Mining for Unknown Unknowns

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Unknown unknowns are future relevant contingencies that lack an ex ante description. While there are numerous retrospective accounts showing that significant gains or losses might have been achieved or avoided had such contingencies been previously uncovered, getting hold of unknown unknowns still remains elusive, both in practice and conceptually. Using Formal Concept Analysis (FCA) - a subfield of lattice theory which is increasingly applied for mining and organizing data - this paper introduces a simple framework to systematically think out of the box and direct the search for unknown unknowns.

There are only two kinds of campaign plans, good ones and bad ones.
The good ones almost always fail through unforeseen circumstances
that often make the bad ones succeed.

- Napoleon Bonaparte -

1 Introduction

As the recent Covid-19 pandemic reminded us, life is filled with unknown unknowns – i.e. contingencies one cannot be aware of ex ante, much less fit into standard risk analysis. In addition to a wealth of examples coming from history and politics, unknown unknowns are now well-documented, and their importance is acknowledged, in many areas of economics and management such as public policy [23], business strategy [5, 11], entrepreneurship [12], contracts and the theory of the firm [40], and security [32].

To be sure, getting hold of such contingencies might allow to achieve significant payoffs or avoid major losses. Substantial research efforts have thus been expended, and notable advances been made, in this direction. To get a rigorous conceptual grasp at the notion of unknown unknowns, one may now draw, notably, from the literatures on Knightian uncertainty (e.g., [4]), undescribable events (e.g., [24]), unforeseen contingencies (e.g., [7, 20]), unawareness (e.g., [35, 34]), and surprises [39, 26]. Yet, for someone who would primarily want to uncover ahead of time the concrete unknown unknowns she might be facing, the task would remain elusive.

This paper will now seek to meet this demand. Similar endeavors have already been tried, and results obtained, in areas where unknown unknowns occur frequently: like C-K Theory [17], TRIZ [19], and

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creativity support systems [13, 44] in innovation management; knowledge spaces [10] in learning; and elicitation methods [21, 31, 38] in engineering. As it turns out, Formal Concept Analysis (FCA), which I will be using here, provides an appropriate language, and especially structure, to build a framework which is both rigorous (it is grounded in lattice theory) and operational (its implementation requires only spreadsheets).

The suggested scheme is sketched in the following Section 2. It is next developed rigorously and with more generality in Section 3. A fourth section contains concluding remarks.

2 An informal account

Consider a 3x3 matrix with horizontal coordinates A, B, C, and vertical coordinates α β, γ. For concreteness, the former might refer to different objects, items or events and the latter to various attributes, characteristics, properties or features. Table 1 shows the features respectively held by each specific item: object A, for instance, possesses attributes α, β.

Objects \ Attributes	α	β	γ
A	×	×	
B			×
C		×	×

Table 1: The existing context

In Formal Concept Analysis (FCA), such a matrix showing relationships between 'objects' (items, events, etc.) and 'attributes' (properties, features, etc.) is called a context.

In practice, FCA users would of course face much more complicated types of contexts, with tables comprising dozens of rows and columns, mitigated relationships between objects and attributes, and (what is crucial for decision-making) value-weighted properties. But this simple example will suffice to convey our main points.

The upshot of a discovery, experiment or invention would be the expanded matrix displayed in Table 2. Two new items – D and E – and an extra characteristic δ were found. The initial objects A and B now bear attributes δ and α respectively, while D possesses properties α and δ, E exhibits features β and γ. This matrix forms a context as well.

Objects \ Attributes	α	β	γ	δ
A	×	×		×
B	×		×	
C		×	×	
D	×			×
E		×	×	

Table 2: The new context

Back in time, the incremental component of Table 2 – i.e. the rows D, E, column δ and the small x's – might have been impossible to describe, much less to anticipate even as random outcomes. They were, so to speak, *unknown unknowns*. Altogether, they make the context displayed in Table 3.

Objects \ Attributes	α	β	γ	δ
A				$\times$
B	$\times$			
C				
D	$\times$			$\times$
E		$\times$	$\times$	

Table 3: The discovery context

Let us now see how someone could have a grasp at Table 3 using known knowns **only**, these known knowns being the data available from Table 1.

In FCA, the primary mode of organizing the data of a context is through the use of 'concepts'. A concept is defined as a list of objects and attributes such that the mentioned objects are precisely the ones that share the listed attributes, and the mentioned attributes are precisely the ones shared by all the listed objects. Examples of concepts in Table 3 are the objects B, D with their common attribute α, event E with features β, γ, items A, D with the shared property δ, and object D with attributes α, δ.

FCA calls an incompletely specified concept, i.e. a list that misses some object and attributes, a preconcept. The list (B;α) is a preconcept of the concept (B,D;α), for instance. Since its object B and attribute α could already be seen in the existing context of Table 1, I shall refer to such specific preconcept as a *seed*.

Now, the relationship between B and α is captured in Table 4, which is actually the negative picture of Table 1. This table constitutes a context as well, and *it is made only of data from Table 1*. Its concepts – which might be called 'anti-concepts', since they are the counterpart of the existing initial concepts – include (B,C;α), (A;γ) and (B;α,β).

Objects \ Attributes	α	β	γ
A			Z
B	Z	Z	
C	Z		

Table 4: The negative existing context

This paper's main result is that *a seed* - like (B;α) - *will always be the pre-concept of some anti-concept* – namely, here, (B,C;α) or (B;α,β). This fact has at least three ramifications.

First, although one cannot say anything about which objects or which attributes will be discovered, the structure of the existing context bears some implications for the structure among discovered objects and attributes.[1] This opens the door for establishing a systematic procedure to get some grasp at, and eventually uncover, unknown unknowns:

[1] I am grateful to an anonymous referee for this observation.

(i) Build the negative picture of the existing context;
(ii) Examine the preconcepts of each anticoncept;
(iii) If a seed is found, dig into it to uncover some concepts in the unknown unknown.

Second, the latter procedure might be seen as an instance of abduction, a mode of reasoning associated with creativity and the generation of ideas [22, 41]. Unlike deduction, which draws the logical ramifications of previously given assertions, or induction, which infers general laws from the observation of recurrent facts, abduction looks for the best justification (which is here a concept of the discovery context) after hitting a singular event (a seed).

Third, as the upcoming section will show, it allows for the deployment of a potentially powerful tool for data exploration and exploitation - namely, Galois connections.

3 Formal Developments

Let's now present the mathematics which underlie the scheme outlined above. Subsection 3.1 revisits the basics of Formal Concept Analysis (FCA). Subsection 3.2 next introduces the notion of 'revelation mappings'. Subsection 3.3, finally, develops the systematic procedure for thinking out of the box. The treatment is meant to be self-contained. Only set-theoretic arguments are used throughout.

3.1 Basic FCA notions

A *formal context* is referred to as a triplet $K = (G,M;R)$, where G is a set of *objects*, M a set of *attributes* these objects may have, and R is a *relation* between G and M, i.e. a subset of the Cartesian product $G \times M$ with the interpretation that $(g,m) \in R$, or gRm, if object g has attribute m.

Denote $\wp(G)$ and $\wp(M)$ the respective power sets (or sets of all subsets) of G and M. Set inclusion $\subseteq$ provides a partial order on the elements of these sets.[2] The following set-to-set functions I_R and E_R defined as

$$\begin{aligned} \text{for S} \quad &\subseteq \quad G,\ I_R(S) = \{m \in M : gRm \ \text{ for all } \ \mathrm{g} \in S\} \\ \text{for T} \quad &\subseteq \quad M,\ E_R(T) = \{g \in G : gRm \ \text{ for all } \ \mathrm{m} \in T\} \end{aligned}$$

are called the *Birkhoff Operators* for G and M respectively. For a set of objects S, $I_R(S)$ - the *intent* of S - gives all the attributes in T which these objects have in common. For a given set of attributes T, $E_R(T)$ - the *extent* of T - gives all the objects in S that share these attributes. In the context displayed in Table 1, $I_R(\mathrm{A,C}) = \{\beta\}$ and $E_R(\alpha,\beta) = \{\mathrm{A}\}$.

A well-known property of the Birkhoff Operators is that of *duality*: knowing $I_R(\cdot)$ completely determines $E_R(\cdot)$, and vice-versa, specifying $E_R(\cdot)$ also defines $I_R(\cdot)$.

A *formal concept* in the context $K = (G,M;R)$ is now a pair of sets $(Q;V)$, with $Q \subseteq G$ and $V \subseteq M$, such that $I_R(Q) = V$ and $E_R(V) = Q$. The extent of a concept $(Q;V)$ is thus Q, while its intent is V.[3]

[2] A set Q is a *partially ordered set* (or *poset*) if there is a relation $\leq$ on Q (called a *partial order*) such that: (i) for $q \in Q$, $q \leq q$ (reflexivity property); (ii) for $q_1, q_2 \in Q$, $q_1 \leq q_2$ and $q_2 \leq q_1$ implies $q_1 = q_2$ (antisymmetry); for $q_1, q_2, q_3 \in Q$, $q_1 \leq q_2$ and $q_2 \leq q_3$ implies $q_1 \leq q_3$ (transitivity).

[3] The way FCA defines a formal concept agrees with the International Standard Organization's ISO 704 definition: "In a concept, one distinguishes its 'intension' and 'extension'. The intension of a concept comprises all attributes thought with it, the extension comprises all objects for which the concept can be predicated. In general, the richer the intension of a concept is, the lesser is its extension, and vice versa."

A *preconcept* in K, finally, is a pair $(P;U)$, with $P \subseteq G$ and $U \subseteq M$, such that $P \subseteq E_R(U)$ or, equivalently, $U \subseteq I_R(P)$. Preconcepts can be ordered as follows [8]: $(P;U) \sqsubseteq (P';U')$, meaning that $(P;U)$ is *less extensive than* $(P';U')$, if $P \subseteq P'$ and $U \subseteq U'$.

3.2 Revelation mappings

From now on, $K = (G,M;R)$ will denote the existing context and $K^+ = (G^+,M^+;R^+)$ the new context after the previous unknown unknowns have been revealed.[4]

Let's then call *revelation mappings* the functions $\Phi : \wp(G^+) \to \wp(M^+)$, $\Psi : \wp(M^+) \to \wp(G^+)$ such that[5]

$$\begin{aligned} \text{for } \mathrm{S}^+ &\subseteq G^+,\ \Phi(S^+) = I_{R^+}(S^+) \setminus \bigcup_{g \in S^+} I_R(g) \\ \text{for } \mathrm{T}^+ &\subseteq M^+,\ \Psi(T^+) = E_{R^+}(T^+) \setminus \bigcup_{m \in T^+} E_R(m) \end{aligned}$$

If one takes a set $S \subseteq G$ of objects from the existing context K, $\Phi(S)$ delivers the set of attributes (old or new) in M^+ which are newly associated with these objects. In Table 2, for instance, $\Phi(\mathrm{B,E}) = \varnothing$ and $\Phi(\mathrm{A}) = \{\delta\}$. Similarly, for a subset of initial attributes $T \subseteq M$, $\Psi(T)$ gives all (and only) the initial or new objects that now possess these attributes. For example, $\Psi(\gamma,\delta) = \varnothing$ but $\Psi(\beta,\gamma) = \{\mathrm{E}\}$.

As for the Birkhoff operators, there is a *duality property* between $\Phi(\cdot)$ and $\Psi(\cdot)$: each one uniquely characterizes the other. These mappings also hold additional features which are spelled out in the upcoming propositions.

First, say that a function $\pi : X \to Y$ between two sets X and Y, partially ordered by $\leq$ and $\preceq$ respectively, is *antitone* (or order-reversing) if, for $p_1, p_2 \in X$, $p_1 \leq p_2$ implies $\pi(p_2) \preceq \pi(p_1)$. A first statement is now at hand.

PROPOSITION 1: The revelation mappings Φ and Ψ are antitone.

PROOF:

First, consider Φ. Take two sets S_1^+, $S_2^+ \in \wp(G^+)$ such that $S_1^+ \subseteq S_2^+$; we must show that $\Phi(S_2^+) \subseteq \Phi(S_1^+)$. If $m \in \Phi(S_2^+)$, then $m \in I_{R^+}(S_2^+)$ so gR^+m for all $g \in S_2^+$. Since $S_1^+ \subseteq S_2^+$, we have that gR^+m for all $g \in S_1^+$, hence $m \in I_{R^+}(S_1^+)$. Now, if $m \notin M$, $m \notin I_R(g)$ for any $g \in S_1^+$; it follows that $m \in I_{R^+}(S_1^+) \setminus \bigcup_{g \in S_1^+} I_R(g) = \Phi(S_1^+)$. Suppose, alternatively, that $m \in M$. Since $m \in \Phi(S_2^+)$, it must be the case that $not(gRm)$ for all $g \in S_2^+$, hence $not(gRm)$ as well for all $g \in S_1^+$ since $S_1^+ \subseteq S_2^+$; it follows again that $m \in \Phi(S_1^+)$. This shows that $\Phi(S_2^+) \subseteq \Phi(S_1^+)$. The same line of reasoning works for Ψ (as can be expected from duality). ■

This property of revelation mappings means that, the more objects or attributes one starts with, the more demanding it is to find new relationships that fit them all. This intuitive result is also instrumental in deriving other important characteristics of revelation mappings.

A key notion to introduce at this point is that of a *Galois connection*.[6] Let X and Y be two sets

[4]Power sets, Birkhoff Operators, formal concepts, and preconcepts are similarly defined on their context of reference, be it K, K^+, or any other context.

[5]Let's agree that $I_R(g) = \varnothing$ when $g \notin G$, and $E_R(m) = \varnothing$ when $m \notin M$.

[6]Since at least Ore (1944)'s seminal article [25], Galois connections have been increasingly employed throughout mathematics and computer science. To go beyond the very short primer offered in this paper, the reader may look at [6, 10, 15], and some of their common references.

partially ordered by $\leq$ and $\preceq$ respectively. A (antitone) *Galois connection* (π, θ) on X and Y is a pair of functions $\pi : X \to Y$ and $\theta : Y \to X$ such that the following equivalent properties are satisfied.
(i) For each $p \in X$, $p \leq \theta\pi(p)$ and for each $q \in Y$, $q \preceq \pi\theta(q)$.
(ii) For $p \in X$ and $q \in Y$, $p \leq \theta(q)$ if and only if $q \preceq \pi(p)$.

It is well-known that the Birkhoff operators (I_R, E_R), (I_{R^+}, E_{R^+}) are antitone Galois connections on, respectively, the power sets $\wp(G)$, $\wp(M)$ and $\wp(G^+)$, $\wp(M^+)$ ordered by set inclusion (see, e.g., [15], p. 13-14). In this case, property (i) means that the attributes common to a given set of objects might be shared by more objects, while the objects that share a given set of attributes might have more attributes in common. Property (ii), on the other hand, says that some objects are among those sharing a given set of attributes if and only if these attributes are among those common to these objects.

As it turns out, the pair of revelation mappings (Φ, Ψ) forms a Galois connection.

PROPOSITION 2: The pair of revelation mappings (Φ, Ψ) is a Galois connection on the power sets $\wp(G^+)$ and $\wp(M^+)$ partially ordered by inclusion.

PROOF:
To see this, take two sets $S^+ \in \wp(G^+)$ and $T^+ \in \wp(M^+)$, and notice that

$$\begin{array}{rcl} S^+ & \subseteq & \Psi(T^+) \\ \text{if and only if } \forall g & \in & S^+, \forall m \in T^+: \ gR^+m \ \text{ and } \ not(gRm) \\ \text{if and only if } \forall m & \in & T^+, \forall g \in S^+: \ gR^+m \ \text{ and } \ not(gRm) \\ \text{if and only if } T^+ & \subseteq & \Phi(S^+) \end{array}$$

■

Proposition 2 underlies a central result. Like any Galois connection ([15], p. 14), (Φ, Ψ) establishes a relation, noted $R^+_{(\Phi,\Psi)}$, between the set of objects G^+ and the set of attributes M^+. This relation is defined as

$$\begin{aligned} R^+_{(\Phi,\Psi)} &= \left\{(g,m) \in G^+ \times M^+ \mid g \in \Psi(m)\right\} \\ &= \left\{(g,m) \in G^+ \times M^+ \mid m \in \Phi(g)\right\} \end{aligned}$$

We can show that $R^+_{(\Phi,\Psi)}$ coincides with $R^+ \setminus R$, the set of all new relationships.

PROPOSITION 3: $R^+_{(\Phi,\Psi)} = R^+ \setminus R$.

PROOF: Observe that $(g,m) \in R^+_{(\Phi,\Psi)}$ if and only if gR^+m and $not(gRm)$, if and only if $(g,m) \in R^+ \setminus R$.

■

3.3 Thinking out of the box

From now on, let $R^+_{(\Phi,\Psi)} = R^+ \setminus R$ be referred to as R^*. The latter relation defines another formal context, the *discovery* context noted $K^* = (G^+, M^+; R^*)$, which is the context of the unknown unknowns. Can K^* be inferred from K, at least partly? We will now see that the answer actually errs on the yes side.

The ordered pair $(X;Y)$ with $X \neq \varnothing$, $Y \neq \varnothing$ is called a *seed* in K for K^* if it is a preconcept in K^* while $X \subseteq G$ and $Y \subseteq M$. As the next statement confirms, the existence of a seed is guaranteed when the existing context harbors at least one new relationship between the original objects and attributes.

PROPOSITION 4: If $R^* \cap (G \times M) \neq \varnothing$, then there is at least one seed in K for K^*.

PROOF:

The assumption implies that there is at least one concept $(Q;V)$ in K^* such that $Q \cap G \neq \varnothing$ and $V \cap M \neq \varnothing$. Since $Q \cap G \subseteq Q = I_{R^*}(V) \subseteq I_{R^*}(V \cap M)$ and $V \cap M \subseteq V = E_{R^*}(Q) \subseteq E_{R^*}(Q \cap G)$, the pair $(Q \cap G; V \cap M)$ is a preconcept in K^*. ∎

As suggested in Section 2, looking for seeds might be a reasonable first step to uncover unknown unknowns. The major reason is that, as we will now demonstrate, *it is possible to characterize the location of seeds*.

First, according to the following proposition, a seed must combine objects and attributes which are a priori unrelated.

PROPOSITION 5: No preconcept (a fortiori concept) in the existing context K can be a seed for the discovery context K^*.

PROOF:

Let $(P;U)$ be a preconcept in K. By definition, $\Phi(P) = I_{R^+}(P) \setminus \bigcup_{g \in P} I_R(g)$. But $U \subseteq I_R(P) = \bigcap_{g \in P} I_R(g) \subseteq \bigcup_{g \in P} I_R(g)$. It follows that $U \nsubseteq \Phi(P)$, hence $(P;U)$ is not a preconcept in K^*.

This result tells us something about how not to look for novelties. A corollary is that a seed in K for K^* must be a pair $(P;U)$, with $P \subseteq G$ and $U \subseteq M$, such that $P \cap (\bigcup_{m \in U} E_R(m)) = \varnothing$ and $U \cap (\bigcup_{g \in P} I_R(g)) = \varnothing$. This suggests working with the negative of the existing context K, noted $\overline{K} = (G, M; \overline{R})$, where the relation $\overline{R} = G \times M \setminus R$ refers to the *reverse relation* $g\overline{R}m$ which holds when object g *does not* have attribute m. The next (key, and somewhat surprising) proposition shows that $\overline{K}$ - which can be obtained using **only** the initial data - is the appropriate 'outbox' in which mining for unknown unknowns might begin.

PROPOSITION 6: A seed is a preconcept of the negative existing context $\overline{K}$.

PROOF:

Let $(P;U)$ be a seed for K^*. Then $U \subseteq \Phi(P) \cap M = M \cap I_{R^+}(P) \setminus \bigcup_{g \in P} I_R(g) \subseteq M \setminus \bigcup_{g \in P} I_R(g)$
$= \bigcap_{g \in P} (I_R(g))^c = I_{\overline{R}}(P)$. ∎

Seeds for the discovery context K^* - which comprises a priori unknown relationships between objects in G and attributes in M - thus happen to point, not only at concepts in K^*, but also at the concepts of the negative existing context $\overline{K}$. This suggests the procedure already outlined in Section 2:

- Take the negative context $\overline{K}$ of K;
- Consider a concept in $\overline{K}$ (i.e. an anti-concept);
- Examine the latter's preconcepts;
- If one of these preconcepts brings out a new relationship between its objects and attributes, then a seed has been found which anticipates some concepts in the discovery context K^*.

Whether this scheme can be fruitful in practice remains to be seen. One hurdle could be computational complexity (see the concluding remarks).

Interestingly, however, Propositions 5 and 6 suggest that concepts in the negative existing context $\overline{K}$ can be constructed using the mappings $\widetilde{\Phi} : \wp(G) \to \wp(M)$ and $\widetilde{\Psi} : \wp(M) \to \wp(G)$ defined as

$$\text{for S} \subseteq G,\ \widetilde{\Phi}(S) = M \setminus \bigcup_{g \in S} I_R(g)$$

$$\text{for T} \subseteq M,\ \widetilde{\Psi}(T) = G \setminus \bigcup_{m \in T} E_R(m)$$

respectively. Comparing the latter expressions with the ones corresponding to the above revelation mappings, the functions $\widetilde{\Phi}$ and $\widetilde{\Psi}$ can be seen as approximations for Φ and Ψ. Whether closer approximations (in a sense to be made precise) can be found, which would then provide a better grasp at unknown unknowns, would be a valuable research topic.

4 Concluding remarks

This paper submitted a new framework and approach to handle unknown unknowns. The scheme has rigorous foundations in lattice theory. It looks widely applicable, furthermore, since it can incorporate various kinds of data – quantitative and qualitative, objective and subjective, financial and non-financial. And it seems to be user-friendly, boiling down to using only spreadsheets.

At this stage, in addition to the extensions suggested at the end of the previous section, other ones could be the following:

First, on a technical note, listing all the concepts of a formal context is generally burdensome.[7] Yet, the search for seeds requires this exercise. Research and development on how to identify concepts in a given context is very much ongoing. Several algorithms and softwares already exist: many (mentioned in [37], for instance) are subject to a patent but others – GALICIA and JALABA, for example – can be freely downloaded. Two promising trends are to take full advantage of negative information (i.e. the information contained in the negative existing context $\overline{K}$), as in [33] or [27], and to assign weights to attributes, as in [3].

Second, the above derivation made minimal assumptions about the use of a priori knowledge, ignoring issues of landscape and timing, and forbidding the use of probabilities. In practice, however, one might be able to tap on probabilistic beliefs based on science, predictive models or sound experience, in order to figure out the plausibility of new relationships between objects and attributes. This endeavor will enhance the search for seeds, hence the prospecting for unknown-unknowns.

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