Risk Sensitive Path Integral Control

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1 Introduction

The objective in risk neutral stochastic optimal control is to minimize an expected cost-to-go. Risk sensitive optimal control generalizes this objective by minimizing an expected exponentiated cost-to-go. Depending on the risk parameter θ, expected exponentiated cost-to-go puts more emphasis on the mode of the distribution of the cost-to-go, or on its tail, and in that way allows for a modelling of risk seeking ($\theta < 0$) or risk averse ($\theta > 0$) behaviour. The case $\theta = 0$ corresponds to risk neutral stochastic optimal control.

Recently, a path integral formalism has been developed for a large class of continuous space-time problems [1]. This formalism contains Linear Quadratic Gaussian (LQG) control as a special case. In our paper [2] we show that path integral control generalizes to the risk sensitive setting, requiring no additional conditions. Consequently, characteristics of path integral control, which include superposition of controls and approximate inference, carry over to the risk sensitive setting.

We can interpret the risk seeking case as optimistic and risk averse as pessimistic. This is illustrated with an optimal control problem of simultaneously reaching a target and avoiding an adjacent threat; see Figure 1. The figures show histograms of the log-probability of the cost. We see that with larger θ, the mode of the distribution shifts to higher costs. On the other hand, the tails of the distribution at the high cost end are thinner with larger θ. This is what is to be expected: small θ is more greedy, aiming at low cost, however at the expense of some outliers with high costs. A larger θ is more cautious, reducing the probability of costly outliers.

Presently, we are using risk sensitive path integral control to model delayed choices in sensorimotor behaviour.

References

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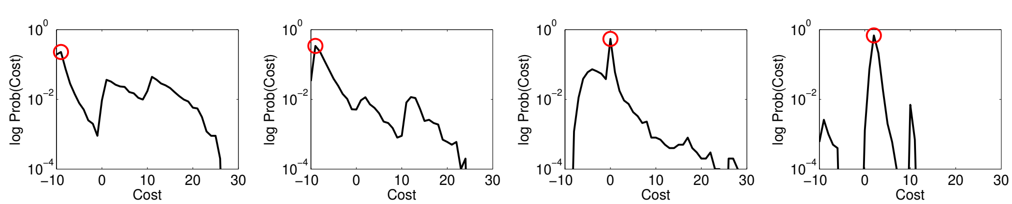

(a) $\theta = -1$ (b) $\theta = 0$ (c) $\theta = 1$ (d) $\theta = 3$

Figure 1: The log-probability of the cost in the case of simultaneously reaching a target (low cost -10) and avoiding an adjacent threat (high cost $+10$) at end time. The mode is marked with $\circ$.