PROCEEDINGS OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 135, Number 9, September 2007, Pages 2803–2809
S 0002-9939(07)08828-4
Article electronically published on May 4, 2007

ORDER-WEAKLY COMPACT OPERATORS FROM VECTOR-VALUED FUNCTION SPACES TO BANACH SPACES

MARIAN NOWAK

(Communicated by Jonathan M. Borwein)

Abstract. Let E be an ideal of L^0 over a σ-finite measure space (Ω, Σ, μ), and let $E^\sim$ stand for the order dual of E. For a real Banach space $(X, \|\cdot\|_X)$ let $E(X)$ be a subspace of the space $L^0(X)$ of μ-equivalence classes of strongly Σ-measurable functions $f : \Omega \longrightarrow X$ and consisting of all those $f \in L^0(X)$ for which the scalar function $\|f(\cdot)\|_X$ belongs to E. For a real Banach space $(Y, \|\cdot\|_Y)$ a linear operator $T : E(X) \longrightarrow Y$ is said to be order-weakly compact whenever for each $u \in E^+$ the set $T(\{f \in E(X) : \|f(\cdot)\|_X \leq u\})$ is relatively weakly compact in Y. In this paper we examine order-weakly compact operators $T : E(X) \longrightarrow Y$. We give a characterization of an order-weakly compact operator T in terms of the continuity of the conjugate operator of T with respect to some weak topologies. It is shown that if $(E, \|\cdot\|_E)$ is an order continuous Banach function space, X is a Banach space containing no isomorphic copy of l^1 and Y is a weakly sequentially complete Banach space, then every continuous linear operator $T : E(X) \longrightarrow Y$ is order-weakly compact. Moreover, it is proved that if $(E, \|\cdot\|_E)$ is a Banach function space, then for every Banach space Y any continuous linear operator $T : E(X) \longrightarrow Y$ is order-weakly compact iff the norm $\|\cdot\|_E$ is order continuous and X is reflexive. In particular, for every Banach space Y any continuous linear operator $T : L^1(X) \longrightarrow Y$ is order-weakly compact iff X is reflexive.

1. Introduction and preliminaries

P. G. Dodds [D] considered order-weakly compact operators from a vector lattice E to a Banach space Y. Recall that a linear operator $T : E \longrightarrow Y$ is called order-weakly compact if the set $T([-u, u])$ is relatively-weakly compact in Y for every $u \in E^+$. Some further properties of these operators can be found in [M, Section 3.4]. M. Duhoux [Du] extended Dodd's results to the setting Y being a locally convex space. Z. Ercan [E] examined some properties of order-weakly compact operators from a vector lattice E to a topological vector space Y.

In this paper, we consider order-weakly compact operators from a vector-valued function space $E(X)$ to a Banach space Y.

Received by the editors December 18, 2003 and, in revised form, May 18, 2006.

2000 *Mathematics Subject Classification.* Primary 47B38, 47B07, 46E40, 46A20.

Key words and phrases. Vector-valued function spaces, Köthe-Bochner spaces, order-bounded operators, order-weakly compact operators, order intervals.

For terminology concerning Riesz spaces and function spaces we refer to [AB1], [AB2] and [KA]. Given a topological vector space (L,τ), by $(L,\tau)^*$ we will denote its topological dual. We denote by $\sigma(L,K)$, $\beta(L,K)$ and $\tau(L,K)$ the weak topology, the strong topology and the Mackey topology, respectively, for a dual system $\langle L,K\rangle$. By $\mathbb{N}$ and $\mathbb{R}$ we will denote the sets of all natural and real numbers, respectively.

Throughout the paper we assume that (Ω,Σ,μ) is a complete σ-finite measure space and L^0 denotes the corresponding space of μ-equivalence classes of all Σ-measurable real-valued functions. Let E be an ideal of L^0 with $\operatorname{supp} E=\Omega$ and let $E^\sim$ stand for the order dual of E.

Let $(X,\|\cdot\|_X)$ be a real Banach space, and let S_X stand for the unit sphere of X. By $L^0(X)$ we denote the set of μ-equivalence classes of all strongly Σ-measurable functions $f:\Omega\longrightarrow X$. For $f\in L^0(X)$ let us set $\widetilde{f}(\omega):=\|f(\omega)\|_X$ for $\omega\in\Omega$. Let

$$E(X)=\{\, f\in L^0(X):\widetilde{f}\in E\,\}.$$

For each $u\in E^+$ the set $D_u=\{\, f\in E(X):\widetilde{f}\le u\,\}$ will be called an *order interval* in $E(X)$.

Following [D] we are now ready to define two classes of linear operators.

Definition 1.1. Let E be an ideal of L^0, and let $(X,\|\cdot\|_X)$ and $(Y,\|\cdot\|_Y)$ be real Banach spaces. A linear operator $T:E(X)\longrightarrow Y$ is said to be *order-weakly compact* (resp. *order-bounded*) whenever for each $u\in E^+$ the set $T(D_u)$ is relatively-weakly compact (resp. norm-bounded) in Y.

Clearly each order-weakly compact operator $T:E(X)\longrightarrow Y$ is order-bounded. Order-bounded operators $T:\ E(X)\longrightarrow Y$ have been considered in [N5].

Now we recall some terminology and results concerning the duality theory of the function spaces $E(X)$ as set out in [B1], [BL], [N1], [N2].

For a linear functonal F on $E(X)$ let us put

$$|F|(f)=\sup\{\,|F(h)|:h\in E(X),\ \widetilde{h}\le\widetilde{f}\,\}\quad\text{for}\quad f\in E(X)\ .$$

The set

$$E(X)^\sim=\{\,F\in E(X)^\#:|F|(f)<\infty\quad\text{for all}\quad f\in E(X)\,\}$$

will be called the *order dual* of $E(X)$ (here $E(X)^\#$ denotes the algebraic dual of $E(X)$).

For $F_1,F_2\in E(X)^\sim$ we will write $|F_1|\le|F_2|$ whenever $|F_1|(f)\le|F_2|(f)$ for all $f\in E(X)$. A subset A of $E(X)^\sim$ is said to be *solid* whenever $|F_1|\le|F_2|$ with $F_1\in E(X)^\sim$ and $F_2\in A$ imply $F_1\in A$.

In particular, for a Banach function space $(E,\|\cdot\|_E)$ the space $E(X)$ provided with the norm $\|f\|_{E(X)}:=\|\widetilde{f}\|_E$ is a Banach space, and it is usually called a *Köthe-Bochner space*. It is well known that $(E(X),\|\cdot\|_{E(X)})^*=E(X)^\sim$ (see [BL, §3, Lemma 12]).

For each $f\in E(X)$ let

$$\rho_f(F)=|F|(f)\quad\text{for}\quad F\in E(X)^\sim\ .$$

We define the *absolute weak topology* $|\sigma|(E(X)^\sim,E(X))$ on $E(X)^\sim$ as a locally convex topology generated by the family $\{\,\rho_f:f\in E(X)\,\}$ of seminorms. Clearly, $|\sigma|(E(X)^\sim,E(X))$ is the topology of uniform convergence on the family of all order intervals D_u, where $u\in E^+$.

Now let $\mathcal{B}_0$ be the family of all absolutely convex subsets of $E(X)$ that absorb every order interval in $E(X)$. Then $\mathcal{B}_0$ is a local base at zero for a locally convex topology τ_0 on $E(X)$ (called an *order-bounded topology*), which is the finest locally convex topology on $E(X)$ for which every order interval is a bounded set (see [Na], [N5]).

The following characterization of τ_0 and order-bounded operators $T : E(X) \to Y$ will be needed (see [N5, Corollary 2.2] and Theorem 2.3).

Theorem 1.2. *The order-bounded topology τ_0 on $E(X)$ coincides with the Mackey topology $\tau(E(X), E(X)^\sim)$, i.e., $\tau_0 = \tau(E(X), E(X)^\sim)$. For a linear operator $T : E(X) \longrightarrow Y$ the following statements are equivalent:*

(i) T *is order-bounded;*
(ii) T *is* $(\tau(E(X), E(X)^\sim), \|\cdot\|_Y)$*-continuous;*
(iii) T *is* $(\sigma(E(X), E(X)^\sim), \sigma(Y, Y^*))$*-continuous.*

Moreover, for a Banach function space $(E, \|\cdot\|_E)$, the statements (i)–(iii) are equivalent to the following:

(iv) T *is* $(\|\cdot\|_{E(X)}, \|\cdot\|_Y)$*-continuous.*

2. Characterization of order–weakly compact operators

Let τ be a linear topology on $E(X)$. Recall that a linear operator $T : E(X) \longrightarrow Y$ is ***τ**-weakly compact* whenever there exists a neighbourhood U of 0 for τ such that the set $T(U)$ is relatively-weakly compact in Y.

In view of the definition of the order-bounded topology τ_0 on $E(X)$ we can easily observe that every τ_0-weakly compact operator $T : E(X) \longrightarrow Y$ is order-weakly compact. Hence, if $(E, \|\cdot\|_E)$ is a Banach function space, then τ_0 coincides with the $\|\cdot\|_{E(X)}$-topology, so every weakly compact operator $T : E(X) \longrightarrow Y$ is order-weakly compact. In particular, a linear operator $T : L^\infty(X) \longrightarrow Y$ is order-weakly compact if and only if T is weakly compact (because the unit ball in $L^\infty(X)$ coincides with the order interval $D_{\mathbb{1}_\Omega}$).

In this section we characterize order-weakly operators $T : E(X) \longrightarrow Y$ in terms of the continuity of the conjugate operator $T^\sim$ with respect to appropriate weak topologies. We start by recalling some concepts and results of the duality theory of the spaces $E(X)$ (see [B1], [BL], [N1], [N2]).

For a linear functional V on $E(X)^\sim$ let us put:

$$|V|(F) = \sup\{\, |V(G)| : G \in E(X)^\sim,\ |G| \le |F|\,\} \quad \text{for} \quad F \in E(X)^\sim .$$

The set

$$(E(X)^\sim)^\sim = \{\, V \in (E(X)^\sim)^\# : |V|(F) < \infty \quad \text{for all} \quad F \in E(X)^\sim \,\}$$

will be called the *order dual of* $E(X)^\sim$ (here $(E(X)^\sim)^\#$ denotes the algebraic dual of $E(X)^\sim$).

For $V_1, V_2 \in (E(X)^\sim)^\sim$ we will write $|V_1| \le |V_2|$ whenever $|V_1|(F) \le |V_2|(F)$ for all $F \in E(X)^\sim$. A subset K of $(E(X)^\sim)^\sim$ is said to be *solid* whenever $|V_1| \le |V_2|$ with $V_1 \in (E(X)^\sim)^\sim$, $V_2 \in K$ imply $V_1 \in K$. A linear subspace L of $(E(X)^\sim)^\sim$ is called an *ideal* of $(E(X)^\sim)^\sim$ if L is a solid subset of $(E(X)^\sim)^\sim$.

For each $f \in E(X)$ let us put

$$\pi_f(F) = F(f) \quad \text{for all} \quad F \in E(X)^\sim .$$

One can show (see [N2]) that for $f \in E(X)$,

$$|\pi_f|(F) = |F|(f) \quad \text{for} \quad F \in E(X)^\sim \quad \text{and} \quad \pi_f \in (E(X)^\sim)^\sim .$$

Thus we have a natural embedding $\pi : E(X) \ni f \longmapsto \pi_f \in (E(X)^\sim)^\sim$. Denote by $E(X)_0$ the ideal of $(E(X)^\sim)^\sim$ generated by the set $\pi(E(X))$ (i.e., $E(X)_0$ is the smallest ideal of $(E(X)^\sim)^\sim$ containing $\pi(E(X)\,)$).

Theorem 2.1 (see [N2, Theorem 3.2])**.** *We have*

$$(E(X)^\sim, |\sigma|(E(X)^\sim, E(X)))^* = E(X)_0$$
$$= \{\, V \in (E(X)^\sim)^\sim : |V| \le |\pi_f| \quad \textit{for some } f \in E(X)\,\}.$$

For $u \in E^+$ and $f \in E(X)$ let:

$$C_u := \pi(D_u) = \{\, \pi_h : h \in E(X),\ \widetilde{h} \le u \,\} \quad (\,= \text{an interval in } \pi(E(X))\,)\ ,$$
$$I_f := \{\, V \in E(X)_0 : |V| \le |\pi_f|\,\} \quad (\,= \text{an interval in } E(X)_0\ )\ .$$

The following properties of C_u and I_f will be needed.

Theorem 2.2 ([N2, Theorem 2.4])**.** *Let* $f \in E(X)$. *Then*

(i) I_f *is* $\sigma(E(X)_0, E(X)^\sim)$*-compact in* $E(X)_0$ *;*
(ii) $C_{\widetilde{f}}$ *is* $\sigma(E(X)_0, E(X)^\sim)$*-dense in* I_f *.*

Assume now that a linear operator $T : E(X) \longrightarrow Y$ is order-bounded. Then in view of Theorem 1.1, T is $(\,\sigma(E(X), E(X)^\sim), \sigma(Y, Y^*)))$-continuous. Hence, we can consider the linear mapping

$$T^\sim :\ Y^* \longrightarrow E(X)^\sim$$

defined by

$$T^\sim(y^*)(f) = y^*(T(f)) \quad \text{for} \quad y^* \in Y \quad \text{and all} \quad f \in E(X)\ .$$

Then $T^\sim$ is $(\,\beta(Y^*, Y),\ \beta(E(X)^\sim,\ E(X)))$-continuous; hence $T^\sim$ is also $(\,\beta(Y^*, Y),\ \ |\sigma|(E(X)^\sim, E(X))\,)$-continuous (because $|\sigma|(E(X)^\sim, E(X)) \subset \beta(E(X)^\sim, E(X))$; see ([N2, §4]). Since $(E(X)^\sim, |\sigma|(E(X)^\sim, E(X)))^* = E(X)_0$ (see Theorem 2.1) and $(Y^*, \beta(Y^*, Y))^* = Y^{**}$, we see that $T^\sim$ is also $(\,\sigma(Y^*, Y^{**}),\ \sigma(E(X)^\sim, E(X)_0)\,)$-continuous. Finally, we consider the linear mapping

$$T^{\sim\sim} :\ E(X)_0 \longrightarrow Y^{**}$$

defined by

$$T^{\sim\sim}(V)(y^*) = V(T^\sim(y^*)) \quad \text{for} \quad V \in E(X)_0 \quad \text{and all} \quad y^* \in Y^*\ ,$$

which is $(\,\sigma(E(X)_0, E(X)^\sim),\ \sigma(Y^{**}, Y^*)\,)$-continuous.

Let $i : Y \ni y \longmapsto i_y \in Y^{**}$ stand for the canonical isometry, i.e.,

$$i_y(y^*) = y^*(y) \quad \text{for} \quad y^* \in Y^*\ .$$

It is seen that $T^{\sim\sim} \circ \pi = i \circ T$, where $\pi : E(X) \longrightarrow (E(X)^\sim)^\sim$ is a natural embedding.

Now we are ready to state a "vector-valued version" of a characterization of order-weakly compact operators $T :\ E \longrightarrow Y$ (see [D, Theorem 4.2]).

Theorem 2.3. *For an order-bounded operator* $T : E(X) \longrightarrow Y$ *the following statements are equivalent:*

(i) T *is order-weakly compact;*

(ii) $T^{\sim\sim}(E(X)_0) \subset i(Y)$;
(iii) $T^{\sim} : (Y^*, \sigma(Y^*, Y)) \longrightarrow (E(X)^{\sim}, \sigma(E(X)^{\sim}, E(X)_0))$ *is continuous.*

Proof. (i) $\Longrightarrow$ (ii) Assume that T is order-weakly compact and let $f \in E(X)$. Since the set $C_{\widetilde{f}}$ is $\sigma(E(X)_0, E(X)^{\sim})$-dense in I_f (see Theorem 2.2), by the $(\sigma(E(X)_0, E(X)^{\sim}),\ \sigma(Y^{**}, Y^*))$-continuity of $T^{\sim\sim}$ we obtain

$$T^{\sim\sim}(I_f) = T^{\sim\sim}(cl_{\sigma(E(X)_o, E(X)^{\sim})} C_{\widetilde{f}}) \subset cl_{\sigma(Y^{**}, Y^*)} T^{\sim\sim}(C_{\widetilde{f}}) \ .$$

But for $h \in E(X)$ and $y^* \in Y^*$ we have

$$T^{\sim\sim}(\pi_h)(y^*) = \pi_h(T^{\sim}(y^*)) = \pi_h(y^* \circ T) = y^*(T(h)) = i_{T(h)}(y^*) \ .$$

Thus putting $A_f = T(\{ h \in E(X) : \widetilde{h} \leq \widetilde{f} \})$ we get

$$T^{\sim\sim}(C_{\widetilde{f}}) = \{ i_{T(h)} : h \in E(X),\ \widetilde{h} \leq \widetilde{f} \} = i(T(\{ h \in E(X) : \widetilde{h} \leq \widetilde{f} \})) = i(A_f) \ .$$

Since T is order-weakly compact, the set A_f is relatively $\sigma(Y, Y^*)$-compact in Y, so the space $(cl_{\sigma(Y,Y^*)} A_f, \sigma(Y, Y^*)|_{cl_{\sigma(Y,Y^*)} A_f})$ is compact. Hence in view of the $(\sigma(Y, Y^*),\ \sigma(Y^{**}, Y^*))$-continuity of i we obtain that the set $i(cl_{\sigma(Y,Y^*)} A_f)$ is $\sigma(Y^{**}, Y^*)$-closed in Y^{**}. It follows that

$$cl_{\sigma(Y^{**}, Y^*)} i(A_f) \subset i(cl_{\sigma(Y,Y^*)} A_f) \subset i(Y) \ ,$$

so finally $T^{\sim\sim}(I_f) \subset i(Y)$. Hence, since $E(X)_0 = \bigcup \{ I_f : f \in E(X) \}$, we conclude that $T^{\sim\sim}(E(X)_0) \subset i(Y)$.

(ii) $\Longrightarrow$ (i) Assume that $T^{\sim\sim}(E(X)_0) \subset i(Y)$ and let $u \in E^+$. Let us set $f = u \otimes x_0$, where $x_0 \in S_X$ and $(u \otimes x_0)(\omega) = u(\omega) x_0$ for $\omega \in \Omega$. Since the mapping $T^{\sim\sim} : E(X)_0 \longrightarrow Y^{**}$ is $(\sigma(E(X)_0, E(X)^{\sim}),\ \sigma(Y^{**}, Y^*))$-continuous and I_f is a $\sigma(E(X)_0, E(X)^{\sim})$-compact subset of $E(X)_0$, we obtain that the set $T^{\sim\sim}(I_f)$ is a $\sigma(i(Y), Y^*)$-compact subset of $i(Y)$. Since $T^{\sim\sim} \circ \pi = i \circ T$ and $\pi(D_u) = C_{\widetilde{f}}$, we get $i(T(D_u)) = T^{\sim\sim}(\pi(D_u)) = T^{\sim\sim}(C_{\widetilde{f}}) \subset T^{\sim\sim}(I_f)$. Hence $i(T(D_u))$ is a relatively compact subset of $(i(Y), \sigma(i(Y), Y^*))$, so $T(D_u)$ is relatively-weakly compact in Y.

(ii) $\Longleftrightarrow$ (iii) See [W, Lemma 11.1.1]. □

3. Order-weakly compact operators from Köthe-Bochner spaces to Banach spaces

In this section we assume that $(E, \|\cdot\|_E)$ is a Banach function space. Then $E(X)^{\sim} = E(X)^*$ $(= (E(X), \|\cdot\|_{E(X)})^*)$. In view of Theorem 1.1 for a Banach space Y a linear operator $T : E(X) \longrightarrow Y$ is order-bounded if and only if T is continuous (i.e., $(\|\cdot\|_{E(X)}, \|\cdot\|_Y)$-continuous).

Recall that a subset A of the Banach space is said to be *conditionally-weakly compact* whenever each sequence in A contains a weakly Cauchy subsequence.

Making use of [N3, Corollary 2.8] we get:

Theorem 3.1. *Let $(E, \|\cdot\|_E)$ be an order continuous Banach function space and X a Banach space containing no isomorphic copy of l^1. Then for each $u \in E^+$ the order interval D_u is conditionally-weakly compact in $E(X)$.*

As an application of Theorem 3.1 we get:

Theorem 3.2. *Let* $(E, \|\cdot\|_E)$ *be an order continuous Banach function space,* X *a Banach space containing no isomorphic copy of* l^1 *and* Y *a weakly sequentially complete Banach space. Then every continuous linear operator* $T : E(X) \longrightarrow Y$ *is order-weakly compact.*

Proof. Let $u \in E^+$. Then by Theorem 3.1, D_u is a conditionally-weakly compact set in $E(X)$. It follows that the set $T(D_u)$ is conditionally-weakly compact in Y. Since Y is supposed to be weakly sequentially complete, we conclude that $T(D_u)$ is relatively-weakly sequentially compact in Y. Hence $T(D_u)$ is relatively-weakly compact in Y. □

In particular, we have:

Corollary 3.3. *Let* X *be a Banach space containing no isomorphic copy of* l^1 *and let* Y *be a weakly sequentially complete Banach space. Then every continuous linear operator* $T : L^p(X) \longrightarrow Y$ $(1 \le p < \infty)$ *is order-weakly compact.*

It is well known that a Lebesgue-Bochner space $L^q(Y)$ $(1 \le q < \infty)$ is weakly sequentially complete whenever a Banach space Y is weakly sequentially complete (see [T, Theorem 11]). Hence, as a consequence of Corollary 3.3 we get:

Corollary 3.4. *Let* X *be a Banach space containing no isomorphic copy of* l^1 *and let* Y *be a weakly sequentially complete Banach space. Then every continuous linear operator* $T : L^p(X) \longrightarrow L^q(Y)$ $(1 \le p < \infty,\ 1 \le q < \infty)$ *is order-weakly compact.*

Now, we present necessary and sufficient conditions for the order-weak compactness of any continuous operator $T : E(X) \longrightarrow Y$ with any Banach space Y. For this purpose we shall need the following characterization of weak compactness of order intervals in $E(X)$.

Theorem 3.5 (see [BL, §4, Corollary 1], [B2, Proposition 2], [N4, Theorem 2.4]). *Let* $(E, \|\cdot\|_E)$ *be a Banach function space and* X *a Banach space. Then the following statements are equivalent:*

(i) *the norm* $\|\cdot\|_E$ *is order continuous and* X *is reflexive;*
(ii) *for each* $u \in E^+$ *the order interval* D_u *is weakly compact.*

Theorem 3.6. *Let* $(E, \|\cdot\|_E)$ *be a Banach function space and* X *a Banach space. Then the following statements are equivalent:*

(i) *the norm* $\|\cdot\|_E$ *is order continuous and* X *is reflexive;*
(ii) *for every Banach space* Y, *any continuous linear operator* $T : E(X) \longrightarrow Y$ *is order-weakly compact.*

Proof. (i) $\Longrightarrow$ (ii) This follows from Theorem 3.5.

(ii) $\Longrightarrow$ (i) Assume that (ii) holds and let $(Y, \|\cdot\|_Y) = (E(X), \|\cdot\|_{E(X)})$. Then the identity operator $\mathrm{id} : E(X) \longrightarrow E(X)$ is order-weakly compact. It follows that for each $u \in E^+$ the set D_u is weakly compact in $E(X)$, so by Theorem 3.5 the norm $\|\cdot\|_E$ is order continuous and X is reflexive. □

In particular, as an application of Theorem 3.6 we get:

Corollary 3.7. *For a Banach space* X *the following statements are equivalent:*

(i) X *is reflexive;*
(ii) *for every Banach space* Y, *any continuous linear operator* $T : L^p(X) \longrightarrow Y$ $(1 \le p < \infty)$ *is order-weakly compact.*

References

[AB1] C. D. Aliprantis, O. Burkinshaw, *Locally solid Riesz spaces*, Academic Press, New York, San Francisco, London, 1978. MR0493242 (58:12271)

[AB2] C. D. Aliprantis, O. Burkinshaw, *Positive operators*, Academic Press, Orlando, San Diego, New York, London, Tokyo, 1985. MR0809372 (87h:47086)

[B1] A. V. Bukhvalov, *On an analytic representation of operators with abstract norm*, Izv. Vyss. Vceb. Zaved., 11 (1975), 21–32 (in Russian). MR0470746 (57:10492)

[B2] A. V. Bukhvalov, *Factorization of linear operators in Banach lattices and in spaces of vector-valued functions*, in Qualitative and approximate methods for investigating operator equations, 168, 34–46, Yaroslav Gos. Univ., Yaroslav, 1982 (in Russian). MR0857126 (87m:47049)

[BL] A. V. Bukhvalov, G. Ya. Lozanowskii, *On sets closed in measure in spaces of measurable functions*, Trans. Moscow Math. Soc., 2 (1978), 127–148.

[D] P. G. Dodds, *o-weakly compact mappings of Riesz spaces*, Trans. Amer. Math. Soc., 214 (1975), 389–402. MR0385629 (52:6489)

[Du] M. Duhoux, *o-weakly compact mappings from a Riesz space to a locally convex space*, Bull. Math. Soc. Sci. Math. R. S. Roumanie, 22 (70) no. 4 (1978), 371–378. MR0522533 (80a:47057)

[E] Z. Ercan, *Interval-bounded operators and order-weakly compact operators of Riesz spaces*, Demonstr. Math., 31, no. 4 (1998), 805–812. MR1677834 (2000g:47046)

[KA] L. V. Kantorovitch, A. V. Akilov, *Functional Analysis*, Nauka, Moscow, 1984 (3^{rd} ed., in Russian). MR0788496 (86m:46001)

[M] P. Meyer-Nieberg, *Banach Lattices*, Springer Verlag, Berlin, Heidelberg, 1991. MR1128093 (93f:46025)

[Na] I. Namioka, *Partially ordered linear topological spaces*, Mem. Amer. Math. Soc., no. 24 (1957). MR0094681 (20:1193)

[N1] M. Nowak, *Duality theory of vector-valued function spaces I*, Comment. Math., Prace Mat., 37 (1997), 195–215. MR1608189 (99h:46061)

[N2] M. Nowak, *Duality theory of vector-valued function spaces II*, Comment. Math., Prace Mat., 37 (1997), 217–230. MR1608185 (99h:46062)

[N3] M. Nowak, *Weak sequential compactness and completeness in Köthe-Bochner spaces*, Bull. Polish Acad. Sci. Math., 47, no. 3 (1999), 209–220. MR1711823 (2000h:46050)

[N4] M. Nowak, *On some topological properties of vector-valued function spaces*, Rocky Mountain J. Math. (to appear).

[N5] M. Nowak, *Order bounded operators from vector-valued function spaces to Banach spaces*, Proc. Conf. Function Spaces VII, Poznań 2003, Banach Center Publ., 68 (2005), 109–114. MR2179004 (2006e:47067)

[T] M. Talagrand, *Weak Cauchy sequences in $L^1(E)$*, Amer. J. Math., 106 (1984), 703–724. MR0745148 (85j:46062)

[W] A. Wilansky, *Modern methods in topological vector spaces*, McGraw-Hill Inc., 1978. MR0518316 (81d:46001)

Faculty of Mathematics, Computer Science and Econometrics, University of Zielona Góra, ul. Szafrana 4A, 65–001 Zielona Góra, Poland

E-mail address: M.Nowak@wmie.uz.zgora.pl