}}$.
- If $\phi_i^* = 1$ and $0 < c_i^* < B_{max}$, then $\omega_i \geq 0$, therefore from (45), we can argue that the optimal solution k_i^* must satisfy $k_i^* \geq v_{s,i}$. Then, the optimal distortion D_i^* is given by $D_i^* = \sigma_i^2 2^{-2k_i^*}$.

V. ILLUSTRATION OF THE RESULTS

In this section, we provide numerical results to illustrate the structure of the optimal distortion and power allocation policy, and to analyze the impact of the delay constraint, energy harvesting, processing and sampling costs on the optimum average distortion. Throughout this section, we consider $N = 10$ time slots. The channel gains are chosen as $\mathbf{h} = [0.4, 0.2, 0.2, 0.5, 0.4, 0.6, 0.9, 0.3, 0.4, 1]$, and the source variances are $\sigma^2 = [0.7, 0.6, 1, 0.5, 0.3, 0.6, 0.2, 0.3, 0.7, 0.5]$. We first set $d = 1$, and consider a battery-run system with initial energy $E = 4$ Joules. We set $\epsilon_p = \epsilon_s = 0$. We illustrate the optimal rate and power allocation for $B_{max} = 0.15$ bits/sample in Fig. 4. In the figure, the dashed line corresponds to $K_i 2^{2B_{max}}$. As shown in Fig. 4, the data buffer size bounds the total sampled data in each time slot and the minimum distortion. The average achievable distortion is computed as $D = 0.45$. The optimal power and distortion allocations are $\mathbf{p}^* = [0.57, 0.23, 1.15, 0.46, 0.11,$

0.38, 0.25, 0, 0.5, 0.23] Joules per transmitted symbol and $\mathbf{D}^* = [0.56, 0.57, 0.81, 0.40, 0.28, 0.48, 0.16, 0.3, 0.56, 0.4]$, respectively.

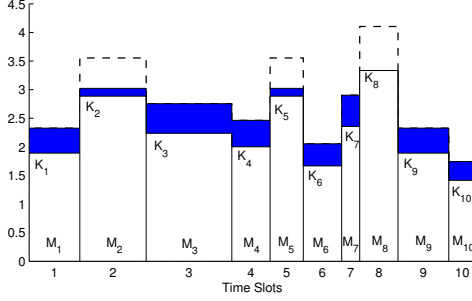


Figure 4. 2D waterfilling for a battery-run system. $E = 4$ Joules, $B_{max} = 0.15$ bits/sample, $\epsilon_p = \epsilon_s = 0$, $\mathbf{h} = [0.4, 0.2, 0.2, 0.5, 0.4, 0.6, 0.9, 0.3, 0.4, 1]$, $\sigma^2 = [0.7, 0.6, 1, 0.5, 0.3, 0.6, 0.2, 0.3, 0.7, 0.5]$, $\mathbf{p}^* = [0.57, 0.23, 1.15, 0.46, 0.11, 0.38, 0.25, 0, 0.5, 0.23]$ Joules per transmitted symbol, and $\mathbf{D}^* = [0.56, 0.57, 0.81, 0.40, 0.28, 0.48, 0.16, 0.3, 0.56, 0.4]$.

Next, we assume an infinite data buffer. We assume the same channel gains and source variances as given above. The 2D waterfilling solution is shown in Fig. 5, resulting in the optimal average distortion $D = 0.448$. The optimal power and distortion allocations are $\mathbf{p}^* = [0.74, 0, 0.48, 0.45, 0, 0.78, 0.04, 0, 0.74, 0.73]$ Joules per transmitted symbol and $\mathbf{D}^* = [0.53, 0.6, 0.9, 0.4, 0.3, 0.4, 0.19, 0.3, 0.53, 0.28]$, respectively.

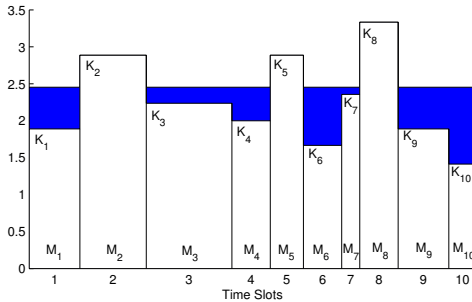


Figure 5. 2D waterfilling for battery-run system. $E = 4$ Joules, $B_{max} \rightarrow \infty$, $\epsilon_p = \epsilon_s = 0$, $\mathbf{h} = [0.4, 0.2, 0.2, 0.5, 0.4, 0.6, 0.9, 0.3, 0.4, 1]$, $\sigma^2 = [0.7, 0.6, 1, 0.5, 0.3, 0.6, 0.2, 0.3, 0.7, 0.5]$, $\mathbf{p}^* = [0.74, 0, 0.48, 0.45, 0, 0.78, 0.04, 0, 0.74, 0.73]$ Joules per transmitted symbol and $\mathbf{D}^* = [0.53, 0.6, 0.9, 0.4, 0.3, 0.4, 0.19, 0.3, 0.53, 0.28]$.

We illustrate the optimal distortion with respect to B_{max} in Fig. 6. We assume the same channel gains and source variances as before, and set $E = 4$ Joules and $\epsilon_p = \epsilon_s = 0$. As shown in Fig. 6, the distortion decreases dramatically when the data buffer size is large. As expected, the distortion, when the delay constraint is $d = 1$, is larger than the case when $d = N$. The figure also shows that the data buffer size has more impact on the distortion when the delay constraint is more relaxed. This is because a relaxed delay constraint allows more flexibility in terms of rate allocation, but this flexibility can be exploited only with a sufficiently large data buffer. In addition, distortion remains constant when the data buffer size $B_{max} \geq 0.31$ bits/sample for $d = 1$, and when $B_{max} \geq 1.12$ bits/sample

for $d = 10$. Since the bit allocation is limited by the available energy, relaxing the data buffer size does not decrease the minimum achievable distortion once all the available energy is optimally allocated to the sources.

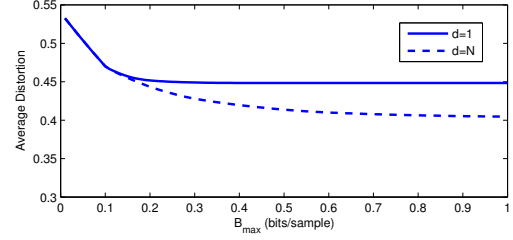


Figure 6. Distortion versus buffer size. $E = 4$ Joules, $\epsilon_p = \epsilon_s = 0$, $\mathbf{h} = [0.4, 0.2, 0.2, 0.5, 0.4, 0.6, 0.9, 0.3, 0.4, 1]$, $\sigma^2 = [0.7, 0.6, 1, 0.5, 0.3, 0.6, 0.2, 0.3, 0.7, 0.5]$.

We investigate the variation of the optimal distortion D with respect to the delay constraint d in Fig. 7. We consider a battery-run system with initial energy $E = 4$ Joules and $\epsilon_p = \epsilon_s = 0$. The optimal distortion values for increasing d , plotted in Fig. 7, show that when $B_{max} = \infty$, the optimal distortion decreases monotonically for $d \leq 4$, and remains constant afterwards. However, when the data buffer size is limited to $B_{max} = 0.15$ bits/sample, relaxing the delay constraint beyond two time slots does not decrease the minimum achievable distortion.

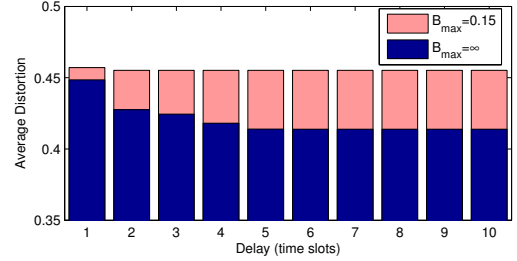


Figure 7. Average distortion D versus delay constraint d . $E = 4$ Joules, $B_{max} = 0.15$ bits/sample, $\epsilon_p = \epsilon_s = 0$, $\mathbf{h} = [0.4, 0.2, 0.2, 0.5, 0.4, 0.6, 0.9, 0.3, 0.4, 1]$, $\sigma^2 = [0.7, 0.6, 1, 0.5, 0.3, 0.6, 0.2, 0.3, 0.7, 0.5]$.

We also investigate the variation of the optimal distortion D with respect to the available total energy. We consider a battery-run system with initial energy $E \in [0, 10]$ Joules and $\epsilon_p = \epsilon_s = 0$. We assume that $B_{max} = 0.15$ bits/sample. As it can be seen from Fig. 8, the achievable distortion decays with the available total energy, and for very low and very high energy levels, the minimum achievable distortion values are the same for $d = 1$ and $d = N$. When the available energy is $E = 0$, no compression is possible, which leads to the maximum distortion independent of the delay constraint d . However, when the available energy in the battery is large, all the samples of source i can be transmitted within the time slot i with minimum achievable distortion $D_i = \sigma_i^2 2^{-2B_{max}}$, and hence, relaxing the delay constraint does not decrease the minimum achievable distortion since we are limited by the data buffer constraint.

Next, we consider an energy harvesting system with energy packets of sizes $E_1 = 1$, $E_6 = 3$, and $E_i = 0$ Joules

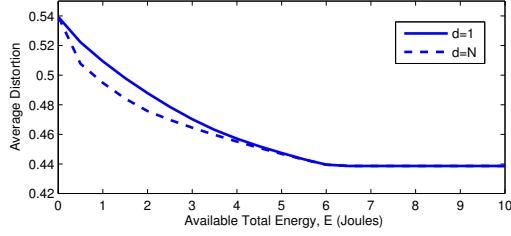


Figure 8. Average distortion D versus available total energy in a battery-run system. $\epsilon_p = \epsilon_s = 0$, $B_{max} = 0.15$ bits/sample, $\mathbf{h} = [0.4, 0.2, 0.2, 0.5, 0.4, 0.6, 0.9, 0.3, 0.4, 1]$, $\sigma^2 = [0.7, 0.6, 1, 0.5, 0.3, 0.6, 0.2, 0.3, 0.7, 0.5]$.

otherwise. We set $\epsilon_p = \epsilon_s = 0$ and $B_{max} = \infty$. The 2D directional waterfilling solution for infinite data buffer size is given in Fig. 9. Note that the water level changes after the fifth time slot because of directional waterfilling. The resulting optimal distortion is $D = 0.45$, larger than the battery-run system with the same total energy (see Fig. 5), since the battery-run system has more flexibility in allocating the available energy over time. The optimal power and distortion allocations are $\mathbf{p}^* = [0.54, 0, 0.15, 0.3, 0, 0.98, 0.13, 0, 1, 0.87]$ Joules per transmitted symbol and $\mathbf{D}^* = [0.57, 0.6, 0.97, 0.43, 0.3, 0.37, 0.17, 0.29, 0.49, 0.26]$, respectively.

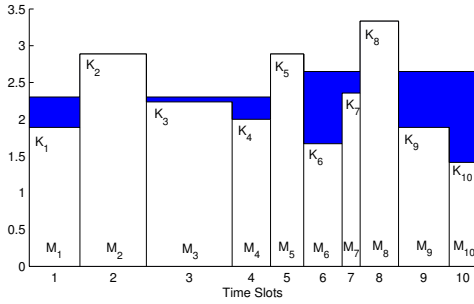


Figure 9. 2D directional waterfilling for an EH system. $E_1 = 1$, $E_6 = 3$, $E_i = 0$ Joules, $B_{max} = \infty$, $\epsilon_p = \epsilon_s = 0$, $\mathbf{h} = [0.4, 0.2, 0.2, 0.5, 0.4, 0.6, 0.9, 0.3, 0.4, 1]$, $\sigma^2 = [0.7, 0.6, 1, 0.5, 0.3, 0.6, 0.2, 0.3, 0.7, 0.5]$. We have $\mathbf{p}^* = [0.54, 0, 0.15, 0.3, 0, 0.98, 0.13, 0, 1, 0.87]$ and $\mathbf{D}^* = [0.57, 0.6, 0.97, 0.43, 0.3, 0.37, 0.17, 0.29, 0.49, 0.26]$.

The effect of the processing cost on the minimum distortion for a battery-run system is illustrated in Fig. 10. We set $E = 4$ Joules and $\epsilon_s = 0$. As seen in the figure, when the data buffer constraint is 0.1 bits/sample and the processing cost is low, the minimum achievable distortion is the same for the delay constrained and unconstrained scenarios. However, as the processing cost increases, the performance degrades under the delay constraint. In addition, when the data buffer size is relaxed, the performance without a delay constraint significantly improves. However, when the processing cost is high, for the strict delay case, relaxing the data buffer size does not decrease the average distortion because high processing cost limits the compression rate.

Finally, we consider the effect of the sampling cost on the minimum distortion for a battery-run system, illustrated in Fig. 11. We set $E = 4$ Joules and $\epsilon_p = 0$. As seen in the figure, when the sampling cost is low, the effect of the limited data buffer on the average achievable distortion is more

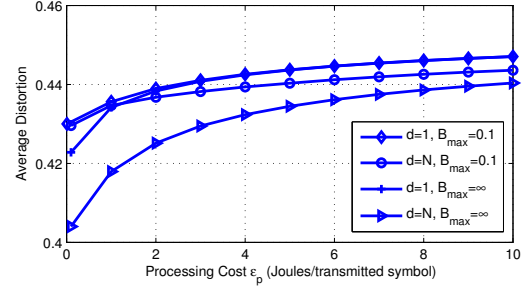


Figure 10. Average distortion D versus the processing energy cost for a battery-run system. $E = 4$ Joules, $\epsilon_s = 0$, $\mathbf{h} = [0.4, 0.2, 0.2, 0.5, 0.4, 0.6, 0.9, 0.3, 0.4, 1]$, $\sigma^2 = [0.7, 0.6, 1, 0.5, 0.3, 0.6, 0.2, 0.3, 0.7, 0.5]$.

significant. However, when we increase the sampling cost, the performance of the system is mostly determined by the delay constraint. As shown in Fig. 10 and Fig. 11, the behavior of the distortion with respect to the sampling cost is similar to that of the processing cost.

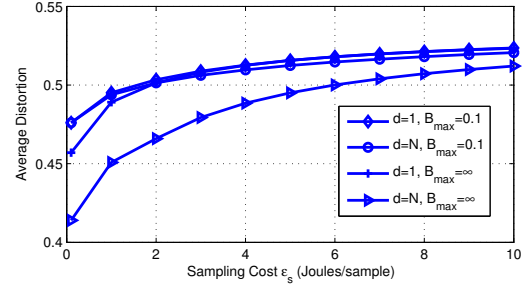


Figure 11. Average distortion D versus sampling energy cost for a battery-run system. $E = 4$ Joules, $\epsilon_p = 0$, $\mathbf{h} = [0.4, 0.2, 0.2, 0.5, 0.4, 0.6, 0.9, 0.3, 0.4, 1]$, $\sigma^2 = [0.7, 0.6, 1, 0.5, 0.3, 0.6, 0.2, 0.3, 0.7, 0.5]$.

VI. CONCLUSIONS

We have investigated source-channel coding for a wireless sensor node under delay, data buffer size and various energy constraints. For a time slotted system, we have considered the scenario in which the samples of a time varying Gaussian source are to be delivered to a destination over a fading channel within d time slots. In addition, we have imposed a finite size data buffer on the compressed samples. In this framework, we have investigated optimal transmission policies that minimize the average mean squared distortion of the samples at the destination for battery operated as well as an energy harvesting system. We have also studied the impact of various additional energy costs, including processing and sampling costs. In each case, we have provided a convex optimization formulation and identified the characteristics of the optimal distortion and power levels. We have also provided numerical results to investigate the impact of energy harvesting, processing and sampling costs. Our results have shown that for an energy harvesting transmitter energy arrivals over time may result in higher average distortion at the destination. In addition, we have observed that relaxing the data buffer constraint induces more dramatic decrease in the average distortion when processing and sampling costs are

low. These results have important implications for the design of energy-limited wireless sensor nodes, and indicate that the optimal system operation and performance can be significantly different when the energy consumption of various other system components, or the arrival of the energy over time are taken into consideration.

APPENDIX

In this appendix, we illustrate Fourier-Motzkin elimination of (1)-(3) for three time slots $N = 3$ when delay constraint is $d = 2$. Rewriting (1)-(3) in terms of $r_i \triangleq \frac{1}{2} \log \left(\frac{\sigma_i^2}{D_i} \right)$ and $c_i \triangleq \frac{1}{2} \log (1 + h_i p_i)$ we get

$$\begin{aligned} R_{1,1} &\leq c_1 \\ R_{1,2} + R_{2,2} &\leq c_2 \\ R_{2,3} + R_{3,3} &\leq c_3 \\ r_1 &\leq R_{1,1} + R_{1,2} \\ r_2 &\leq R_{2,2} + R_{2,3} \\ r_3 &\leq R_{3,3} \\ R_{1,1} + R_{1,2} &\leq B_{max} \\ R_{1,2} + R_{2,2} + R_{2,3} &\leq B_{max} \\ R_{2,3} + R_{3,3} &\leq B_{max}, \end{aligned}$$

where $R_{1,1} \geq 0$, $R_{1,2} \geq 0$, $R_{2,2} \geq 0$, $R_{2,3} \geq 0$, $R_{3,3} \geq 0$, $r_i \geq 0$, and $c_i \geq 0$.

We have upper and lower bounds on $R_{1,1}$ as $\max\{0, r_1 - R_{1,2}\} \leq R_{1,1} \leq \min\{c_1, B_{max} - R_{1,2}\}$. Therefore, eliminating $R_{1,1}$ and the redundant inequalities, we obtain:

$$\begin{aligned} r_1 &\leq c_1 + R_{1,2} \\ R_{1,2} + R_{2,2} &\leq c_2 \\ R_{2,3} + R_{3,3} &\leq c_3 \\ r_2 &\leq R_{2,2} + R_{2,3} \\ r_3 &\leq R_{3,3} \\ r_1 &\leq B_{max} \\ R_{1,2} + R_{2,2} + R_{2,3} &\leq B_{max} \\ R_{2,3} + R_{3,3} &\leq B_{max} \end{aligned}$$

The upper and lower bounds on $R_{1,2}$ are $\max\{0, r_1 - c_1\} \leq R_{1,2} \leq \min\{c_2 - R_{2,2}, B_{max} - R_{2,2} - R_{2,3}\}$. Therefore, eliminating $R_{1,2}$ and the redundant inequalities, we obtain:

$$\begin{aligned} r_1 + R_{2,2} &\leq c_1 + c_2 \\ R_{2,2} &\leq c_2 \\ R_{2,3} + R_{3,3} &\leq c_3 \\ r_2 &\leq R_{2,2} + R_{2,3} \\ r_3 &\leq R_{3,3} \\ r_1 + R_{2,2} + R_{2,3} &\leq c_1 + B_{max} \\ r_1 &\leq B_{max} \\ R_{2,2} + R_{2,3} &\leq B_{max} \\ R_{2,3} + R_{3,3} &\leq B_{max} \end{aligned}$$

The upper and lower bounds on $R_{2,2}$ are $\max\{0, r_2 - R_{2,3}\} \leq R_{2,2} \leq \min\{c_2, c_1 + c_2 - r_1, B_{max} - R_{2,3}, c_1 +$

$B_{max} - r_1 - R_{2,3}\}$. Eliminating $R_{2,2}$ and the redundant inequalities, we obtain:

$$\begin{aligned} r_1 &\leq c_1 + c_2 \\ r_2 &\leq c_2 + R_{2,3} \\ r_1 + r_2 &\leq c_1 + c_2 + R_{2,3} \\ R_{2,3} + R_{3,3} &\leq c_3 \\ r_3 &\leq R_{3,3} \\ r_i &\leq B_{max}, \quad i = 1, 2 \\ r_1 + R_{2,3} &\leq B_{max} + c_1 \\ r_1 + r_2 &\leq B_{max} + c_1 \\ R_{2,3} + R_{3,3} &\leq B_{max} \end{aligned}$$

The upper and lower bounds on $R_{2,3}$ are $\max\{0, r_2 - c_2, r_1 + r_2 - c_1 - c_2\} \leq R_{2,3} \leq \min\{B_{max} + c_1 - r_1, c_3 - R_{3,3}, B_{max} - R_{3,3}\}$. Eliminating $R_{2,3}$ and the redundant inequalities, we obtain:

$$\begin{aligned} R_{3,3} + r_1 + r_2 &\leq c_3 + c_2 + c_1 \\ r_1 &\leq c_1 + c_2 \\ R_{3,3} &\leq c_3 \\ R_{3,3} + r_2 &\leq c_3 + c_2 \\ r_3 &\leq R_{3,3} \\ r_i &\leq B_{max}, \quad i = 1, 2 \\ R_{3,3} &\leq B_{max} \\ R_{3,3} + r_2 &\leq B_{max} + c_2 \\ R_{3,3} + r_1 + r_2 &\leq B_{max} + c_2 + c_1 \\ r_1 + r_2 &\leq B_{max} + c_1 \end{aligned}$$

Finally, we have upper and lower bounds on $R_{3,3}$ as $\max\{0, r_3\} \leq R_{3,3} \leq \min\{c_3, c_3 + c_2 - r_2, B_{max}, B_{max} + c_2 - r_2, B_{max} + c_1 + c_2 - r_1 - r_2, c_3 + c_2 + c_1 - r_1 - r_2\}$. Eliminating $R_{3,3}$ and the redundant inequalities, we obtain:

$$\begin{aligned} r_3 &\leq c_3 \\ r_2 + r_3 &\leq c_2 + c_3 \\ r_1 + r_2 + r_3 &\leq c_1 + c_2 + c_3 \\ r_1 &\leq c_1 + c_2 \\ r_1 + r_2 &\leq c_1 + B_{max} \\ r_1 + r_2 + r_3 &\leq c_1 + c_2 + B_{max} \\ r_2 + r_3 &\leq c_2 + B_{max} \\ r_i &\leq B_{max}, \quad i = 1, 2, 3. \end{aligned}$$

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