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NONLINEAR AND QUANTUM OPTICS

Quantum Memory for Images with Feedback

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Abstract—A spatially multimode scheme of the quantum memory for light is presented that can be considered to be a thin hologram, whose long-lived spin subsystem of a multiatomic ensemble stores image peculiarities, such as the multimode superposition and entangled quantum states, which is unattainable for ordinary holography. In the recording process, the measurement of polarization parameters of the transmitted light wave and the feedback are used for the action on an ensemble of fixed atoms (obtained, for example, by means of the laser cooling). The information capacity of a quantum hologram with a feedback will considerably exceed that of a spatially singlemode quantum memory.

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INTRODUCTION

The development of a quantum interface between light and matter [1] is one of the current problems of quantum optics and quantum information. Through a quantum interface, an exchange of quantum states between light and matter (quantum memory) can be organized and/or entangled quantum states of light and matter can be generated, which forms the basis for many schemes of quantum information. The quantum memory for light allows the high-fidelity exchange (recording, storage, and readout) of quantum states between the light and long-lived degrees of freedom of the matter. The quantum memory will be a necessary component of long-range quantum information transfer schemes (quantum repeaters) and quantum computer networks.

To obtain the quantum memory based on atomic ensembles, various physical mechanisms were proposed [2], for example, nondemolition quantum interaction [3, 4], electromagnetically induced transparency [5], and Raman scattering processes in lambda-schemes of atomic levels [6, 7]. Below, we consider the multimode parallel quantum memory based on the nondemolition interaction, which was used in singlemode demonstrations of the quantum memory [8] and teleportation [9].

Multimode parallel quantum protocols ensuring the transfer of a quantum state between light fields were already proposed for the quantum holographic teleportation [10, 12] and the quantum dense coding of optical images [13]. These quantum image processing protocols are based on the use of broadband spatially multimode beams in the Einstein–Podolsky–Rosen entangled quantum state.

In [14], we proposed a model of the multimode parallel quantum memory for light, whose input signal is a wave front (optical image) distributed in space and time. Long-lived spin degrees of freedom of enlarged atomic ensembles, which were still used only for the singlemode memory, can also be used for quantum holograms if the translation motion is suppressed (with the aid of laser cooling) because they can store many spatial modes, which is their advantage over single-atom memory. In contrast to the singlemode case, the spatial profile of the input image can be arbitrary and unknown to the observer in the general case. At the classical level, the field of a particular image is specified by an arbitrary superposition of orthogonal spatial modes. The ideal quantum memory stores and outputs an input image, keeping quantum states of all spatial modes in every detail.

In [14], we studied the scheme of recording and readout of a thin quantum hologram with the use of the nondemolition quantum interaction at two passages of a signal wave through a material layer. This protocol can be considered as an extension of the scheme of [3] proposed for the singlemode memory to the spatially multimode case.

In this paper, we consider a different scheme of parallel quantum memory based on a thin hologram. In this scheme, the nondemolition interaction is used for the action on an atomic ensemble in the process of recording, polarization parameters of the transmitted light wave are measured, and the control feedback is applied to atoms. In the spatially singlemode case, the feedback can be implemented through the spin rotation in an auxiliary magnetic field. This scheme with a feedback was proposed and implemented in [8]. However, this method fails to modulate the control signal with the

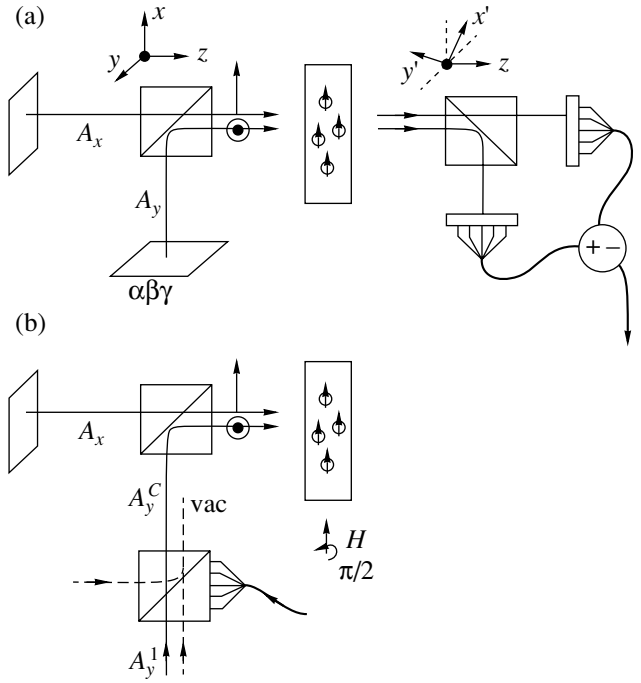

Fig. 1. Two stages of recording of a quantum hologram with the aid of the nondemolition interaction: (a) first passage of light and measurement and (b) second passage with the use of the feedback.

spatial resolution sufficient for the recording of an optical image in the memory. That is why we consider here the scheme, in which the feedback is performed through the nondemolition interaction with the control light wave modulated by the feedback signal. Quantum fluctuations arising in this scheme of the quantum memory for light are considered. It is shown that this scheme principally allows the high-fidelity recording of a quantum state of the light field having many spatial degrees of freedom or pixels (if we deal with the description of the field in the near zone).

NONDEMOLITION INTERACTION OF A LIGHT WAVE WITH ATOMS

The scheme illustrating the process of recording of a quantum hologram is shown in Fig. 1.

Let us consider an ensemble of randomly located atoms with a spin of 1/2 in the ground and excited states. The long-lived spin of the ground state of an atom $\mathbf{J}^a$ is initially oriented in the vertical direction x. A classical off-resonant x-polarized plane wave with the frequency ω_0 and the slowly varying amplitude A_x (taken as real), propagates to the right in the z direction. An input signal is represented by a weak quantized y-polarized field with the amplitude $A_y(\mathbf{r}, t) \ll A_x$ and with the same carrier frequency and the mean direction of propagation. In what follows, we consider this multimode input field in the paraxial approximation. The quantum nondemolition interaction between light and matter gives rise to two main effects: (i) the Faraday rotation of the light polarization induced by the longitudinal z-component of the collective spin of atoms and (ii) the rotation of the atomic spin caused by unequal light shifts of magnetic sublevels $m_z = \pm 1/2$ of the ground state at different intensities of contributions of orthogonal circular polarizations to the total light wave. The corresponding part of the Hamiltonian can be written as follows [14]:

$$H = \frac{2\pi k_0 |d|^2}{\omega_{eg} - \omega_0} \int_V d\mathbf{r} \sum_a J_z^a S_z(\mathbf{r}, t) \delta(\mathbf{r} - \mathbf{r}_a).$$

Here, ω_{eg} is the frequency of the nonperturbed atomic transition; d is the dipole transition matrix element; and $k_0 = \omega_0 c$. The projection of the Stokes vector onto the z axis is determined as

$$S_z(\mathbf{r}, t) = -iA_x[A_y(\mathbf{r}, t) - A_y^\dagger(\mathbf{r}, t)].$$

The slowly varying amplitude A_y is specified as

$$A_y(z, \boldsymbol{\rho}, t) = [1/(2\pi)^3] \int dk_z \int d\mathbf{q} \sqrt{\omega(k)/k_0}\, a_y(\mathbf{k}) \times \exp[i(\mathbf{q} \cdot \boldsymbol{\rho} + (k_z - k_0)z - (\omega(k) - \omega_0)t)],$$

where $a_y(\mathbf{k})$ and $a_y^\dagger(\mathbf{k})$ are the annihilation and creation operators of a photon with the wave vector $\mathbf{k} = (\mathbf{q}, k_z)$ satisfying the standard commutation relations

$$[a_y(\mathbf{k}), a_y^\dagger(\mathbf{k}')] = (2\pi)^3 \delta(\mathbf{k} - \mathbf{k}'),$$
$$[a_y(\mathbf{k}), a_y(\mathbf{k}')] = 0.$$

In the paraxial approximation, for slow amplitudes, the following commutation relations appear [15]:

$$[A_y(z, \boldsymbol{\rho}, t), A_y^\dagger(z, \boldsymbol{\rho}', t')] = \delta(t - t')\delta(\boldsymbol{\rho} - \boldsymbol{\rho}'), \quad (1)$$

where $\mathbf{r} = (\boldsymbol{\rho}, z)$. Space-dependent canonic variables for the input field are defined as quantities averaged over the interaction time T

$$X_L(\boldsymbol{\rho}) = (1/\sqrt{2T}) \int_T dt[A_y(0, \boldsymbol{\rho}, t) + A_y^\dagger(0, \boldsymbol{\rho}, t)],$$
$$P_L(\boldsymbol{\rho}) = (1/i\sqrt{2T}) \int_T dt[A_y(0, \boldsymbol{\rho}, t) - A_y^\dagger(0, \boldsymbol{\rho}, t)]. \quad (2)$$

These quantities satisfy the canonic commutation relations

$$[X_L(\boldsymbol{\rho}), P_L(\boldsymbol{\rho}')] = i\delta(\boldsymbol{\rho} - \boldsymbol{\rho}'). \quad (3)$$

Below, we neglect the diffraction along the length of the atomic ensemble L, thus assuming that the Rayleigh length, which is related to the linear size of a pixel $\sqrt{S}$ is much longer than L, that is, $S \gg L\lambda$. The field averaged over the area of this pixel propagates at the length

of an atomic ensemble roughly as a longitudinal wave, and slow amplitudes averaged over this area behave closely to the one-dimensional description. The inverse linear size of a pixel gives the width of the spatial frequency band to which this description is applicable. For a thin atomic layer located at the origin of coordinates $z = 0$, we introduce the surface density of the collective spin $J(\boldsymbol{\rho}) = \Sigma_a J^a \delta(\boldsymbol{\rho} - \boldsymbol{\rho}_a)$. The commutation relation averaged over random atomic positions for the components y and z of the collective spin can be written as

$$\overline{[J_y(\boldsymbol{\rho}), J_z(\boldsymbol{\rho}')]} = i\sum_a \langle J_x^a\rangle \overline{\delta(\boldsymbol{\rho} - \boldsymbol{\rho}_a)\delta(\boldsymbol{\rho}' - \boldsymbol{\rho}_a)}^a$$

$$= in_a\langle J_x^a\rangle\delta(\boldsymbol{\rho} - \boldsymbol{\rho}').$$

Here, n_a is the mean surface density of atoms. Canonic variables for the spin subsystem are introduced as

$$X_A(\boldsymbol{\rho}) = J_y(\boldsymbol{\rho})/\sqrt{n_a\langle J_x^a\rangle}, \quad P_A(\boldsymbol{\rho}) = J_z(\boldsymbol{\rho})/\sqrt{n_a\langle J_x^a\rangle}.$$

As can be easily seen, the averaged commutation relations for these variables are analogous to the canonic commutation relations (3) for the field subsystem. It is convenient to introduce canonic variables for atoms and for light, because the transfer of a quantum state from a subsystem of one physical nature to that of another physical nature is easiest to describe when both subsystems are represented by similar variables.

RECORDING AND READOUT WITH THE USE OF FEEDBACK

The recording into the quantum memory and the readout from it can be performed in different ways. We already considered the two-passage scheme [14], when the signal wave passes the atomic ensemble twice during the recording. After the first passage, the atomic spin is turned through the angle $\pi/2$ around the axis x, and the state of polarization of the signal is changed by an anisotropic plate so that other canonic variables of the light and atoms interact during the second passage. In this case, the field and the matter exchange quantum states accurate to the additive noise. For the readout, the exchange procedure is repeated.

In the experiment [8], in which the spatially singlemode quantum memory was implemented, the feedback through an auxiliary magnetic field was used in place of the second passage of the light through atoms. The feedback signal was determined by the measured polarization of the light passed through the atomic ensemble. This form of the feedback does not fit the spatially multimode memory, since it does not allow the action on atoms with the high spatial resolution, which is necessary for the recording of the optical image. Below, we consider the feedback through the light wave, when this restriction is lifted.

The quantum recording of light consists of three stages: (i) nondemolition quantum interaction of the signal light wave with atoms during the first passage through the atomic layer, (ii) spatially resolved measurement of the polarization of the transmitted light, rotation of spins through the angle $\pi/2$ around the axis x and preparation of the transverse modulated auxiliary light wave, and (iii) nondemolition interaction of the auxiliary wave with atoms during the second passage.

The transformation of atomic and light variables for the recording time T as a result of the light passage through the thin (such that the diffraction can be neglected) layer consists of the mutual rotation of the Stokes vectors and the spin moment of the lower level around the longitudinal axis [3, 8]. Taking into account the transverse dimension of the system, we have [14]

$$X_L^{W(1)}(\boldsymbol{\rho}) = X_L^{W(\text{in})}(\boldsymbol{\rho}) + \kappa P_A^{W(\text{in})}(\boldsymbol{\rho}),$$

$$P_L^{W(1)}(\boldsymbol{\rho}) = P_L^{W(\text{in})}(\boldsymbol{\rho}),$$

$$X_A^{W(1)}(\boldsymbol{\rho}) = X_A^{W(\text{in})}(\boldsymbol{\rho}) + \kappa P_L^{W(\text{in})}(\boldsymbol{\rho})\left(1 + \frac{\delta J_x(\boldsymbol{\rho})}{n_a\langle J_x^a\rangle}\right), \quad (4)$$

$$P_A^{W(1)}(\boldsymbol{\rho}) = P_A^{W(\text{in})}(\boldsymbol{\rho}).$$

Here, the superscripts (1) and W correspond to the first stage of the recording. The interaction constant is represented in the form $\kappa^2 = \alpha_0\eta = 1$, where α_0 is the resonance optical density and η is the probability of spontaneous emission over the recording time T calculated per one atom [4]. Since the condition $\eta \ll 1$ is necessary to neglect the spontaneous emission, the other ordinary condition $\alpha_0 = \lambda^2 n_a/2\pi \gg 1$ should be true.

The right-hand side of the expression for $X_A^{W(1)}(\boldsymbol{\rho})$ includes the contribution responsible for fluctuations of the atomic density in the cross section. These fluctuations lead to the situation that the local value of the classical x-component of the collective spin may differ from the mean value. After a small rotation of the spin around the z axis, the arising response of observable $J_y(\boldsymbol{\rho})$ begins to depend on fluctuations of the atomic density. The influence of these fluctuations depends on the atomic density and the pixel area S. For the random arrangement of atoms, we have

$$\frac{\delta J_x(\boldsymbol{\rho})}{n_a\langle J_x^a\rangle} \sim \frac{1}{\sqrt{n_a S}}.$$

If the conditions $\lambda^2 n_a/2\pi \gg 1$ and $S \gg L\lambda$, where $L \gg \lambda$, are fulfilled, the number of atoms in a pixel is large, and we can neglect fluctuations of the atomic density.

The interaction of atoms with the light yields the record of the $P^{W(\text{in})}$ quadrature amplitudes of atoms (light) onto the $X^{W(1)}$ quadrature amplitudes of light (atoms). For the complete recording of the quantum state, it is necessary to transfer the quadrature amplitudes $X^{W(\text{in})}$.

For this purpose, after the passage of the signal field through the atomic ensemble, a symmetric homodyne

detector measures (with the necessary spatial resolution) the perturbed $X_L^{W(1)}(\boldsymbol{\rho})$ quadrature amplitude of the light, which is proportional to the y-component of the Stokes vector

$$S_y(\mathbf{r}, t) = A_x[A_y(\mathbf{r}, t) + A_y^{\dagger}(\mathbf{r}, t)],$$

accumulated within the time T. By the definition of the Stokes parameters, which have the meaning of the intensity differences at the observation in three complementary bases, for this purpose we should measure and accumulate (in time) the intensity difference of the waves linearly polarized in the (x', y') basis rotated by the angle $\pi/4$ relative to the (x, y) basis

$$X_L^M(\boldsymbol{\rho}) \equiv S_y^M(\boldsymbol{\rho})/\sqrt{2A_x^2T}$$

$$= (1/\sqrt{2A_x^2T})\int_T dtA_x[A_y(\mathbf{r}, t) + A_y^{\dagger}(\mathbf{r}, t)] = X_L^{W(1)}(\boldsymbol{\rho}).$$

After measurement, all spins are rotated around the x axis through the angle $\pi/2$ to obtain $X_A^{W(2)} = -P_A^{W(1)}$, $P_A^{W(2)} = X_A^{W(1)}$ (the superscript (2) corresponds to this stage of the control action). This is performed with the aid of the $\pi/2$ pulse of the auxiliary magnetic field directed along x.

Then, the control field with the quadrature amplitude $P_L^C(\boldsymbol{\rho}) \sim X_L^M(\boldsymbol{\rho})$ is generated to be directed into the atomic layer at the second passage of the light. The field transfers the measured amplitude $X_L^M(\boldsymbol{\rho})$ to observed amplitude $X_A^{W(2)}(\boldsymbol{\rho})$ during the nondemolition interaction with atoms.

The control field with the given admixture of a circular polarization, which is necessary for the spin rotation, can be obtained by mixing (on a polarization splitter) a strong reference classical field of the x-polarization and a relatively weak (but still strong enough to be considered as the nearly classical) light wave of the y-polarization shifted in phase by $\pi/2$ relative to the reference one. The amplitude of the weak wave is modulated in the cross section by the value of the measured real observable $X_L^M(\boldsymbol{\rho})$ such that the modulation is imposed on the control quadrature component of the total field.

To suppress the contribution of vacuum fluctuations of the control field to the y-polarization, the interaction constant or the interaction time should be decreased. A possible way is to detune the control field further from the resonance with the electronic transition. Thus, the interaction is determined by the interaction constant κ_2 and (or) the interaction time T_2. As a result of the action of the control field, the atomic variables transform with allowance for Eq. (4) from their initial values $X_A^{W(2)}(\boldsymbol{\rho})$ to final ones $X_A^{W(\text{out})}(\boldsymbol{\rho})$ according to

$$\begin{aligned} X_A^{W(\text{out})}(\boldsymbol{\rho}) &= X_A^{W(2)}(\boldsymbol{\rho}) + \kappa_2 P_L^C(\boldsymbol{\rho}), \\ P_A^{W(\text{out})}(\boldsymbol{\rho}) &= P_A^{W(2)}(\boldsymbol{\rho}). \end{aligned} \tag{5}$$

The first equation (5) shows that the variable of the input field $X_A^{W(\text{in})}(\boldsymbol{\rho})$ is transferred into the quadrature amplitude of the atomic moment $X_L^{W(\text{out})}(\boldsymbol{\rho})$. Previously, as a result of the direct action (4), the variable of the input field $P_A^{W(\text{in})}(\boldsymbol{\rho})$ was written in the amplitude of the atomic moment $P_L^{W(\text{out})}(\boldsymbol{\rho})$.

Let us discuss quantum fluctuations of the control signal in greater detail. To generate this signal, a plane laser wave of the y-polarization with the amplitude $A_y^1 + A_y^{1(\text{vac})}(\boldsymbol{\rho}, t)$ is directed at input *1* of a high-Q mirror of the splitter. Here, the contribution $A_y^{1(\text{vac})}(\boldsymbol{\rho}, t)$ takes into account minimal quantum fluctuations of the amplitude, which correspond to the field in the Glauber coherent state. The splitter mirror is described by the controlled amplitude transmission coefficients of the field $\tau(\boldsymbol{\rho})$ (from input *1*) and $\rho(\boldsymbol{\rho})$ (from input *2*). The field of vacuum fluctuations $A_y^{2(\text{vac})}(\boldsymbol{\rho}, t)$ comes from the dark idle input *2* to the same output in the y-polarization. The transmission coefficients are related by the unitarity conditions $|\tau(\boldsymbol{\rho})|^2 + |\rho(\boldsymbol{\rho})|^2 = 1$. For definiteness, we believe that the transmission coefficient is real and its value is specified with the aid of the feedback such that it is proportional to the measured signal

$$\tau(\boldsymbol{\rho}) = \beta X_L^M(\boldsymbol{\rho}).$$

The classical amplitude of the incident wave on the polarization mixer is taken in the form $A_y^1 = i|A_y^1|$, which means the phase shift of $\pi/2$ relative to the reference wave A_x. The control field is

$$\begin{aligned} A_y^C(\boldsymbol{\rho}, t) &= \tau(\boldsymbol{\rho})[A_y^1 + A_y^{1(\text{vac})}(\boldsymbol{\rho}, t)] \\ &+ \rho(\boldsymbol{\rho})A_y^{2(\text{vac})}(\boldsymbol{\rho}, t). \end{aligned} \tag{6}$$

It can be easily checked that, as a consequence of unitarity, the field (6) satisfies the commutation relation for free fields, and its fluctuation part is vacuum (checked in the action on the initial vacuum state of the fields $A_y^{1,2(\text{vac})}(\boldsymbol{\rho}, t)$). Thus, the canonic variables of the control field averaged over the interaction time are

$$\begin{aligned} X_L^C(\boldsymbol{\rho}) &= X_L^{C(\text{vac})}(\boldsymbol{\rho}), \\ P_L^C(\boldsymbol{\rho}) &= gX_L^M(\boldsymbol{\rho}) + P_L^{C(\text{vac})}(\boldsymbol{\rho}), \end{aligned} \tag{7}$$

where $g = \sqrt{2T_2}A_y^1\beta$.

Since the spatial resolution of the quantum hologram is finite and our consideration is carried out in the paraxial approximation, it is necessary to introduce the procedure of smoothing in the cross section of the light wave. To describe the near field, we relate orthogonal spatial modes with field amplitudes averaged over the surface S_i of square pixels of the area S, $i = 1, \ldots, N$. Averaged quadrature amplitudes P of the control field (that is, canonic variables for pixels) are introduced as

$$P_L^C(i) = (1/\sqrt{S})\int_{S_i} d\boldsymbol{\rho} P_L^C(\boldsymbol{\rho}) \tag{8}$$

and analogous quantities are introduced for X. Using Eqs. (1) and (2), for the vacuum fluctuation of the control quadrature amplitude averaged in this way, we find

$$\begin{aligned}\langle P_L^{C(\mathrm{vac})}(\boldsymbol{\rho})P_L^{C(\mathrm{vac})}(\boldsymbol{\rho}')\rangle &= \delta(\boldsymbol{\rho}-\boldsymbol{\rho}')/2,\\ \langle P_L^{C(\mathrm{vac})}(i)P_L^{C(\mathrm{vac})}(j)\rangle &= \delta_{i,j}/2.\end{aligned} \tag{9}$$

In Eq. (7), the contribution of the fluctuation part of the control amplitude $P_L^C(i)$ (in the rms sense) related to the average value of the same amplitude is estimated as

$$\sqrt{ST_2\overline{(A_y^C)^2}}/2. \tag{10}$$

Here, $A_y^C \sim \beta X_L^M(\rho)A_y^1$. The denominator of this relation includes the root of the number of photons of the control field with the y-polarization being, on the average, in the averaging volume ST_2. If this component of the control field is not too weak (its degeneracy parameter is much greater than unity), the vacuum fluctuation is relatively small. This estimate is typical for cases of the superposition of the signal and vacuum fluctuations. Since the estimate depends on the time and on the averaging area, the spatial restriction is imposed on the model here due to the photon noise. This restriction, as is clear from the above-said, can be lifted by an increase in the intensity of the control field (or by squeezing its amplitude P).

Let us explicitly present the contributions to the quadrature amplitudes of the atomic moment once the recording procedure is accomplished. For this purpose, we substitute Eqs. (4) and (7) with allowance for the spin rotation into the result (5). Thus, we obtain

$$\begin{aligned}X_A^{W(\mathrm{out})}(\boldsymbol{\rho}) &= \kappa_2 g X_L^{W(\mathrm{in})}(\boldsymbol{\rho})\\ &+ (\kappa\kappa_2 g - 1)P_A^{W(\mathrm{in})}(\boldsymbol{\rho}) + \kappa_2 P_L^{C\mathrm{vac}}(\boldsymbol{\rho}),\\ P_A^{W(\mathrm{out})}(\boldsymbol{\rho}) &= X_A^{W(\mathrm{in})}(\boldsymbol{\rho}) + \kappa P_L^{W(\mathrm{in})}(\boldsymbol{\rho}).\end{aligned}$$

The exchange of the observable values between the light and atoms, that is, memory, is achieved at the matching between constants such that $\kappa = 1$ and $\kappa_2 g = 1$. In this case, the initial fluctuation of the atomic P-quadrature is compensated. The recording is not ideal because the atomic quadrature P keeps the value of the initial fluctuation of the corresponding atomic quadrature X and the contribution of vacuum fluctuations of the control field, whose smallness is determined by Eq. (10), penetrates the atomic quadrature X through the feedback.

The goal of the quantum memory protocol is the transfer of the input quantum state of the light at the stage of recording to the output state of the light at the stage of readout. Recording to the quantum memory was described above. The readout is performed exactly in the same way (only with the replacement of light with atoms and vice versa). This leads to the penetration of additional noise from the initial fluctuations X of the quadrature of the reading-out light, as well as the feedback noises. As was already mentioned, the quality of the memory at the stage of recording can be increased, if atoms in the squeezed spin state (with the suppressed fluctuations of the quadrature X) are used. The same is true for the stage of readout. That is, the reading-out light can be used in the squeezed state. In [14], we studied how the squeezing of the reading-out light influences the complete cycle of recording–readout.

The transfer of the state of the light to atoms (without readout) is of independent interest for quantum logical operations to be performed at material degrees of freedom, that is, for the quantum information processing. Let us consider the fidelity of the recording into the quantum memory. At the optimal matching of recording parameters, when $\kappa = 1$ and $\kappa_2 g = 1$, the atomic canonical variables per one pixel (that is, averaged analogously to Eq. (8)) can be represented in the form

$$\begin{aligned}X_A^{W(\mathrm{out})}(i) &= X_L^{W(\mathrm{in})}(i) + F_X(i),\\ P_A^{W(\mathrm{out})}(i) &= P_L^{W(\mathrm{in})}(i) + F_P(i).\end{aligned} \tag{11}$$

Equations (11) are analogous to those describing the quantum holographic teleportation of an optical image [10–12]. However, the explicit form of noise contributions is specific to our model of the memory

$$F_X(i) = P_L^{C(\mathrm{vac})}(i)/g, \quad F_P(i) = X_A^{W(\mathrm{in})}(i). \tag{12}$$

As follows from Eq. (12), the larger is the feedback transfer coefficient compared to unity $g \gg 1$, the more efficient is the suppression of vacuum fluctuations of the control field.

The quality of the transfer of a quantum state $|\psi^{(\mathrm{in})}\rangle \longrightarrow |\psi^{(\mathrm{out})}\rangle$ is characterized by the fidelity parameter $F = \left|\langle\psi^{(\mathrm{in})}|\psi^{(\mathrm{out})}\rangle\right|^2$. We assume that the field to be recorded is in a spatially multimode coherent state. For an image represented by the superposition of

the fields at N pixels, the fidelity is described by the equation [12]

$$F_N = [\det(\delta_{ij} + C^X(i,j))\det(\delta_{ij} + C^P(i,j))]^{-1/2},$$

where the noise correlation matrices are determined as

$$C^X(i,j) = \langle F_X(i)F_X(j)\rangle, \quad C^P(i,j) = \langle F_P(i)F_P(j)\rangle.$$

As was shown in [12], the fidelity of the transfer of a quantum state for simple multipixel arrays is scaled roughly as the Nth degree of $F_{\text{av}} = (F_N)^{1/N}$, which is referred to as the average fidelity per pixel. If the quantum state of spatial modes of the collective spin before the recording is vacuum, then, analogously to Eq. (9), we have $C^P(i,j) = \delta_{i,j}/2$ and the average fidelity of the recording is

$$F_{\text{av}} = [(1 + 1/2g)(1 + 1/2)]^{-1/2}.$$

In the limit of the efficient feedback, the contribution of vacuum fluctuations of the control field can be neglected, and the upper limit of the fidelity of the recording into the quantum memory is $F_{\text{av}} = \sqrt{2/3} = 0.82$. This value is higher than the classical limit for the recording of coherent states, which, according to [16], is equal to 0.5. If the used initial states of spatial modes of the collective spin are ideally squeezed in the quadrature X, the ideal transfer of a quantum state of the light to an atomic ensemble is achieved and $F_{\text{av}} = 1$.

One of the important requirements for a quantum hologram, which follows from the possibility to record the quantum state of many spatial modes of the light field, is the possibility to store entangled quantum states. In the limit of the ideal fidelity ($F = 1$), the quantum hologram would be capable of storing all quantum peculiarities of a state, including the entanglement. In the case that the recording fidelity per pixel calculated for the class of coherent states satisfies the condition $F > 0.67$, the memory adds less than one unit of vacuum noise $\langle F^2_{X,P}(i)\rangle = 1/2$ (see Eq. (9)) per pixel. As follows from the entanglement condition $\langle(\Delta X(i) - \Delta X(j))^2\rangle + \langle(\Delta P(i) - \Delta P(j))^2\rangle < 2$ proposed in [17], the quantum hologram in this case can store a two-modal Gaussian entangled state of fields in two pixels. The problem of storing more general classes of entangled quantum states of the light calls for further investigations.

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