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Multi-Objective Crystal Structure Algorithm (MOCryStAl): Introduction and Performance Evaluation

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ABSTRACT In many optimization problems, the main goal is to improve a single performance index in which a minimum or maximum value of this index fully reflects the quality of the response obtained from a system. However, in some cases, it is impossible to rely solely on a single index, so a multi-objective optimization problem with multiple performance indicators is considered where the values of all of them should be optimized simultaneously. The mentioned process requires a multi-objective optimization algorithm that can deal with the complexity of problems with simultaneous indexes. This paper presents the multi-objective version of a recently proposed metaheuristic algorithm called Crystal Structure Algorithm (CryStAl) which was inspired by the principles underlying the formation crystal structures. For the performance evaluation of this algorithm which is called MOCryStAl, the benchmark problems of the Completions on Evolutionary Computation (CEC) on multi-objective optimization, called CEC-09, are utilized. Some real-world engineering design problems are used to evaluate the efficiency of the proposed approach. The results demonstrate that the proposed methods can provide excellent results in dealing with the considered multi-objective problems.

INDEX TERMS Multi-objective optimization, metaheuristic, crystal structure algorithm (CryStAl), completions on evolutionary computation (CEC), real-world engineering design problem.

I. INTRODUCTION

Optimization is the art of finding the best answer among a set of possible solutions under some predefined conditions. It is used for decision-making in various areas such as engineering, management, economics, and finance [1]–[4]. The complexity and interdependency of advanced engineering systems require an analyst with, at least, a general understanding of the system to assist in the optimization of production, laboratory, store, or service system. Besides, the interaction of the subsystems should be considered in the study of a system to ensure its integrity and optimality. In addition, the technical specifications and limitations of system components as well as existing uncertainties should be determined and considered when defining the sought-after goals. These goals often require multidisciplinary optimization and modeling

approaches to obtain design solutions. Metaheuristic algorithms are a type of searching techniques in which some upper-level strategies are utilized for finding the best solution to a problem. Genetic Algorithm (GA) [5], Particle Swarm Optimizer (PSO) [6], Ant colony Optimization (ACO) [7], Chaos Game Optimization (CGO) [8], Atomic Orbital Search (AOS) [9], Dynamic Water Strider Algorithm (DWSA) [10], Material Generation Algorithm (MGA) [11], Stochastic Paint Optimizer (SPO) [12], Flow Direction Algorithm (FDA) [13], Advanced Charged System Search (ACSS) [14], and Hybrid Invasive Weed Optimization-Shuffled Frog-leaping Algorithm (IWO-SFLA) [15] are some well-known metaheuristic algorithms. Besides, some of the improved and hybridized metaheuristics have also been proposed for different applications [16]–[28].

Multi-objective optimization is an area of multi-criteria decision-making in which multiple objective functions need to be optimized simultaneously. Multi-objective optimization

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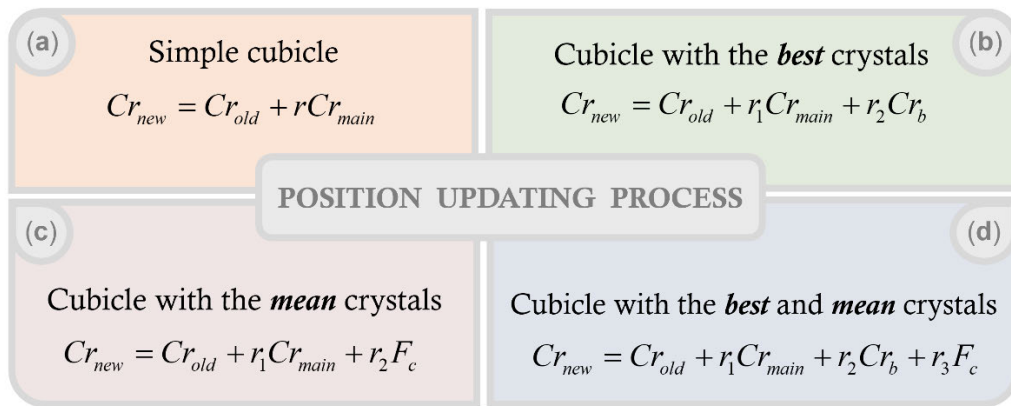


FIGURE 1. Position updating processes of cubicles containing different crystals.

TABLE 1. Parameters setting of all algorithms.

Parameters	MOPSO	MSSA	MOMVO	MOCryStAl
Mutation Probability (P_w or pro)	0.5	-	-	-
Population Size (N_{pop})	100	100	100	100
Archive Size (N_{rep} or TM)	100	100	100	100
Number of Adaptive Grid (N_{grid})	30	30	30	30
Personal Learning Coefficient (C_1)	1	-	-	-
Global Learning Coefficient (C_2)	2	-	-	-
Inertia weight (w)	0.4	-	-	-
Beta	4	4	4	4
Gamma	2	2	2	2

is also known by other names such as multi-objective programming, vector optimization, multi-criteria optimization, multi-attribute optimization, and Pareto optimization. Multi-objective optimization methods are used in many branches of science and engineering. These methods are particularly used to achieve optimal decisions in problems in which striking a balance between two or more conflicting goals is in perspective. In most engineering applications, processes, and systems, designers make decisions based on different conflicting goals. For example, in a typical vehicle design process, in addition to aiming to achieve the highest attainable performance, engineers make effort to minimize fuel consumption and environmental pollution at the same time. In such cases, because more than one objective functions must be considered, it is necessary to consider the application of multi-objective optimization methods. The most important feature of such methods is that they provide system engineers and designers with more than one candidate solution (i.e. a possible answer to the problem) each of which will show the balance between the different objective functions.

In recent decades, researchers have proposed the multi-objective versions of some well-known metaheuristic algorithms where the searching techniques of corresponding single-objective algorithms have been modified to deal with multiple objective functions. The multi-objective version of

the Genetic Algorithm (GA) [29] as Non-Dominated Sorting Genetic Algorithm (NSGA), proposed by Srinivas and Deb, alongside the improved version of this method called NSGA-II [30], are amongst the primary multi-objective optimization methods in the area of evolutionary computation.

Furthermore, researchers in the field of artificial intelligence have proposed a range of multi-objective developments such as the Multiple Objective Particle Swarm Optimization (MOPSO) by Coello and Lechuga [31], Multi-Objective Evolutionary Algorithm (MOEA) by Zhang and Li [32], Multi-Objective Ant Colony Optimization (MOACO) by Alaya *et al.* [33], and Multi-Objective Simulated Annealing (MOSA) by Smith *et al.* [34]. Besides, the multi-objective versions of some other recently proposed metaheuristic algorithms have also been proposed, such as the multi-objective seagull optimization algorithm [35], multi-objective forest optimization algorithm [36], multi-objective whale optimization algorithm with differential evolution [37], multi-objective crow search algorithm [38], and Multi-objective Slap Swarm Algorithm (MSSA) [39].

Based on the Crystal Structure Algorithm (CryStAl) developed recently by Talatahari *et al.* [40], here we propose the multi-objective version of CryStAl, abbreviated as MOCryStAl, as a multi-objective metaheuristic algorithm. In this algorithm, the geometric principles of crystal

Algorithm 1 Pseudo-Code of CryStAl [19]

```

procedure Crystal Structure Algorithm (CryStAl)
    Create random values for initial positions ( $x_i^j$ )
    of initial crystals( $Cr_i$ )
    Evaluate fitness values for each crystal
    while ( $t < \text{maximum number of iterations}$ )
        for  $i = 1$ : number of initial crystals
            Create  $Cr_{main}$ 
            Create new crystals by Eq. 4
            Create  $Cr_b$ 
            Create new crystals by Eq. 5
            Create  $F_c$ 
            Create new crystals by Eq. 6
            Create new crystals by Eq. 7
            if new crystals violate boundary conditions
                Control the position constraints for new
                crystals and amend them
            end if
            Evaluate the fitness values for new crystals
            Update Global Best (GB) if a better solution
            is found
        end for
         $t = t + 1$ 
    end while
    Return GB
end procedure

```

structures, including the concepts of lattice and basis in the configuration of crystals, are in perspective. For the performance evaluation of this algorithm, the benchmark problems of the Completions on Evolutionary Computation (CEC) on multi-objective optimization, called CEC-09 [41], are utilized. Some real-world engineering design problems are used to evaluate the efficiency of the proposed approach. The results demonstrate that MOCryStAl is capable of providing excellent results in dealing with the considered multi-objective problems.

A. CRYSTAL STRUCTURE ALGORITHM (CryStAl)

Crystalline solids and their rich structural symmetries have inspired the conception and design of many man-made structures, mechanisms, and artworks [42]–[54]. The crystal structure algorithm (CryStAl) is a recently proposed metaheuristic algorithm in which the geometric principles of crystal structures are in perspective. As described in [40], the Bravais model is one of the frequently referenced configurations of crystals in which a periodic crystal structure is determined by an infinite lattice shape in which any lattice point is defined by the positions of related lattice points with a vector $r = \sum n_i a_i$, where n_i is an integer, a_i is the shortest vector along the principal crystallographic directions, and i is the number of crystal corners. At first, the initialization process of the algorithm is formulated in which a random generation of the

Algorithm 2 Pseudo-Code of MOCryStAl

```

procedure Multi-Objective Crystal Structure
Algorithm (CryStAl)
    Create random values for initial positions ( $x_i^j$ ) of
    initial crystals( $Cr_i$ )
    Evaluate fitness values for each crystal
    find The GB
    obtain The non-dominated answers and create the archive
    while ( $t < \text{maximum number of iterations}$ )
        for  $i = 1$ : number of initial crystals
            Create  $Cr_{main}$ 
            Create new crystals by Eq. 4
            Create  $Cr_b$ 
            Create new crystals by Eq. 5
            Create  $F_c$ 
            Create new crystals by Eq. 6
            Create new crystals by Eq. 7
            if new crystals violate boundary conditions
                Control the position constraints for new
                crystals and amend them
            end if
            Evaluate the fitness values for new crystals
        end for
        evaluate The objective function for all crystals
        obtain The non-dominated solution answers
        update The archive according to the found non-
        dominated answers
        If the archive is full
            use the grid mechanism to delete the cur-
            rent archives
            add the new answer to the archive
        endif
        If any of the new added solutions to the
        archive is located outside the hypercubes
            update the grids
        endif
         $t = t + 1$ 
    end while;
    Return Archive
end procedure

```

initial candidate solutions are determined as follows:

$$Cr = \begin{bmatrix} Cr_1 \\ Cr_2 \\ \vdots \\ Cr_i \\ \vdots \\ Cr_n \end{bmatrix} = \begin{bmatrix} x_1^1 & x_1^2 & \dots & x_1^j & \dots & x_1^d \\ x_2^1 & x_2^2 & \dots & x_2^j & \dots & x_2^d \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ x_i^1 & x_i^2 & \dots & x_i^j & \dots & x_i^d \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ x_n^1 & x_n^2 & \dots & x_n^j & \dots & x_n^d \end{bmatrix}, \begin{cases} i = 1, 2, \dots, n \\ j = 1, 2, \dots, d \end{cases} \quad (1)$$

$$x_i^j(0) = x_{i,\min}^j + \xi(x_{i,\max}^j - x_{i,\min}^j), \quad \begin{cases} i = 1, 2, \dots, n \\ j = 1, 2, \dots, d \end{cases} \quad (2)$$

TABLE 2. Bi-objective CEC-09 benchmark functions.

Function	Mathematical formulation	D	Range
UF1	$f_1 = x_1 + \frac{2}{ J_1 } \sum_{j \in J_1} [x_j - \sin(6\pi x_1 + \frac{j\pi}{n})]^2, \quad f_2 = 1 - \sqrt{x_1} + \frac{2}{ J_2 } \sum_{j \in J_2} [x_j - \sin(6\pi x_1 + \frac{j\pi}{n})]^2$ $J_1 = \{j j \text{ is odd and } 2 \leq j \leq n\}, \quad J_2 = \{j j \text{ is even and } 2 \leq j \leq n\}$	30	$x_1 \in [0,1]$ $x_i \in [-1,1]$ $i = 1, \dots, D$
UF2	$f_1 = x_1 + \frac{2}{ J_1 } \sum_{j \in J_1} y_j^2, \quad f_2 = 1 - \sqrt{x_1} + \frac{2}{ J_2 } \sum_{j \in J_2} y_j^2$ $J_1 = \{j j \text{ is odd and } 2 \leq j \leq n\}, \quad J_2 = \{j j \text{ is even and } 2 \leq j \leq n\}$ $y_j = \begin{cases} x_j - [0.3x_1^2 \cos(24\pi x_1 + \frac{4j\pi}{n}) + 0.6x_1] \cos(6\pi x_1 + \frac{j\pi}{n}) & \text{if } j \in J_1 \\ x_j - [0.3x_1^2 \cos(24\pi x_1 + \frac{4j\pi}{n}) + 0.6x_1] \cos(6\pi x_1 + \frac{j\pi}{n}) & \text{if } j \in J_2 \end{cases}$	30	$x_1 \in [0,1]$ $x_i \in [-1,1]$ $i = 1, \dots, D$
UF3	$f_1 = x_1 + \frac{2}{ J_1 } (4 \sum_{j \in J_1} y_j^2 - 2 \prod_{j \in J_1} \cos(\frac{20y_j\pi}{\sqrt{j}}) + 2)$ $f_2 = \sqrt{x_1} + \frac{2}{ J_2 } (4 \sum_{j \in J_2} y_j^2 - 2 \prod_{j \in J_2} \cos(\frac{20y_j\pi}{\sqrt{j}}) + 2)$ $J_1 \text{ and } J_2 \text{ are the same as those of UF1; } y_j = x_j - x_1^{0.5(1.0+\frac{3(j-2)}{\pi-2})}, \quad j = 2, 3, \dots, n$	30	$x_i \in [0,1]$
UF4	$f_1 = x_1 + \frac{2}{ J_1 } \sum_{j \in J_1} h(y_j), \quad f_2 = 1 - x_2 + \frac{2}{ J_2 } \sum_{j \in J_2} h(y_j)$ $J_1 \text{ and } J_2 \text{ are the same as those of UF1; } y_j = x_j - \sin(6\pi x_1 + \frac{j\pi}{n}), \quad j = 2, 3, \dots, n$ $h(t) = \frac{ t }{1 + e^{2 t }}$	30	$x_1 \in [0,1]$ $x_i \in [-2,2]$ $i = 1, \dots, D$
UF5	$f_1 = x_1 + (\frac{1}{2N} + \varepsilon) \sin(2N\pi x_1) + \frac{2}{ J_1 } \sum_{j \in J_1} h(y_j)$ $f_2 = 1 - x_1 + (\frac{1}{2N} + \varepsilon) \sin(2N\pi x_1) + \frac{2}{ J_2 } \sum_{j \in J_2} h(y_j)$ $J_1 \text{ and } J_2 \text{ are identical to those of UF1; } \varepsilon > 0, \quad y_j = x_j - \sin(6\pi x_1 + \frac{j\pi}{n}), \quad j = 2, 3, \dots, n$ $h(t) = 2t^2 - \cos(4\pi t) + 1$	30	$x_1 \in [0,1]$ $x_i \in [-1,1]$ $i = 1, \dots, D$
UF6	$f_1 = x_1 + \max\{0, 2(\frac{1}{2N} + \varepsilon) \sin(2N\pi x_1) \} + \frac{2}{ J_1 } (4 \sum_{j \in J_1} y_j^2 - 2 \prod_{j \in J_1} \cos(\frac{20y_j\pi}{\sqrt{j}}) + 1)$ $f_2 = 1 - x_1 + \max\{0, 2(\frac{1}{2N} + \varepsilon) \sin(2N\pi x_1) \} + \frac{2}{ J_2 } (4 \sum_{j \in J_2} y_j^2 - 2 \prod_{j \in J_2} \cos(\frac{20y_j\pi}{\sqrt{j}}) + 1)$ $J_1 \text{ and } J_2 \text{ are identical to those of UF1; } \varepsilon > 0, \quad y_j = x_j - \sin(6\pi x_1 + \frac{j\pi}{n}), \quad j = 2, 3, \dots, n$	30	$x_1 \in [0,1]$ $x_i \in [-1,1]$ $i = 1, \dots, D$
UF7	$f_1 = \sqrt[5]{x_1} + \frac{2}{ J_1 } \sum_{j \in J_1} y_j^2, \quad f_2 = 1 - \sqrt[5]{x_1} + \frac{2}{ J_2 } \sum_{j \in J_2} y_j^2$ $J_1 \text{ and } J_2 \text{ are identical to those of UF1; } \varepsilon > 0, \quad y_j = x_j - \sin(6\pi x_1 + \frac{j\pi}{n}), \quad j = 2, 3, \dots, n$	30	$x_1 \in [0,1]$ $x_i \in [-1,1]$ $i = 1, \dots, D$

where n is the number of initial candidate solutions or crystals, d is the dimension of the problem; $x_i^j(0)$ is the initial position of the crystals; $x_{i,\min}^j$ and $x_{i,\max}^j$ are the minimum and maximum bounds in the considered problem; and ξ is a randomly generated number in the interval of $[0,1]$.

Based on the concept of ‘basis’ in crystallography, all the crystals at the corners are considered as the main crystals, Cr_{main} , determined randomly by considering the initially-created crystals (candidate solutions). It should be noted that the random selection process for each step is determined by omitting the current Cr . The crystal with the best configuration is determined as Cr_b while the mean values of randomly-selected crystals are denoted by F_c .

The position updating process for the crystals, in which the basis and lattice principles are utilized, are presented in Fig.1, where Cr_{new} represents the new position, Cr_{old} represents the old position, and r , r_1 , r_2 , and r_3 denote randomly-generated coefficient.

A boundary control flag is determined for considering the violating variables during the optimization process. In contrast, a maximum number of iterations or objective function evaluation can be considered for termination purposes. The pseudo-code of CryStAl is presented in Algorithm 1.

B. MULTI-OBJECTIVE CRYSTAL (MOCryStAl)

The Multi-Objective Crystal Structure Algorithm, abbreviated as MOCryStAl, is developed in this paper with the

TABLE 3. Tri-objective CEC-09 benchmark functions.

Function	Mathematical formulation	D	Range
UF8	$f_1 = \cos(0.5x_1\pi) \cos(0.5x_2\pi) + \frac{2}{ J_1 } \sum_{j \in J_1} (x_j - 2x_2 \sin(2\pi x_1 + \frac{j\pi}{n})^2)$	30	$x_1 \in [0,1]$ $x_2 \in [0,1]$ $x_i \in [-2,2]$ $i = 1, \dots, D$
	$f_2 = \cos(0.5x_1\pi) \sin(0.5x_2\pi) + \frac{2}{ J_2 } \sum_{j \in J_2} (x_j - 2x_2 \sin(2\pi x_1 + \frac{j\pi}{n})^2)$		
	$f_3 = \sin(0.5x_1\pi) + \frac{2}{ J_3 } \sum_{j \in J_3} (x_j - 2x_2 \sin(2\pi x_1 + \frac{j\pi}{n})^2)$		
	$J_1 = \{j 3 \leq j \leq n, \text{ and } j - 1 \text{ is a multiplication of } 3\}$		
	$J_2 = \{j 3 \leq j \leq n, \text{ and } j - 2 \text{ is a multiplication of } 3\}$		
	$J_3 = \{j 3 \leq j \leq n, \text{ and } j \text{ is a multiplication of } 3\}$		
UF9	$f_1 = 0.5[\max\{0, (1 + \varepsilon)(1 - 4(2x_1 - 1)^2)\} + \frac{2}{ J_1 } \sum_{j \in J_1} (x_j - 2x_2 \sin(2\pi x_1 + \frac{j\pi}{n})^2)$	30	$x_1 \in [0,1]$ $x_2 \in [0,1]$ $x_i \in [-2,2]$ $i = 1, \dots, D$
	$f_2 = 0.5[\max\{0, (1 + \varepsilon)(1 - 4(2x_1 - 1)^2)\} + 2x_1]x_2 + \frac{2}{ J_2 } \sum_{j \in J_2} (x_j - 2x_2 \sin(2\pi x_1 + \frac{j\pi}{n})^2)$		
	$f_3 = 1 - x_2 + \frac{2}{ J_3 } \sum_{j \in J_3} (x_j - 2x_2 \sin(2\pi x_1 + \frac{j\pi}{n})^2)$		
	$J_1 = \{j 3 \leq j \leq n, \text{ and } j - 1 \text{ is a multiplication of } 3\}$		
	$J_2 = \{j 3 \leq j \leq n, \text{ and } j - 2 \text{ is a multiplication of } 3\}$		
	$J_3 = \{j 3 \leq j \leq n, \text{ and } j \text{ is a multiplication of } 3\}, \varepsilon = 0.1$		
UF10	$f_1 = \cos(0.5x_1\pi) \cos(0.5x_2\pi) + \frac{2}{ J_1 } \sum_{j \in J_1} [4y_j^2 - \cos(8\pi y_j) + 1]$ $f_2 = \cos(0.5x_1\pi) \sin(0.5x_2\pi) + \frac{2}{ J_2 } \sum_{j \in J_2} [4y_j^2 - \cos(8\pi y_j) + 1]$ $f_3 = \sin(0.5x_1\pi) + \frac{2}{ J_3 } \sum_{j \in J_3} [4y_j^2 - \cos(8\pi y_j) + 1]$ $J_1 = \{j 3 \leq j \leq n, \text{ and } j - 1 \text{ is a multiplication of } 3\}$ $J_2 = \{j 3 \leq j \leq n, \text{ and } j - 2 \text{ is a multiplication of } 3\}$ $J_3 = \{j 3 \leq j \leq n, \text{ and } j \text{ is a multiplication of } 3\}$	30	$x_1 \in [0,1]$ $x_2 \in [0,1]$ $x_i \in [-2,2]$ $i = 1, \dots, D$

goal of solving some problems more effectively. In order to perform multi-objective optimization by MOCryStAl, we integrate three components. The employed components are very similar to those of (MOPSO) by Coello and Lechuga [31]. This algorithm has three capabilities for performing multi-objective optimizations as follows:

- (i) **Archive:** The archive contains all of the non-dominated Pareto optimum responses collected so far. When a solution is meant to be archived or the archive is full, an archive controller is a vital component of the archive. In addition, the archive will only accept a limited amount of participants. Non-dominated solutions obtained up to this step are compared to the existing archive during the training process of an iteration.
- (ii) **Leader selection:** The leader selection feature facilitates the selection of the current best place solutions from the archive as the leaders of the search process.
- (iii) **Grid mechanism:** The grid mechanism features omit one of the current archive members and adds the new solution to the archive.

The employed external archive effectively saves the best non-dominated solutions obtained so far. The grid mechanism and selection leader component maintain the diversity of the archive during optimization. The probability of removing a solution rises in direct proportion to the hypercube's total number of solutions (segments). If the archive is full, the most crowded segments are chosen from which a solution is randomly deleted to make space for the new solution. In an exceptional circumstance, a solution is added outside the hypercubes. In this situation, all components are extended to include the new solutions. As a result, the components of several alternative solutions can be changed.

In the hope of finding a solution that is close to the global optimum, the search leaders guide the other search agents to the more promising areas of the search space. The solutions in a multi-objective search space, however, cannot easily be compared due to the Pareto optimality theories discussed above. The procedure for selecting leaders addresses this problem. As previously stated, there is a database of the best non-dominated solutions found recently. The leader selection component chooses the least congested

TABLE 4. Multimodal benchmark functions with fixed dimensions.

Function	Mathematical formulation	D	Range
ZDT1	$F_1 = x_1, \quad F_2 = g \left(1 - \sqrt{F_1/g} \right), \quad g = 1 + \frac{9}{d-1} \sum_{i=2}^d x_i$	30	$x_i \in [0,1]$
ZDT2	$F_1 = x_1, \quad F_2 = g \left(1 - (F_1/g)^2 \right), \quad g = 1 + \frac{9}{d-1} \sum_{i=2}^d x_i$	30	$x_i \in [0,1]$
ZDT3	$F_1 = x_1, \quad F_2 = g \left(1 - \sqrt{F_1/g} - F_1/g \sin(10\pi F_1) \right), \quad g = 1 + \frac{9}{d-1} \sum_{i=2}^d x_i$	30	$x_i \in [0,1]$
ZDT4	$F_1 = x_1, \quad F_2 = g \left(1 - \sqrt{F_1/g} \right), \quad g = 1 + 10(d-1) + \sum_{i=2}^d (x_i^2 - 10\cos(4\pi x_i))$	10	$x_1 \in [0,1]$ $x_i \in [-5,5]$ $i = 1, \dots, D$
ZDT6	$F_1 = 1 - \exp(-4x_1) \sin^6(6\pi x_1), F_2 = g \left(1 - (F_1/g)^2 \right), \quad g = 1 + 9 \left(\frac{\sum_{i=2}^d x_i}{d-1} \right)^{0.25}$	10	$x_i \in [0,1]$
DTLZ2	$F_1 = (1+g) \cos \left(x_1 \left(\frac{\pi}{2} \right) \right) \cos \left(x_2 \left(\frac{\pi}{2} \right) \right), F_2 = (1+g) \cos \left(x_1 \left(\frac{\pi}{2} \right) \right) \sin \left(x_2 \left(\frac{\pi}{2} \right) \right),$ $F_3 = (1+g) \sin \left(x_1 \left(\frac{\pi}{2} \right) \right), g = \sum_{i=3}^d (x_i - 0.5)^2$	12	$x_i \in [0,1]$
DTLZ4	$F_1 = (1+g) \cos \left(x_1^\pi \left(\frac{\pi}{2} \right) \right) \cos \left(x_2^\pi \left(\frac{\pi}{2} \right) \right), F_2 = (1+g) \cos \left(x_1^\pi \left(\frac{\pi}{2} \right) \right) \sin \left(x_2^\pi \left(\frac{\pi}{2} \right) \right),$ $F_3 = (1+g) \sin \left(x_1^\pi \left(\frac{\pi}{2} \right) \right), g = \sum_{i=3}^d (x_i - 0.5)^2$	12	$x_i \in [0,1]$

TABLE 5. Statistical results of the CEC-09 benchmark functions for the IGD performance metric.

Functions		Algorithm			
		MOPSO	MSSA	MOMVO	MOCryStAl
UF1	Ave	5.3291E-03	4.3420E-03	3.6662E-03	1.6280E-02
	SD	2.9590E-03	3.2262E-04	5.1992E-04	5.3280E-03
UF2	Ave	4.1586E-03	3.6126E-03	2.8279E-03	2.8179E-03
	SD	4.8128E-04	9.4827E-04	3.2623E-04	1.6863E-04
UF3	Ave	1.7231E-02	1.4033E-02	1.4925E-02	1.2304E-02
	SD	1.0806E-03	2.5380E-03	1.3921E-03	4.7659E-04
UF4	Ave	2.8277E-03	4.1039E-03	3.2478E-03	2.0779E-03
	SD	2.5695E-04	5.3604E-04	1.9370E-04	1.8818E-04
UF5	Ave	3.4235E-01	2.3030E-01	2.0559E-01	6.7967E-01
	SD	1.5317E-01	5.3692E-02	6.4855E-02	8.2121E-02
UF6	Ave	2.9255E-02	1.3870E-02	1.6393E-02	7.0633E-02
	SD	9.1394E-03	1.3150E-03	6.8389E-03	1.3778E-02
UF7	Ave	4.3417E-03	3.8381E-03	8.1363E-03	1.5255E-02
	SD	2.4234E-03	3.4544E-04	6.4896E-03	4.7925E-03
UF8	Ave	9.2776E-03	8.0705E-03	4.9482E-03	1.2713E-03
	SD	1.6147E-03	4.1968E-03	2.2834E-03	1.0998E-03
UF9	Ave	1.2300E-02	1.1846E-02	4.3669E-03	1.2111E-02
	SD	2.9460E-03	9.5640E-03	7.8011E-04	3.7146E-03
UF10	Ave	6.3301E-02	1.5437E-02	1.1965E-02	7.1624E-03
	SD	1.3765E-02	4.8236E-03	5.8420E-03	1.4274E-04

regions of the search space and offers the best particles as non-dominated solutions. Each hypercube is chosen using a roulette-wheel methodology expressed by the following equation

$$P_i = \frac{C}{N_i} \quad (3)$$

TABLE 6. Statistical results of the CEC-09 benchmark functions for the GD performance metric.

Functions		Algorithm			
		MOPSO	MSSA	MOMVO	MOCryStAl
UF1	Ave	2.7430E-02	3.4966E-02	3.7175E-03	1.3323E-01
	SD	2.5390E-02	3.1994E-02	2.7354E-03	7.4915E-02
UF2	Ave	2.0656E-02	1.1215E-02	5.7929E-03	4.1005E-03
	SD	4.6831E-03	2.1213E-03	2.3059E-03	1.8970E-02
UF3	Ave	9.6010E-02	7.2062E-02	5.7714E-02	2.8737E-02
	SD	2.3482E-02	1.0267E-02	1.6501E-02	2.9560E-03
UF4	Ave	9.2957E-03	1.0169E-02	1.0073E-02	1.0003E-02
	SD	9.2862E-04	3.4790E-03	9.4967E-04	7.7198E-04
UF5	Ave	4.2733E-01	2.8272E-01	2.1457E-01	1.5649E-01
	SD	2.9837E-01	1.5805E-01	6.2058E-02	2.2386E-01
UF6	Ave	2.0278E-01	1.1532E-01	3.3546E-02	7.5767E-01
	SD	1.6569E-01	5.0468E-02	1.5241E-02	1.1255E-01
UF7	Ave	1.7938E-02	9.1316E-03	3.3843E-03	1.1190E-01
	SD	1.3813E-02	1.2322E-02	2.3377E-03	4.0419E-02
UF8	Ave	2.6587E-01	3.3914E-02	5.6416E-02	2.5020E-02
	SD	8.2205E-02	1.3307E-02	8.0552E-02	6.6786E-02
UF9	Ave	4.0311E-01	5.1960E-02	9.3547E-02	2.3578E-01
	SD	7.9920E-02	2.1763E-02	5.1567E-02	6.3155E-02
UF10	Ave	1.3828E+00	6.9388E-01	1.9043E-01	1.3534E-01
	SD	2.1017E-01	2.0777E-01	1.4655E-01	1.3789E-01

where C is a constant number higher than one and N is the variety of acquired Pareto optimal answers in the i th section.

As can be concluded from Eq. (3), less congested hypercubes have a higher probability of suggesting new leaders. The chance of selecting a hypercube to select leaders from improves as the number of obtained solutions in the hypercube decreases. Importantly, the MOCryStAl approach is

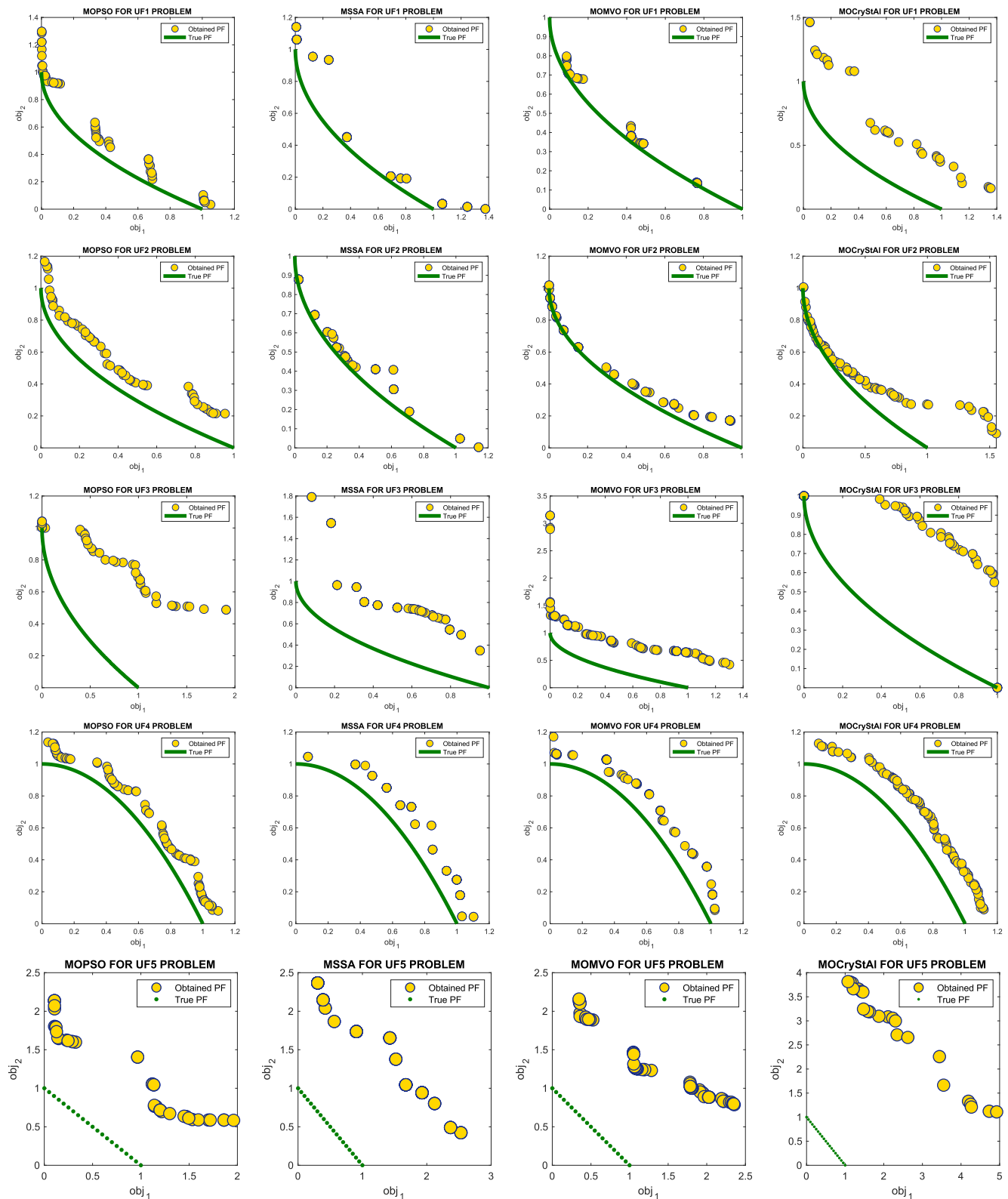


FIGURE 2. True and obtained Pareto fronts for the CEC-09 problems (UF1-5).

based on the CryStAl method for convergence. If one chooses a solution from the archive, the CryStAl approach will most likely be able to increase its already good consistency.

However, finding the Pareto optimal solutions among a large variety of responses is generally challenging. By using archive maintenance with the leader function collection,

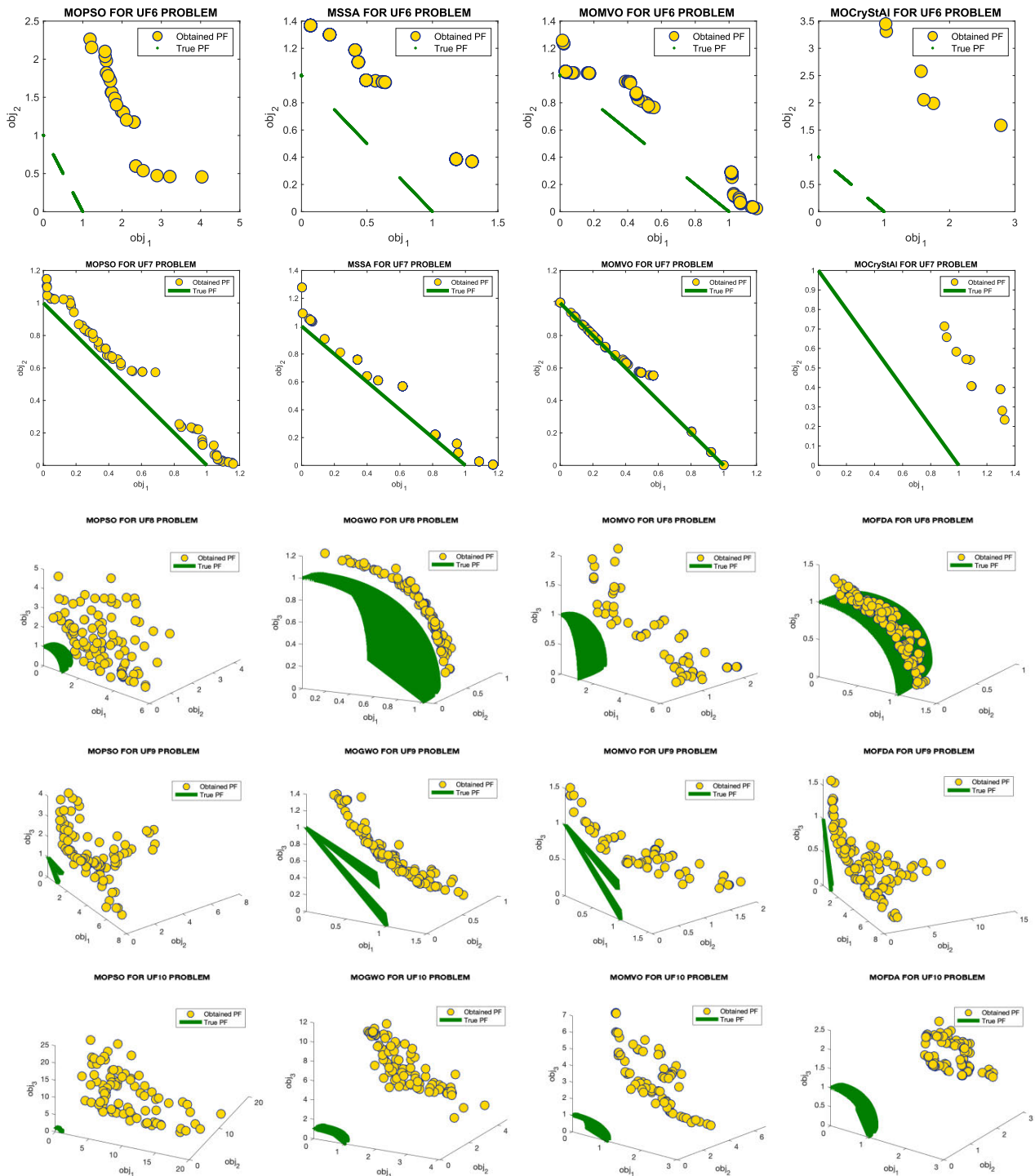


FIGURE 3. True and obtained Pareto fronts for the CEC-09 problems (UF6-10).

the problems can be solved. The pseudo-code of MOCryStAl is presented in Algorithm 2.

II. RESULTS AND DISCUSSION

In this section, the efficiency of the suggested technique is evaluated using performance measures and case studies, including unconstrained and constrained bi- and tri-objective mathematics (CEC-09) and real-world engineering design

problems. The ability of multi-objective optimizers to handle problems with non-convexity and non-linearity is tested using these problems and mathematical functions. MATLAB 2021a was used to code the algorithm. On the computer, the following features are used to carry out the current work: the CPU is 2.3 GHz (an Intel Core i9 computer platform), and the RAM is 16 GB 2400 MHz DDR4 with Macintosh (macOS BigSur).

TABLE 7. Statistical results of the CEC-09 benchmark functions for the MS performance metric.

Functions		Algorithm			
		MOPSO	MSSA	MOMVO	MOCryStAl
UF1	Ave	9.7019E-01	9.6206E-01	7.7791E-01	1.9321E+00
	SD	3.8257E-01	1.1721E-01	1.7104E-01	1.6783E-01
UF2	Ave	1.2037E+00	1.2007E+00	9.7535E-01	1.3190E+00
	SD	1.5132E-01	5.7151E-02	1.1893E-01	1.6796E-01
UF3	Ave	1.3035E+00	1.0923E+00	1.9468E+00	1.0000E+00
	SD	7.1625E-01	9.2630E-02	8.1063E-01	0.0000E+00
UF4	Ave	1.0295E+00	1.0049E+00	1.0315E+00	1.0515E+00
	SD	2.9802E-02	8.6663E-03	1.3932E-02	2.3119E-02
UF5	Ave	6.3155E-01	1.2035E+00	8.2445E-01	2.9287E+00
	SD	7.2675E-01	8.6410E-01	1.1108E+00	8.5137E-01
UF6	Ave	8.7827E-01	1.0298E+00	4.5000E-01	3.9580E+00
	SD	1.2085E+00	3.7343E-02	4.3436E-01	1.8757E+00
UF7	Ave	1.0892E+00	1.0202E+00	6.3955E-01	1.4254E+00
	SD	2.5804E-01	4.9259E-02	3.9756E-01	5.0745E-01
UF8	Ave	5.5041E+00	9.3689E-01	1.4782E+00	3.6836E+00
	SD	1.4655E+00	2.1879E-01	9.9244E-01	9.6799E-01
UF9	Ave	7.6184E+00	1.4259E+00	1.7140E+00	7.6701E+00
	SD	1.6088E+00	4.8994E-01	6.9704E-01	1.5627E+00
UF10	Ave	6.5767E+00	6.3441E+00	2.4780E+00	1.0993E+01
	SD	2.3597E+00	1.4704E+00	1.7208E+00	3.2403E+00

TABLE 8. Statistical results of the CEC-09 benchmark functions for the S performance metric.

Functions		Algorithm			
		MOPSO	MSSA	MOMVO	MOCryStAl
UF1	Ave	1.5649E-02	1.5649E-02	3.9366E-03	5.3134E-02
	SD	9.9871E-03	9.9871E-03	1.6573E-03	2.1338E-02
UF2	Ave	1.8565E-02	1.8565E-02	8.2146E-03	3.3431E-03
	SD	3.1440E-03	3.1440E-03	3.5129E-03	1.0653E-03
UF3	Ave	2.6957E-02	2.6957E-02	5.6488E-02	1.3653E-02
	SD	1.5713E-02	1.5713E-02	8.4649E-02	3.7988E-03
UF4	Ave	1.0182E-02	1.0182E-02	8.2315E-03	1.0008E-02
	SD	1.7583E-03	1.7583E-03	4.7707E-03	1.2861E-03
UF5	Ave	2.5756E-02	2.5756E-02	6.0884E-03	2.3523E-01
	SD	3.3090E-02	3.3090E-02	6.0779E-03	9.0539E-02
UF6	Ave	4.3768E-02	4.3768E-02	5.5990E-03	2.6542E-01
	SD	7.4660E-02	7.4660E-02	4.1791E-03	1.7950E-01
UF7	Ave	1.6883E-02	1.6883E-02	6.1381E-03	5.3850E-02
	SD	8.2967E-03	8.2967E-03	8.7410E-03	1.7464E-02
UF8	Ave	2.7757E-01	2.7757E-01	2.4682E-01	2.1572E-01
	SD	7.9069E-02	7.9069E-02	4.9334E-02	7.5459E-02
UF9	Ave	3.7998E-01	3.7998E-01	3.3556E-01	3.0649E-01
	SD	8.9375E-02	8.9375E-02	3.2153E-02	2.2372E-01
UF10	Ave	1.0998E+00	1.0998E+00	1.1918E-01	1.0315E-01
	SD	1.9502E-01	1.9502E-01	6.6767E-02	6.1896E-01

TABLE 9. Statistical results of the ZDT and DTLZ benchmark functions for the GD performance metric.

Functions		Algorithm			
		MOPSO	MSSA	MOMVO	MOCryStAl
ZDT1	Ave	1.7932E-03	1.2035E-02	7.9682E-03	2.5915E-04
	SD	5.0496E-03	1.9168E-03	1.2073E-02	4.0091E-04
ZDT2	Ave	1.4749E-01	1.1652E-02	5.3988E-03	1.6997E-04
	SD	6.5211E-02	2.1916E-03	2.9609E-04	4.7636E-04
ZDT3	Ave	2.0489E-04	1.6015E-02	1.0954E-02	2.0043E-04
	SD	2.7322E-05	2.0859E-03	1.5715E-02	1.6713E-04
ZDT4	Ave	4.1299E+00	6.0099E-01	1.2363E+00	1.8557E+01
	SD	3.9519E+00	3.1030E-01	7.0133E-01	1.1926E+01
ZDT6	Ave	2.5805E-02	2.1047E-02	1.0183E-02	1.0071E-02
	SD	5.8008E-02	9.6106E-03	8.4995E-03	3.0359E-03
DTLZ2	Ave	6.8600E-03	9.2866E-03	3.1626E-03	2.1556E-01
	SD	9.0878E-04	2.4582E-03	5.5846E-04	7.8364E-03
DTLZ4	Ave	1.0097E-02	1.3120E-02	4.6253E-03	1.9957E-01
	SD	2.4570E-03	3.8723E-03	6.4434E-04	2.5649E-02

A. PERFORMANCE METRICS

The following four measures are used to assess the results of the algorithms:

TABLE 10. Statistical results of the ZDT and DTLZ benchmark functions for the IGD performance metric.

Functions		Algorithm			
		MOPSO	MSSA	MOMVO	MOCryStAl
ZDT1	Ave	8.0879E-04	4.5908E-03	1.5496E-03	2.3138E-04
	SD	1.4764E-03	1.1201E-03	1.0733E-04	1.3159E-05
ZDT2	Ave	5.2023E-02	5.2059E-03	1.9238E-03	1.6173E-03
	SD	9.5007E-03	1.6220E-03	5.1549E-04	1.0133E-04
ZDT3	Ave	2.6241E-04	5.1290E-03	1.6411E-03	1.3612E-03
	SD	3.9884E-05	8.9560E-04	2.1512E-04	1.7230E-03
ZDT4	Ave	7.0747E-01	1.8230E-01	2.4965E-01	8.8854E-01
	SD	3.1784E-01	9.5457E-01	1.3922E-01	1.0955E-01
ZDT6	Ave	8.2281E-03	2.0670E-03	5.0062E-04	3.5820E-04
	SD	2.4862E-02	8.0527E-04	1.9857E-04	1.6232E-04
DTLZ2	Ave	4.6940E-04	3.0425E-03	8.6496E-04	6.8243E-03
	SD	2.6098E-05	3.7517E-04	1.4210E-04	1.5835E-03
DTLZ4	Ave	1.6840E-03	8.3377E-03	4.0415E-03	1.8634E-02
	SD	9.4057E-05	1.1001E-03	8.4951E-04	4.8505E-03

TABLE 11. Statistical results of the ZDT and DTLZ benchmark functions for the MS performance metric.

Functions		Algorithm			
		MOPSO	MSSA	MOMVO	MOCryStAl
ZDT1	Ave	9.9870E-01	9.0233E-01	9.2574E-01	9.9964E-01
	SD	2.3779E-02	1.0585E-01	8.6425E-01	1.4642E-02
ZDT2	Ave	7.4311E-02	7.7491E-01	9.4126E-01	2.8457E-01
	SD	2.3499E-01	1.0567E-01	1.9721E-02	4.2613E-01
ZDT3	Ave	9.9865E-01	8.7507E-01	1.2699E+00	1.3872E+00
	SD	1.7888E-03	6.5264E-02	5.8528E-01	1.1748E-01
ZDT4	Ave	4.6863E-01	1.4795E+00	1.6088E-01	9.4680E+00
	SD	1.4819E+00	1.7705E+00	7.1679E-02	9.5831E+00
ZDT6	Ave	1.1170E+00	1.3361E+00	1.5084E+00	3.6306E+00
	SD	5.3983E-01	1.8535E-01	2.6233E-01	1.5512E+00
DTLZ2	Ave	1.0976E+00	1.8014E-01	9.9553E-01	4.7022E+00
	SD	9.0869E-02	1.6986E-01	8.1926E-02	5.2823E-01
DTLZ4	Ave	1.2437E+00	2.5352E-01	8.2379E-01	4.1525E+00
	SD	1.4187E-01	2.5166E-01	5.5562E-02	8.6314E-01

TABLE 12. Statistical results of the ZDT and DTLZ benchmark functions for the S performance metric.

Functions		Algorithm			
		MOPSO	MSSA	MOMVO	MOCryStAl
ZDT1	Ave	1.1319E-02	1.2165E-02	5.1442E-02	1.1004E-02
	SD	1.0762E-03	3.3123E-03	1.2884E-01	3.1249E-03
ZDT2	Ave	7.3295E-04	1.2250E-02	1.4373E-02	7.1414E-04
	SD	2.3178E-03	6.3466E-03	6.5898E-03	1.2421E-03
ZDT3	Ave	1.3270E-02	1.4228E-02	9.1613E-02	1.1944E-02
	SD	2.3561E-03	6.5709E-03	1.6310E-01	8.5127E-03
ZDT4	Ave	4.3804E-03	2.2121E-03	4.5664E-03	5.8396E-01
	SD	1.3852E-02	1.8168E-03	2.2788E-03	7.6084E-01
ZDT6	Ave	1.8109E-02	1.7684E-02	6.1993E-02	1.8063E-02
	SD	3.0071E-02	1.2219E-02	3.7249E-02	9.5264E-02
DTLZ2	Ave	5.7741E-02	9.1604E-03	6.1739E-02	4.0846E-01
	SD	4.9507E-03	5.5147E-03	1.0072E-02	9.1371E-02
DTLZ4	Ave	6.1316E-02	1.5061E-02	4.5515E-02	2.9569E-01
	SD	5.3260E-03	7.5177E-03	5.4484E-03	5.2568E-03

(i) **Generational distance (GD):** *GD* is an index for determining the sum of adjacent distances of candidate solutions in different sets achieved by different optimization algorithms as a credible indicator to measure the convergence behavior of many-objective metaheuristic optimization algorithms.

$$GD = \left(\frac{1}{n_{pf}} \sum_{i=1}^{n_{pf}} dis_i^2 \right)^{\frac{1}{2}} \quad (4)$$

(ii) **Spacing (S):** *S* is the overall distance between candidate solutions in different sets achieved by different

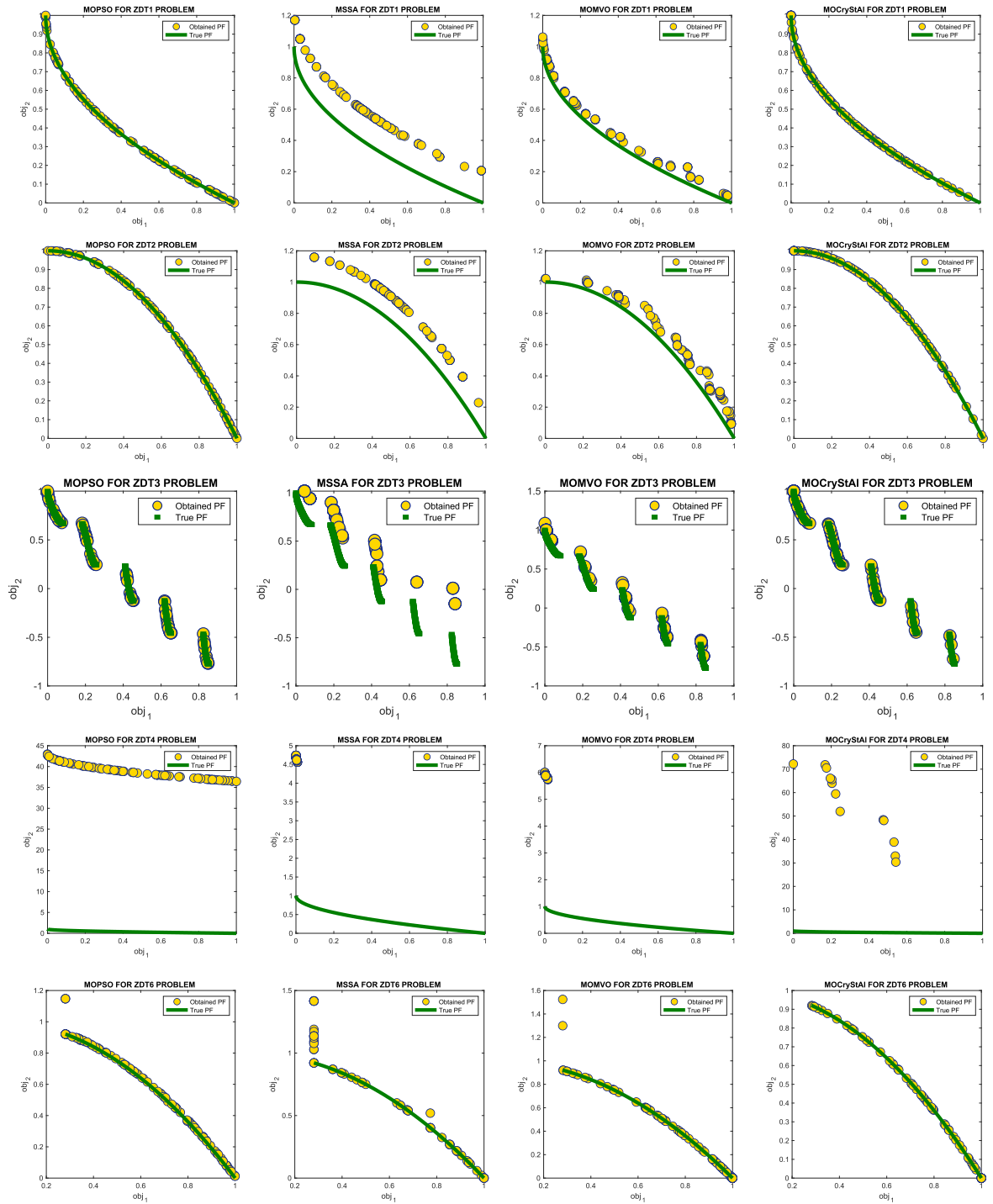


FIGURE 4. True and obtained Pareto fronts for ZDT problems.

optimization algorithms.

$$S = \left(\frac{1}{n_{pf}} \sum_{i=1}^{n_{pf}} (d_i - \bar{d})^2 \right)^{\frac{1}{2}} \quad \text{where } \bar{d} = \frac{1}{n_{pf}} \sum_{i=1}^{n_{pf}} d_i \quad (5)$$

(iii) **Maximum Spread (MS):** MS denotes the spread of candidate solutions in the different solution sets concerning

both numbers of distinct optimum choices.

$$MS = \left[\frac{1}{m} \sum_{i=1}^m \left[\frac{\min(f_i^{max}, F_i^{max}) - \max(f_i^{min}, F_i^{min})}{F_i^{max} - F_i^{min}} \right]^2 \right]^{\frac{1}{2}} \quad (6)$$

iv. **Inverted Generational Distance (IGD):** IGD is a metric for the quality evaluation of the approximations to the Pareto

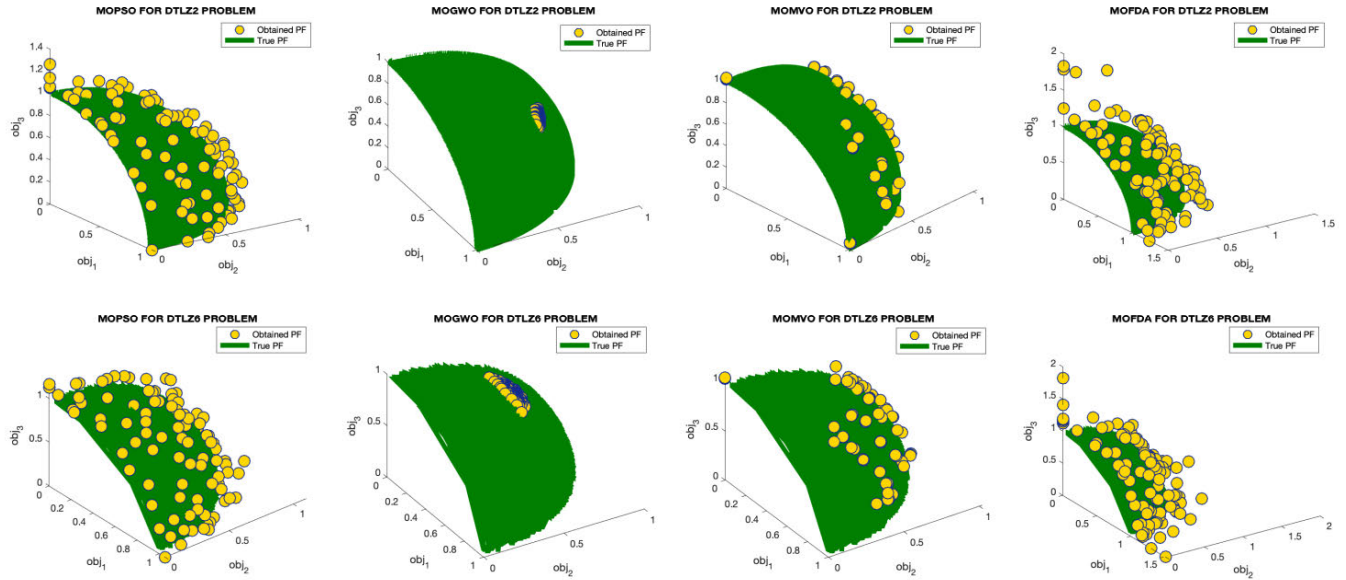


FIGURE 5. True and obtained Pareto fronts for DTLZ problems.

TABLE 13. Statistical results of the engineering problems for the GD performance metric.

Functions		Algorithm			
		MOPSO	MSSA	MOMVO	MOCryStAl
BNH	Ave	3.3894E-02	6.4575E-02	8.5875E-02	2.9356E-02
	SD	2.1392E-03	1.5035E-02	4.0751E-02	1.6599E-03
CONSTR	Ave	8.3098E-04	1.6878E-03	9.2588E-04	1.5219E-03
	SD	3.3369E-05	5.8950E-04	2.4850E-04	5.9590E-04
DISK BREAKER	Ave	2.3045E-03	6.2545E-03	1.5855E-01	1.5745E-03
	SD	5.6273E-03	3.3423E-03	2.4372E-01	1.6833E-03
4-BAR TRUSS	Ave	1.4095E+01	7.7966E+00	1.1017E+01	6.8970E+00
	SD	5.1580E-01	3.5486E+00	4.6552E+00	1.2099E+00
WELDED BEAM	Ave	1.1946E-02	6.3467E-03	1.5031E-02	6.6659E-02
	SD	1.9956E-03	3.0983E-03	4.7477E-03	1.1624E-01
OSY	Ave	3.5765E+00	9.4637E-01	8.4098E-01	6.7807E-01
	SD	2.5250E+00	1.9612E-01	1.4591E-01	3.4919E-01
SPEED REDUCER	Ave	8.2516E+01	7.8657E+00	9.2613E+00	6.7694E+00
	SD	9.8695E+01	3.6189E+00	1.6724E+00	1.6132E+00
SRN	Ave	3.1617E-02	4.1566E-02	3.3373E-01	1.1112E-02
	SD	1.0695E-02	2.4971E-02	1.4098E-01	4.7566E-03

front achieved by different many-objective algorithms.

$$IGD = \frac{\sqrt{\sum_{i=1}^n d_i^2}}{n} \quad (7)$$

B. EXPERIMENTAL SETUP

This section compares MOPSO, MSSA, and MOMVO to MOCryStAl, and the best figure of a collection of Pareto optimal is shown. Table 1 lists the initial settings for each algorithm. Each experiment had a maximum of 1000 iterations and 100 populations. As illustrated in Tables 2-4 and appendices A and B, this section puts the proposed algorithm to the test in 25 different case studies, including seventeen unconstrained and constrained bi- and tri-objective mathematical problems, as well as eight real-world engineering design problems.

1) DISCUSSION OF THE CEC-09 TEST FUNCTION

In Table 5, the comparative and statistical results of different multi-objective approaches alongside the proposed algorithm, namely MOCryStAl, are presented. The mentioned comparison metrics such as the GD, S, MS, and IGD are utilized in dealing with the CEC-09 problems. It turned out that MOCryStAl can outrank the other approaches, with regard to the IGD index, in five of the problems while the other methods, such as MSSA, also produce very competitive results.

Concerning GD, which are calculated in Table 6, MOCryStAl can provide acceptable results in most cases, including the five complex CEC-09 problems. The average results of UF8 in MOCryStAl have total differences of 90%, 80%, and 28% from MOPSO, MSSA, and MOMVO, respectively, which demonstrates the capability of the proposed

TABLE 14. Statistical results of the engineering problems for the IGD performance metric.

Functions		Algorithm			
		MOPSO	MSSA	MOMVO	MOCryStAl
BNH	Ave	9.6868E-04	7.5631E-03	3.2191E-03	1.5496E-03
	SD	1.7147E-04	4.5344E-03	3.3695E-04	1.3504E-04
CONSTR	Ave	5.1838E-04	2.1706E-03	1.8516E-03	1.1648E-03
	SD	5.5618E-05	7.3799E-04	3.6526E-04	2.6604E-04
DISK	Ave	5.8831E-04	2.5113E-03	1.3515E-03	3.6085E-04
BREAKE	SD	5.5995E-05	1.1615E-03	2.1737E-04	2.1021E-05
4-BAR	Ave	2.0010E-02	2.1434E-02	2.1020E-02	2.0005E-02
TRUSS	SD	3.9632E-05	3.8292E-04	4.6879E-04	3.5863E-05
WELDED	Ave	5.9705E-04	4.8242E-03	2.1075E-03	1.3180E-03
BEAM	SD	4.6341E-05	3.5852E-03	4.2017E-04	3.4394E-04
OSY	Ave	1.4663E-02	7.7532E-03	5.8358E-03	4.2891E-03
	SD	8.6917E-03	1.2021E-03	7.8106E-04	2.4468E-03
SPEED	Ave	6.0305E-02	1.4933E-02	8.7559E-03	3.1325E-03
	SD	7.2130E-02	5.7981E-03	2.1149E-03	1.1989E-02
SRN	Ave	4.5146E-04	2.2308E-03	1.1274E-03	3.3603E-04
	SD	1.1656E-04	1.5266E-03	2.5526E-04	6.6580E-05

TABLE 15. Statistical results of the engineering problems for the MS performance metric.

Functions		Algorithm			
		MOPSO	MSSA	MOMVO	MOCryStAl
BNH	Ave	1.0000E+00	7.6222E-01	9.6908E-01	1.0000E+00
	SD	0.0000E+00	1.3378E-01	2.7128E-02	0.0000E+00
CONSTR	Ave	9.9384E-01	9.0536E-01	9.7543E-01	9.9474E-01
	SD	6.8603E-03	4.8380E-02	1.7403E-02	2.8125E-02
DISK	Ave	9.9928E-01	7.9510E-01	1.3230E+00	1.0001E+00
BREAKE	SD	1.1417E-03	1.3013E-01	4.6277E-01	1.0642E-03
4-BAR	Ave	1.4876E+00	1.2019E+00	1.4014E+00	1.5436E+00
TRUSS	SD	5.4502E-04	1.8532E-01	9.2764E-02	8.7530E-02
WELDED	Ave	1.0072E+00	7.9085E-01	1.0552E+00	1.0709E+00
BEAM	SD	6.1053E-02	1.4645E-01	9.0721E-02	1.2073E-01
OSY	Ave	3.2390E-01	6.2048E-01	7.1746E-01	8.1700E-01
	SD	3.4284E-01	8.0930E-02	3.8964E-02	2.7281E-01
SPEED	Ave	2.3456E-01	7.2004E-01	8.0454E-01	8.8453E-01
	SD	2.9012E-02	6.9034E-02	3.9588E-02	1.6080E-01
SRN	Ave	9.0900E-01	7.0583E-01	9.2751E-01	9.7667E-01
	SD	4.5463E-02	1.5109E-01	4.4769E-02	1.9187E-02

TABLE 16. Statistical results of the engineering problems for the S performance metric.

Functions		Algorithm			
		MOPSO	MSSA	MOMVO	MOCryStAl
BNH	Ave	1.0901E+00	1.2546E+00	1.0487E+00	1.0362E+00
	SD	1.3631E-01	3.9410E-01	7.7663E-01	1.3606E-01
CONSTR	Ave	5.8740E-02	5.5908E-02	5.0134E-02	1.1085E-01
	SD	7.8936E-03	1.4886E-02	2.7727E-02	2.6254E-02
DISK	Ave	1.1452E-01	1.2768E-01	2.7320E-01	1.3453E-01
BREAKE	SD	1.3022E-02	7.0776E-02	3.9542E-01	1.3002E-02
4-BAR	Ave	5.3605E+00	6.1160E+00	4.8245E+00	1.1012E+00
TRUSS	SD	2.6169E-01	1.6605E+00	3.3581E+00	4.6044E+00
WELDED	Ave	2.3432E-01	1.8912E-01	2.0967E-01	3.6825E-01
BEAM	SD	2.5702E-02	8.7588E-02	1.1261E-01	1.6472E-01
OSY	Ave	1.1278E+00	1.3502E+00	1.7991E+00	9.7125E-01
	SD	1.4675E+00	6.2967E-01	3.4639E-01	2.1108E+00
SPEED	Ave	3.4532E+01	1.3952E+01	2.2500E+01	8.9121E+00
	SD	4.4378E+00	9.8199E+00	4.7232E+00	7.6068E+00
SRN	Ave	2.2396E+00	2.2721E+00	2.7418E+00	2.0058E+00
	SD	4.7324E-01	9.3573E-01	1.0405E+00	3.6260E-01

multi-objective algorithm in dealing with these sorts of complex problems.

Based on the results in Tables 7 and 8 in which the MS and S are presented, the ability of the MOCryStAl in outranking the other approaches are demonstrated. In Figs. 2 and 3, the true and obtained Pareto fronts for CEC-09 problems are represented by means of the proposed MOCryStAl method in

which this algorithm can create better solutions with closer distance to the Pareto front.

2) DISCUSSION OF THE ZDT AND DTLZ TEST FUNCTIONS

This section presents the statistical results of different methods alongside the proposed MOCryStAl algorithm in dealing with the multimodal benchmark functions with

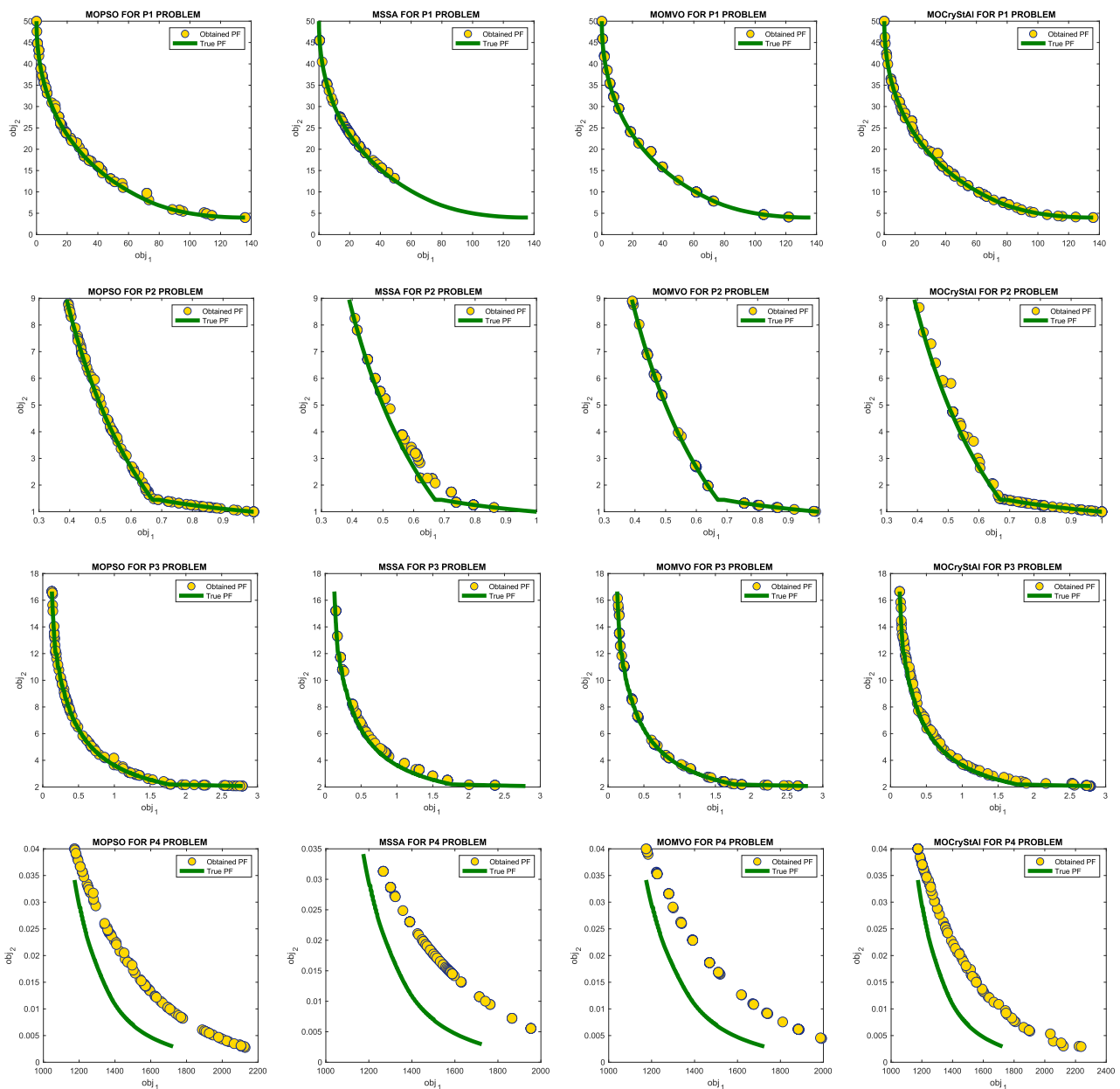


FIGURE 6. True and obtained Pareto fronts for engineering design problems (BNH, CONSTR, DISK BRAKE, and 4-BAR TRUSS).

fixed-dimension. In Table 9, the comparative results of the GD performance metric are demonstrated. The capability of the proposed methods in outranking the other multi-objective algorithms in four of these problems is demonstrated.

Regarding the IGD metric in dealing with the multimodal benchmark functions with fixed-dimension, the MOPSO, MSSA, and MOMVO are capable of providing best results for only one or two of the considered test problems. At the same time, the proposed MOCryStAl is capable of outranking the other methods in four of these complex test problems which demonstrate its capability in dealing with these sorts of complex problems.

In other metrics such as the MS and S indices, MOCryStAl is able to provide even more acceptable results than the IGD

index, while this algorithm is capable of outranking the other methods in most of the considered test problems.

The true and obtained Pareto fronts for the ZDT and DTLZ problems by means of the proposed MOCryStAl method are illustrated in Fig. 4 and Fig. 5, respectively. It can be seen that this algorithm can create better solutions with a closer distance to the Pareto front.

3) DISCUSSION OF THE ENGINEERING PROBLEMS

Based on the fact that the novel multi-objective algorithms should be evaluated by means of some difficult real-world optimization problems, the capability of the proposed MOCryStAl is assessed through some other optimization problems. In these cases, the GD (Table 13), IGD (Table 14),

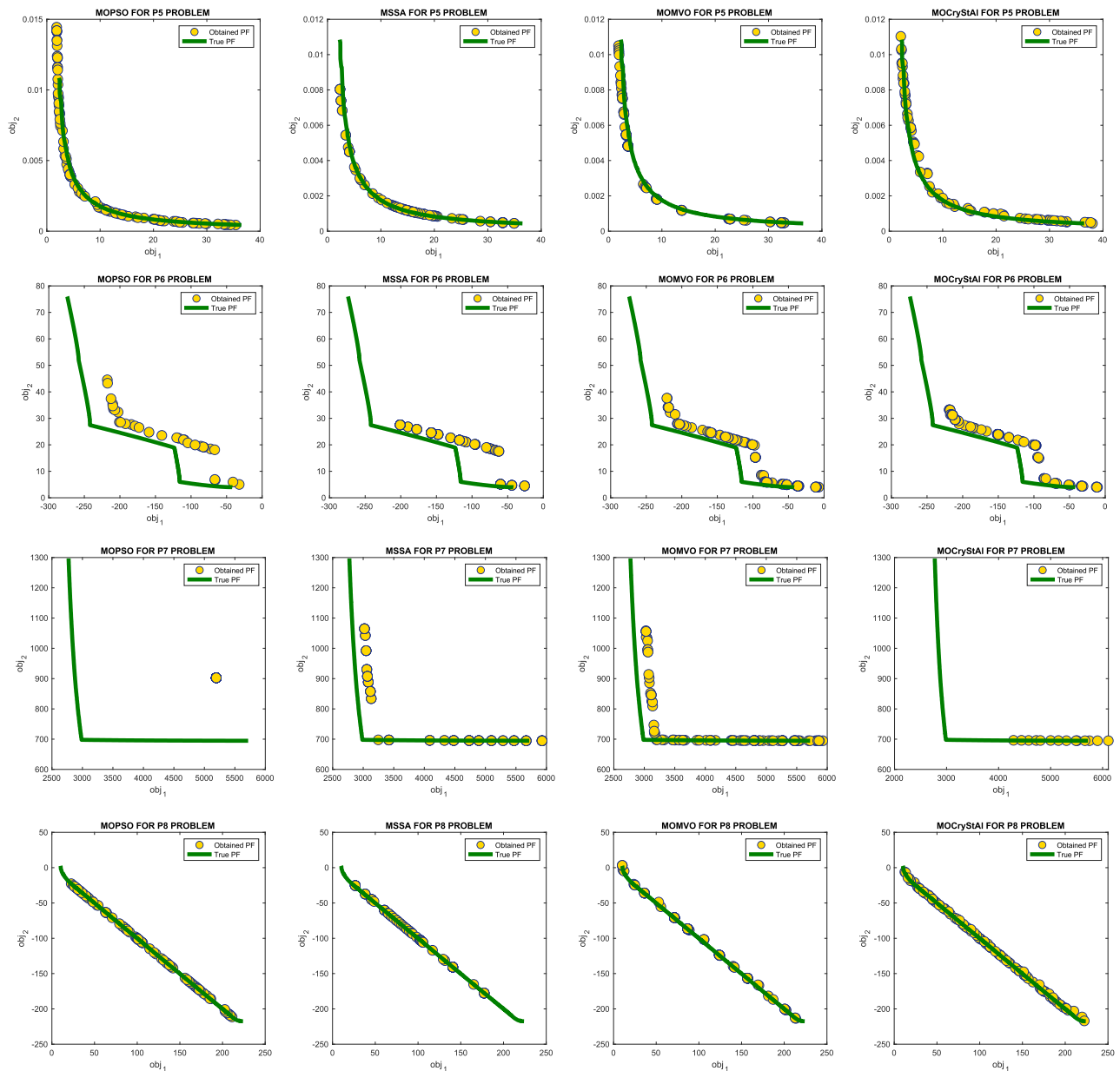


FIGURE 7. True and obtained Pareto fronts for engineering design problems (WELDED BEAM, OSY, SPEED REDUCER, and SRN).

MS (Table 15), and S (Table 16) metrics are also utilized for performance evaluation while the results demonstrate the superiority of the proposed algorithm in most of the cases. In Fig. 6 and Fig. 7, the true and obtained Pareto fronts for these problems are represented through the proposed MOCryStAl method. This algorithm can create better solutions with a closer distance to the Pareto front.

III. CONCLUSION AND FUTURE WORK

This paper presented the multi-objective version of the Crystal Structure Algorithm (CryStAl) as a recently proposed metaheuristic algorithm inspired by some geometric principles of crystal structures including the lattice and basis in the

configuration of crystals. For the performance evaluation of this algorithm, the benchmark problems of the Completions on Evolutionary Computation (CEC) on multi-objective optimization called CEC-09 were utilized. Some real-world engineering design problems were used to evaluate the efficiency of the proposed MOCryStAl approach. This paper demonstrates that MOCryStAl is capable of outranking the other approaches considering the IGD index in five of the CEC-09 problems while the other methods such as MSSA also produce very competitive results. Concerning GD, the average results of UF8 in MOCryStAl had total differences of 90%, 80%, and 28% from the results of MOPSO, MSSA, and MOMVO, respectively, which demonstrate the capability of

the proposed multi-objective algorithm in dealing with such challenging problems. By considering the true and obtained Pareto fronts for the CEC-09, ZDT, and DTLZ problems, it is concluded that the proposed MOCryStAl method can create better solutions with a closer distance from the Pareto front.

APPENDIX A: ONSTRAINED MULTI-OBJECTIVE TEST PROBLEMS (USED IN THIS PAPER)

CONSTR

There are two constraints and two design variables in this problem, which have a convex Pareto front.

$$\text{Minimize } f_1(x) = x_1 \quad (\text{A.1})$$

$$\text{Minimize } f_2(x) = (1 + x_2)/x_1 \quad (\text{A.2})$$

$$\text{where } g_1(x) = 6 - (x_2 + 9x_1) \quad (\text{A.3})$$

$$g_2(x) = 1 + x_2 - 9x_1 \quad (\text{A.4})$$

$$0.1 \leq x_1 \leq 1, 0 \leq x_2 \leq 5$$

SRN

Srinivasan and Deb [55] suggested a continuous Pareto optimal front for the following problem:

$$\text{Minimize } f_1(x) = 2 + (x_1/2)^2 + (x_2/1)^2 \quad (\text{A.5})$$

$$\text{Minimize } f_2(x) = 9x_1(x_2/1)^2 \quad (\text{A.6})$$

$$\text{where } g_1(x) = x_1^2 + x_2^2 - 255 \quad (\text{A.7})$$

$$g_2(x) = x_1 - 3x_2 + 10 \quad (\text{A.8})$$

$$-20 \leq x_1 \leq 20, -20 \leq x_2 \leq 20$$

BNH

Binh and Korn [56] were the first to propose this problem as follows:

$$\text{Minimize } f_1(x) = 4x_1^2 + 4x_2^2 \quad (\text{A.9})$$

$$\text{Minimize } f_2(x) = (x_1 - 5)^2 + (x_2 - 5)^2 \quad (\text{A.10})$$

$$\text{where } g_1(x) = (x_1 - 5)^2 + x_2^2 - 25 \quad (\text{A.11})$$

$$g_2(x) = 7.7 - (x_1 - 8)^2 - (x_2 + 3)^2 \quad (\text{A.12})$$

$$0 \leq x_1 \leq 5, 0 \leq x_2 \leq 3$$

OSY

Osyczka and Kundu [57] proposed five distinct regions for the OSY test issue. There are also six constraints and six design variables to consider as follows:

$$\text{Minimize } f_1(x) = x_1^2 + x_2^2 + x_3^2 + x_4^2 + x_5^2 + x_6^2 \quad (\text{A.13})$$

$$\text{Minimize } f_2(x) = [25(x_1 - 2)^2 + (x_2 - 1)^2 + (x_3 - 1)^2 + (x_4 - 4)^2 + (x_5 - 1)^2] \quad (\text{A.14})$$

$$\text{where } g_1(x) = 2 - x_1 - x_2 \quad (\text{A.15})$$

$$g_2(x) = -6 + x_1 + x_2 \quad (\text{A.16})$$

$$g_3(x) = -2 - x_1 + x_2 \quad (\text{A.17})$$

$$g_4(x) = -2 + x_1 - 3x_2 \quad (\text{A.18})$$

$$g_5(x) = -4 + x_4 + (x_3 - 3)^2 \quad (\text{A.19})$$

$$g_6(x) = 4 - x_6 - (x_5 - 3)^2 \quad (\text{A.20})$$

$$0 \leq x_1 \leq 10, 0 \leq x_2 \leq 10, 1 \leq x_3 \leq 5$$

$$0 \leq x_4 \leq 6, 1 \leq x_5 \leq 5, 0 \leq x_6 \leq 10 \quad (\text{A.21})$$

APPENDIX B: CONSTRAINED MULTI-OBJECTIVE ENGINEERING PROBLEMS (USED IN THIS PAPER)

THE FOUR-BAR TRUSS DESIGN PROBLEM

The 4-bar truss design problem [58], in which the structural volume (f_1) and displacement (f_2) of a 4-bar truss should be minimized, is a well-known problem in the structural optimization field. There are four design variables ($x_1 - x_4$) connected to the cross-sectional area of members 1, 2, 3, and 4, as shown in the equations below:

$$\text{Minimize } f_1(x) = 200 \times (2 \times x(1) + \text{sqrt}(2 \times x(2)) + \text{sqrt}(x(3)) + x(4)) \quad (\text{B.1})$$

$$\text{Minimize } f_2(x) = 0.01 \times \left(\frac{2}{x(1)} \right) + \left(\frac{2 \times \text{sqrt}(2)}{x(2)} \right) - ((2 \times \text{sqrt}(2))/x(3)) + (2/x(4))$$

$$1 \leq x_1 \leq 3, 1.4142 \leq x_2 \leq 3$$

$$1.4142 \leq x_3 \leq 3, 1 \leq x_4 \leq 3 \quad (\text{B.2})$$

THE WELDED BEAM DESIGN PROBLEM

Ray and Liew [59] suggested four constraints for the welded beam design issue. In this issue, the fabrication cost (f_1) and beam deflection (f_2) of a welded beam should be minimized. The thickness of the weld (x_1), the length of the clamped bar (x_2), the height of the bar (x_3), and the thickness of the bar (x_4) are the four design variables.

$$\text{Minimize } f_1(x) = 1.10471 \times x(1)^2 \times x(2) + 0.04811 \times x(3) \times x(4) \times (14 + x(2)) \quad (\text{B.3})$$

$$\text{Minimize } f_2(x) = 65856000/(30 \times 10^6 \times x(4) \times x(3)^3) \quad (\text{B.4})$$

$$\text{where } g_1(x) = \tau - 13600 \quad (\text{B.5})$$

$$g_2(x) = \sigma - 30000 \quad (\text{B.6})$$

$$g_3(x) = x(1) - x(4) \quad (\text{B.7})$$

$$g_4(x) = 6000 - P$$

$$0.125 \leq x_1 \leq 5, 0.1 \leq x_2 \leq 10$$

$$0.1 \leq x_3 \leq 10, 0.125 \leq x_4 \leq 5 \quad (\text{B.8})$$

$$\text{where } q = 6000 \times \left(14 + \frac{x(2)}{2} \right); D = \text{sqrt} \left(\frac{x(2)^2}{4} + \frac{x(1) + x(3)}{4} \right) \quad (\text{B.9})$$

$$J = 2 \times \left(x(1) \times x(2) \times \text{sqrt}(2) \times \left(\frac{x(2)^2}{12} + \frac{(x(1) + x(3))^2}{4} \right) \right) \quad (\text{B.10})$$

$$\alpha = \frac{6000}{\text{sqrt}(2) \times x(1) \times x(2)} \quad (\text{B.11})$$

$$\beta = Q \times \frac{D}{J} \quad (\text{B.12})$$

DISK BRAKE DESIGN PROBLEM

Ray and Liew [59] proposed the disc brake design issue, which has multiple constraints. Stopping time (f_1) and brake mass (f_2) for a disc brake are the two objectives to be minimized. The inner radius of the disc (x_1), the outer radius of the disc (x_2), the engaging force (x_3), and the number of friction surfaces (x_4) as well as five constraints, are shown in the following equations:

$$\text{Minimize } f_1(x) = 4.9 \times (10)^{(-5)} \times (x(2))^{(2)} - x(1)^{(2)} \times (x(4) - 1) \quad (\text{B.13})$$

$$\text{Minimize } f_2(x) = (9.82 \times (10)^{(6)}) \times (x(2))^{(2)} - x(1)^{(2)} \times (x(2))^{(3)} - x(1)^{(3)} \times x(4) \times x(3) \quad (\text{B.14})$$

$$g_1(x) = 20 + x(1) - x(2) \quad (\text{B.15})$$

$$g_2(x) = 2.5 + (x(4) + 1) - 30 \quad (\text{B.16})$$

$$g_3(x) = (x(3)) / (3.14 \times (x(2)^2 - x(1)^2)) - 0.4 \quad (\text{B.17})$$

$$g_4(x) = (2.22 \times (10)^{(-3)} \times x(3) \times (x(2)^3 - x(1)^3)) / ((x(2)^2 - x(1)^2)^2) - 1 \quad (\text{B.18})$$

$$g_5(x) = 900 - (2.66 \times (10)^{(-2)} \times x(3) \times x(4) \times (x(2)^3 - x(1)^3)) / ((x(2)^2 - x(1)^2)^2) \\ 55 \leq x_1 \leq 80, 75 \leq x_2 \leq 110 \\ 1000 \leq x_3 \leq 3000, 2 \leq x_4 \leq 20 \quad (\text{B.19})$$

SPEED REDUCER DESIGN PROBLEM

The weight (f_1) and stress (f_2) of a speed reducer should be minimized in the speed reducer design issue, which is well-known in the field of mechanical engineering [58] and [60]. There are seven design variables: gear face width (x_1), teeth module (x_2), number of teeth of a pinion (x_3 integer variable), distance between bearings 1 (x_4), distance between bearings 2 (x_5), the diameter of shaft 1 (x_6), and the diameter of shaft 2 (x_7), as well as eleven constraints.

$$\text{Minimize } f_1(x) = 0.7854 \times x(1) \times x(2)^2 \times (3.3333 \times x(3)^2 + 14.9334 \times x(3)) \dots - 43.0934 - 1.508 \times x(1) \times (x(6)^2 + x(7)^2) \quad (\text{B.20})$$

$$\text{Minimize } f_2(x) = ((\text{sqrt}(((745 \times x(4))/x(2) \times x(3))))^2 + 19.9e6)/(0.1 \times x(6)^3)) \quad (\text{B.21})$$

$$\text{where } g_1(x) = 27/(x(1) \times x(2)^2 \times x(3)) - 1 \quad (\text{B.22})$$

$$g_2(x) = 397.5/(x(1) \times x(2)^2 \times x(3)^2) - 1 \quad (\text{B.23})$$

$$g_3(x) = (1.93 \times x(4)^3) / (x(2) \times x(3) \times x(6)^4) - 1 \quad (\text{B.24})$$

$$g_4(x) = (1.93 \times x(5)^3) / (x(2) \times x(3) \times x(7)^4) - 1 \quad (\text{B.25})$$

$$g_5(x) = ((\text{sqrt}((745 \times x(4))/x(2) \times x(3)))^2 + 16.9e6)/(110 \times x(6)^3)) - 1 \quad (\text{B.26})$$

$$g_6(x) = ((\text{sqrt}((745 \times x(4))/x(2) \times x(3)))^2 + 157.5e6)/(85 \times x(7)^3)) - 1 \quad (\text{B.27})$$

$$g_7(x) = ((x(2) \times x(3))/40)1 \quad (\text{B.28})$$

$$\tau = \text{sqrt} \left(\alpha^2 + 2 \times \alpha \times \beta \times \frac{x(2)}{2 \times D} + \beta^2 \right) \quad (\text{B.29})$$

$$\sigma = \frac{504000}{x(4) \times x(3)^2} \quad (\text{B.30})$$

$$\text{tmpf} = 4.013 \times \frac{30 \times 10^6}{196} \quad (\text{B.31})$$

$$P = \text{tmpf} \times \text{sqrt} \left(x(3)^2 \times \frac{x(4)^6}{36} \right) \times \left(1 - x(3) \times \frac{\text{sqrt} \left(\frac{30}{48} \right)}{28} \right) \quad (\text{B.32})$$

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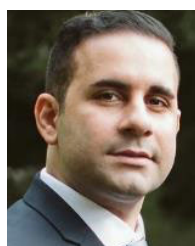
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