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Editors

Sequential Monte Carlo Methods in Practice

Foreword by Adrian Smith

With 168 Illustrations



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