

# Narrative Structure in Inquiry-based Learning

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## Abstract

Our goal with this paper is three-fold. We want to increase awareness of inquiry-based learning by presenting the strategy we use to develop and implement lessons and activities. We describe our approach to structuring lessons in mathematics in a way that engages the students by using language and constructs they are familiar with from other non-science classes. Finally, we include samples from an exploration of celtic knots that we think works well to engage students in the inquiry process, making use of these ideas.

**Keywords:** inquiry-based learning, narrative structure, celtic knot

## 1 Experiences with inquiry-based learning

This paper is based on many years of teaching liberal arts math courses to a wide variety of students, most of whom are filling a requirement for a mathematics course but whose majors do not require any specific body of knowledge.

As mathematicians, we want our students to experience the joy and beauty of mathematics as we do. We want them to learn the process of analyzing a situation and finding a way to understand and classify the outcomes. Unfortunately, many of these students do not want to do either of these. For some of them, their past experience in a math class was neither joyful nor beautiful, and they see no value to solving problems in math. They would tell us that math is hard and that they will never need it. They believe math is a series of steps that one has to learn and memorize to solve a problem someone else wants solved. They see no room for creativity or experimentation in the process. They believe all of mathematics is already known and have never experienced the excitement of mathematical discovery for themselves. However, they continue to enroll in our classes. Some of the reasons we get for registering include:

- I was never really interested in math, but I need a math course to graduate.
- I'm a senior, and I need a math course to graduate.
- I hate math, but my advisor told me that I need a math course to graduate.
- I like math, and the title sounded interesting.
- I'm going to teach math and I want to learn topics and approaches to engage my students.

Our approach in these liberal arts mathematics courses is to introduce topics the students have not seen before and have them explore and discover the mathematics involved. Beginning as in the scientific method, we introduce

a question, have them explore some examples and make a prediction, introduce additional examples to test and refine their theory, but conclude by requiring them to explain (informally) why their final conjecture makes sense. The mathematics does not need to be complicated but should not be trivial.

We encourage students to work in groups while exploring. For the weak students, this alleviates much of their fear of being wrong. They find that some of their classmates share their apprehension regarding math but in a group of like-minded students, they are often willing to offer insightful comments about how to approach a topic. Instead of getting stuck and giving up, the students explain as much as they can which is usually enough to give someone else an idea of how to clarify the concept or what to try next. Even when that “next idea” does not work out, the students find themselves engaging in mathematical exploration.

Many students have responded well to this approach. The students learn that they can discern and justify patterns in math, in nature, and sometimes even in their lives. They improve their reasoning and analytical skills without the stress of having to apply several years of past knowledge to approach a new topic. They find that their long suppressed mathematical intuition really can help them understand situations and make sense of what should happen next. The math-avoidant students find themselves releasing some of the tension, bordering on hatred, that they feel for mathematics.

The stronger students often jump into using the discovery approach and really enjoy the material and the discovery process. These students have succeeded in mathematics before and are a joy to work with in the classroom. They bring lots of ideas and often explore beyond the given examples without encouragement. We try to arrange the working groups so there is one of these students in each group. It is when the weaker and/or math-avoidant students

dominate the classroom that the approach described in the next section has made a significant improvement to student attitudes toward learning mathematics, and thus their willingness to persist through the necessary steps to gain genuine understanding.

## 2 Using narrative structure to design lessons

When using material designed for exploration, most of our students succeed. However, some were still not enjoying the process. These students felt the units we had developed to lead them to explore and discover mathematics were “busy work” or just something they had to get through. Some student groups would find the pattern or rule for a situation and then rush straight into the next problem without reflecting on what they had discovered or even realizing that they had made a significant discovery. Other groups would complain that it was too hard to figure things out. As much as they didn’t like being taught a process and told to follow it blindly without true comprehension, they would ask us for exactly that experience. They would say, just tell me the rule — don’t make me find it.

We realized that these students weren’t understanding the approach in part because they didn’t have a framework for the activities or the language and vocabulary to appreciate it. Instead of changing the approach, we changed the way we talked about it. We did not make the mathematics any easier or the approach less inquiry-based, but rather we changed the expectations the students had about what the process should be. Students from literature, theater and music all understand the concept of conflict and resolution in a work of art. We make use of the dramatic structure first described by Aristotle [1]. According to this, the plot of a story should consist of three parts:

- The beginning, in which the setting, the characters, and the situation they find themselves are introduced.
- The middle, in which the main character is driven from his initial static position through various complications and obstacles. Throughout this part, the tension mounts. Typically, each of these obstacles are surmounted only to lead to another difficulty, culminating in the *climax*, the main crisis.
- The end, in which the climax and any loose ends of the story are resolved during the *denouement*. The accumulated tension rapidly dissipates and the reader is left (ideally) with a feeling of satisfaction.

Aristotle said, “A well-constructed plot, therefore, must neither begin nor end at haphazard, but conform to these principles.” Music students use a slightly different vocabulary, but the structure is the same: Introduce the theme or motif, repeat it, alter it, add conflict, build to the final climax, and finally resolve the piece.

Compare this process with the use of the scientific inquiry method in mathematics: We introduce a question, have them explore some examples and make a prediction, introduce additional examples to test and refine their theory, and then have them explain why their final conjecture makes sense. This approach can be rephrased as: Let the students explore and discover mathematics by introducing a setting and a group of characters, posing a question to create conflict, and leading them to resolve the conflict with carefully chosen exercises. The language of narrative structure is what we suggest giving students to help them relate to inquiry-based learning. Even in our first experiment with this concept, the attitudes and expectations of the students improved

dramatically with a new framework to help them understand the procedure and goals of the lesson.

It is significant that we did not introduce the analogy with a narrative structure at the beginning of the course. Students spent the first part of the semester using the inquiry method to explore topics. Most of them were getting the correct results but not really connecting to the idea that they were doing and discovering mathematics. They were just answering the questions in the order they were asked. Struggling with the material got the students ready to appreciate the framework as much as the framework helped the students make sense of the struggle.

We had discussion days in this class every couple of weeks so when they came in a few days before midterm break, they weren't shocked to be put into a discussion circle. We introduced the idea of a "story arc" as the structure that introduces a topic, character or theme, carries that theme into conflict in the middle of the work, and then resolves the conflict returning the character to a new status quo. (We later learned that this is not the correct term for the narrative structure we had adopted.) Students in all majors could relate to this concept and it was particularly familiar for the music, literature, and drama students. However, they were all surprised when we started relating the units they had been working on and struggling with to the same process. The billiard trajectories or star polygons or celtic knots they were learning to create were the characters and settings. The questions asking them to analyze how the billiard balls bounced or which pocket the ball ended up in or how many strands were in a knot formed the conflict— often several rising levels of conflict as we asked harder and harder questions about more complicated configurations. Figuring out, testing, and refining rules to predict behavior was equivalent to resolving the conflict and establishing a new and deeper un-

derstanding. The students learned to look at the objects and relationships of a new topic as a new cast and setting and to look at new questions as new plot points. They began to expect conflict and look forward to the resolution. They reflected on their accomplishments and were even heard to utter the phrase “Ah, story arc” when they “got it” throughout the rest of the semester. Note that the literary concept of story arc does play a significant role in the experience we want students to have. In narrative, a story arc consists of several episodes woven together to form a longer story. The in-depth explorations that we feel best engage students cover two or three classes. Students then have time to reflect on the characters they have met and the dilemmas those characters are facing before the final resolution.

By using a familiar structure to explain the mathematical process, the lessons became something they were willing to try. Students started looking at the objects as characters. They expected to be involved in the conflict of the story. They looked at the given examples as foreshadowing and hints to resolve the conflict and clarify the complications. Most importantly, they expected a resolution that made sense.

### **3 Celtic knots**

How does one create this experience for students? In literature or music, the dramatic events or crises are used to stimulate the emotions of the audience, raising the level of tension and engaging them in the story. Used effectively, the reader or listener begins to live within the story. This involvement in the story is what we wanted to craft for the students. How does one introduce dramatic events and crisis points without giving away the whole story? Thoughtfully chosen topics, precisely introduced objects, carefully crafted exercises, and

gently leading questions are essential.

When we choose a topic, we try to find things that will interest the students. Celtic knots work well because almost all of our students have seen them and many are wearing them as jewelry or tattoos. The art of the patterns is visually attractive and the notion that math is related to something so familiar is intriguing [4].

First we introduce the basic characters of the story. We show some examples and coax the students into making note of some characteristics: the alternation of overpasses and underpasses, how some knots close on themselves while others have open ends (often decorated), and that some knots are formed by a single strand while others contain multiple strands. Examples of such knots are shown in figure 1, in which the knot on the left is a closed knot with three strands, while the knot on the right has two loose ends and two strands.

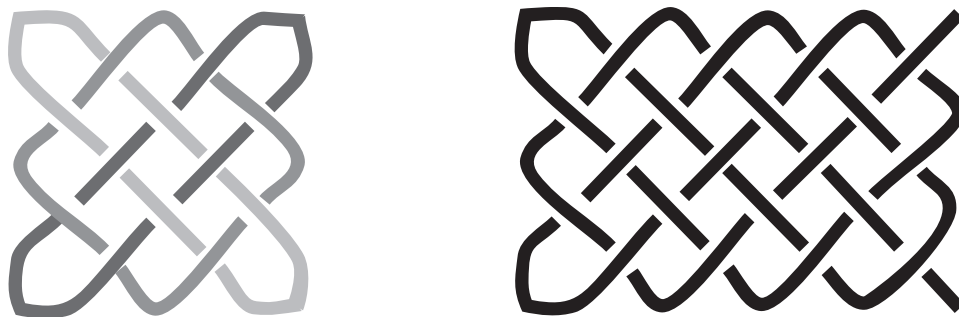


Figure 1: Examples of rectangular celtic knots.

We then give instructions for how to draw a knot on a printed grid so students can create their own examples to explore. These instructions introduce the first levels of conflict as students have to figure out how to relate the given dimensions for the grid to the paper in front of them.

Once the students have drawn several knots, they have enough characters to begin analyzing the plot. They have already resolved the mini-crisis of learning to draw. Pipe cleaners and a geoboard are useful for having the students create a few models that they can actually hold. The weaving with physical objects lets them consider whether they can always alternate overpasses and underpasses. Once that hurdle has been mastered, they can consider whether there are loose ends. Traditionally, loose ends get formed into dragon heads and tails and are an integral part of the design. If we adopt the convention that the upper left corner is always a turn (as is always possible unless the knot has four loose ends), we can ask students to determine the location of any loose ends. They are also asked how many strands are in each knot. As they address the question of how many strands are required for a particular grid size, making pipe cleaner knots with different colors for separate strands introduces a more visual approach to solving the problem.

Finding the rules that answer the questions on loose ends and strands is the resolution to the story. In order to analyze the questions and deduce the rules, the students must have enough data, in the form of a number of examples, some that we provide but most of which they must draw or build. If they are working in groups, different dimensions can be assigned to each group member which gives them additional data and makes sure that all of the students understand the construction process. The solution to the question of loose ends needs only simple facts about which dimensions of the grid are even and odd, and the number of strands is a simple combination of these dimensions. These formulae are well within the reach of all the students, once they organize their data and are brave enough to venture a guess (and then to check their proposed solution).

Additional questions can increase the level of complexity and create new

conflicts. One such problem that we introduce for this unit involves bars which make the strands turn before reaching the edge of the grid. The effect of such a bar is to split what had been a single strand into multiple strands, or to join two strands to form a single strand. Figure 2 shows a grid with bars and the resulting celtic knot.

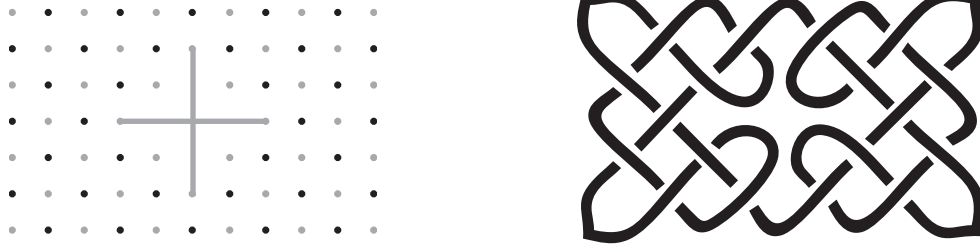


Figure 2: Example of a rectangular celtic knot with bars.

A further conflict involves changing the grid shape and is usually reserved for more motivated students. Much of the jewelry students wear involves triangular or circular patterns. As students try to change grid shape, they run into more mathematics. Strong students enjoy these challenges so this option is often left for an individual final project, but we usually exhibit a few of these knots to show that the lesson can be extended and that an important aspect of mathematics is that one is never finished, but every concept can be extended and every problem gives rise to new questions.

Consider the two triangular knots of figure 3 drawn from [5]. Note that the knot shown below on the right can be formed by bending a rectangular knot to fit an equilateral triangle, while the one on the left is formed by a true triangular grid.

In figure 4, we give two more complicated examples drawn from [2]. On the left below is an annular knot, while on the right there is a circular knot.

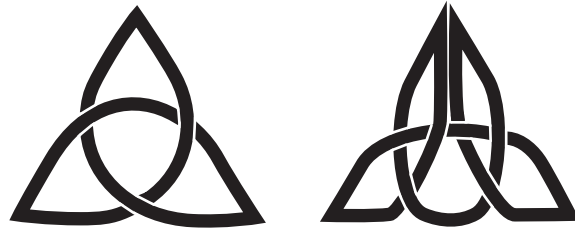


Figure 3: Examples of triangular celtic knot.

Note that since only two strands may cross at a time, the circular knot has a woven hexagon in the center, rather than a six-fold crossing.

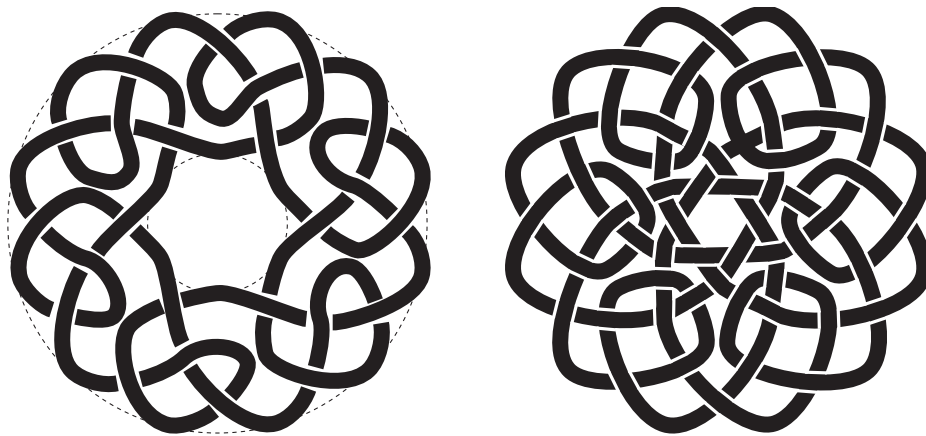


Figure 4: Examples of annular and circular celtic knots.

An excellent advanced final project for mathematics or mathematics education majors is to present the proof of the formula students found for the number of strands in a simple rectangular knot. A reference for this is [3].

## 4 Conclusion

In implementing inquiry-based learning, the layout of the lesson is important. We have found that the most successful units of learning make use of the narrative structure we have described. Further, students seem to be less frustrated and more willing to engage with the material once they have an understanding of the intent of the process in familiar terms. Since they have ample exposure to classical narrative, adapting this to a mathematics lesson makes the process and objectives more palatable and getting results more satisfying. Our students have been enthusiastic about struggling with mathematics as part of the drama of understanding and solving/resolving the conflict of new concepts.

## References

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