

Article

MIMO Over-the-Air Computation for Distributed Estimation

Pangun Park ^{1,*} , Hyejeon Shin ² and Piergiuseppe Di Marco ³¹ Department of Radio and Information Communications Engineering, Chungnam National University, Daejeon 34134, Republic of Korea² Dental Clinic Center, Kyungpook National University, Daegu 41940, Republic of Korea³ Department of Information Engineering, Computer Science and Mathematics, University of L'Aquila, 67100 L'Aquila, Italy

* Correspondence: pgpark@cnu.ac.kr; Tel.: +82-42-821-6862

Abstract: MIMO over-the-air computation (MIMO-AirComp) is a recently proposed technique that leverages the superposition property of the multiple access channel to compute the target multifunction of various applications. This article presents how the MIMO-AirComp principle can be applied to the state estimation problem using distributed sensing data. The representative target function is explicitly formulated as a nomographic function matched to the structure of the multiple access channel with the proper processing function. The proposed framework efficiently computes the target multifunction by coordinating local preprocessing at each node, aggregation through the wireless channel, and postprocessing at the fusion center. We analyze and demonstrate that the proposed approach significantly improves the computation throughput for the distributed estimation application. Specifically, the proposed MIMO-AirComp framework outperforms the conventional separated communication and computation approach when the network system relies on noisy measurements obtained by the densely deployed sensors.

Keywords: over-the-air computation; MIMO; distributed estimation



Citation: Park, P.; Shin, H.; Di Marco, P. MIMO Over-the-Air Computation for Distributed Estimation. *Appl. Sci.* **2023**, *13*, 1593. <https://doi.org/10.3390/app13031593>

Academic Editor: Christos Bouras

Received: 2 January 2023

Revised: 19 January 2023

Accepted: 24 January 2023

Published: 26 January 2023



Copyright: © 2023 by the authors. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (<https://creativecommons.org/licenses/by/4.0/>).

1. Introduction

Many estimation applications rely on real-time data collection and processing from spatially deployed wireless devices, which are used for time-critical inference and decisions [1–3]. In particular, some emerging services, such as surveillance, networked control, and distributed artificial intelligence, are interested in computing a target function using distributed measurements rather than collecting all individual measurements from an enormous number of devices [4–6]. Furthermore, some latency-critical control systems require a very low delay to estimate the system state in the order of 1–10 ms with a large number of nodes 10^2 – 10^3 [7]. Thus, these services primarily need high computation throughput, defined as the rate of computing outputs from nodes to a Fusion Center (FC), rather than the communication throughput of conventional wireless networks [8]. The FC (the nodes, respectively) can be an access point, base station, parameter server, or central controller (the user equipment, edge device, and sensor, respectively), depending on the application domain [1,9].

Even with new techniques, a conventionally separated communication and computation principle encounters fundamental challenges due to the stringent throughput/delay requirements and the heavy computational load due to the limited spectrum bandwidth with resource-constrained devices [2,5,10]. For instance, the 5G standardization has recently introduced new wireless communication and computing techniques such as massive Multiple-Input Multiple-Output (MIMO), millimeter-wave, network slicing, and edge computing [4,11]. However, despite these breakthrough technologies, practical 5G networks only achieve an average data rate of around 150–200 Mb/s with a delay of 10–100 ms [12].

One of the interesting information-theoretical results shows that the superposition property of the multiple access channel enables efficient aggregation of the distributed

signals [5,13]. For instance, the simultaneous transmission automatically computes a linear combination of the measurements, exploiting the interference among the signals. Thus, the wireless channel operates as a distributed computer for various target functions of emerging applications. The over-the-air computation (AirComp) has been developed to leverage the inherent broadcast nature of the wireless channel to efficiently fuse the distributed data [14]. In particular, the recently proposed MIMO-AirComp enables multifunction computation through spatial multiplexing while improving the link reliability under a harsh channel condition [15,16]. The MIMO-AirComp is a new communication and computation design approach to improve the computation throughput while reducing the spectrum usage and the delay for the massive Internet of Things (IoT).

This article bridges the gap between the MIMO-AirComp and distributed estimation applications. We present how the representative target function of the distributed estimation problem is converted to the nomographic function with the proper processing function. The proposed framework consists of the preprocessing function of each node, the collaborative aggregation of the wireless channel, and the postprocessing function of the FC. The analysis and preliminary results demonstrate that such a design improves the computation throughput of the distributed estimation. Furthermore, we show the operating condition for using the proposed MIMO-AirComp compared to the conventional separated design approach for the overall system performance.

Most existing works investigate the optimization problem of the transmit and receive beamforming gain to minimize the computation error of the AirComp [15,17–19]. Some recent surveys have only provided a general overview of the AirComp for various applications [12,20]. On the other hand, several distribution estimation techniques have mainly focused on the design of the local processing function without considering the communication aspects [21,22]. In contrast, this paper explicitly considers the target function of the distributed estimation problem with the MIMO-AirComp framework.

The rest of the paper is organized as follows. Section 2 briefly presents the technical background of the MIMO-AirComp. In Section 3, we present the MIMO-AirComp framework to compute various target multifunctions over the fading channel. Section 4 illustrates how the proposed MIMO-AirComp framework efficiently computes the target function of the distributed estimation. Section 5 evaluates the performance of the proposed approach compared to the conventional separated approach. Finally, we conclude the paper in Section 6.

2. MIMO-AirComp Fundamentals

We consider K wireless nodes, each having a set of data $\mathbf{y}_k = [y_{k,1}, \dots, y_{k,L_y}]^T \in \mathbb{R}^{L_y}, \forall k \in \{1, \dots, K\}$ and a single FC as depicted in Figure 1. This paper assumes a centralized star network topology, where all K nodes contend to send data to the FC, which is the data sink. Each node generates L_y heterogeneous sensing measurements. Multiple antenna arrays are equipped on all devices, where each node has N_{tx} transmit antennas, whereas the FC has N_{rx} receive antennas. Assuming symbol-level synchronization, all nodes concurrently send their symbol vectors via their arrays.

The application aims to compute L_f target functions using all of the set of data $\mathcal{D} = \{y_{k,l} \mid y_{k,l}, \forall k = 1, \dots, K, l = 1, \dots, L_y\}$

$$\mathbf{f}(\mathcal{D}) : \mathbb{R}^{K \times L_y} \rightarrow \mathbb{R}^{L_f}. \quad (1)$$

In theoretical computer science, every real-valued multivariate function can be represented in its nomographic form as a function of a finite sum of univariate functions [15,23].

Definition 1 (Nomographic Function). The function $r_l(d_{1,l}, \dots, d_{K,l}), \forall l \in \{1, \dots, L_d\}$ is a nomographic function, if there is a set of preprocessing functions $\alpha_{k,l} : \mathbb{R} \rightarrow \mathbb{R}, \forall k \in \{1, \dots, K\}, \forall l \in \{1, \dots, L_d\}$ and a postprocessing function $\beta_l : \mathbb{R} \rightarrow \mathbb{R}, \forall l \in \{1, \dots, L_d\}$, such that

$$r_l(d_{1,l}, \dots, d_{K,l}) = \beta_l \left(\sum_{k=1}^K \alpha_{k,l}(d_{k,l}) \right), \forall l \in \{1, \dots, L_d\}. \quad (2)$$

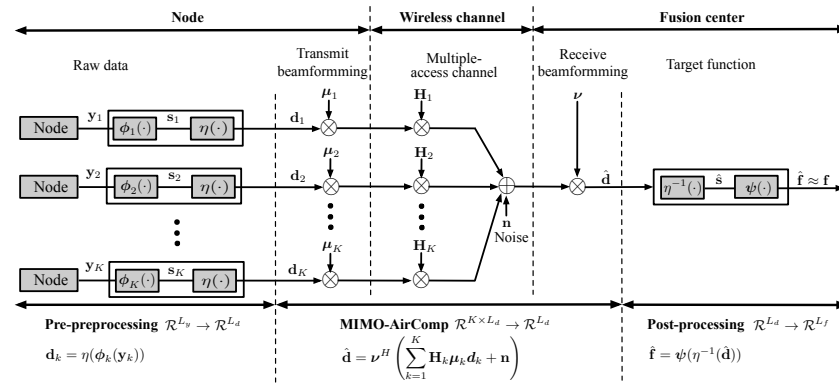


Figure 1. System diagram of the proposed MIMO-AirComp framework consisting of the local preprocessing of nodes, the aggregation of the wireless channel, and the postprocessing at the FC.

The AirComp allows the devices to simultaneously transmit their signal vectors to harness the superposition property of the interference in order to compute the nomographic function [13,14]. Thus, it theoretically completes the computation of the nomographic function of the distributed data in one single time unit (i.e., in a symbol level).

Any continuous function of n variables can be represented as a summation form with a maximum of $(2n + 1)$ nomographic component functions [23]. In other words, the AirComp requires at most $(2n + 1)$ channels to compute any function of n variables. However, many emerging applications still face fundamental technical challenges due to the stringent throughput/delay requirements over the limited spectrum bandwidth. In Section 4, we explicitly convert the target function of the distributed estimation to the nomographic function with proper processing functions.

3. The Proposed MIMO-AirComp Framework

Figure 1 illustrates the detailed operations of the proposed MIMO-AirComp framework to compute various target functions over the fading channel with additive noise. The target function is computed by exploiting the collaboration between multiple nodes, the wireless channel, and the FC. Each node preprocesses the raw measurement to input to the aggregation function of the wireless channel, whereas the FC applies the postprocessing function on the superimposed received signal.

- **Preprocessing at the node:** The local preprocessing function of the nodes converts the raw data $\mathbf{y}_k \in \mathbb{R}^{L_y}$ to the multimodal symbol vector $\mathbf{d}_k \in \mathbb{R}^{L_d}$ in order to input the MIMO-AirComp. The application-dependent function $\phi_k : \mathbb{R}^{L_y} \rightarrow \mathbb{R}^{L_d}$ is applied to the raw data $\mathbf{y}_k$ and generates $\mathbf{s}_k$ to distribute the computation load of the target function. In addition, the source encoder $\eta : \mathbb{R}^{L_d} \rightarrow \mathbb{R}^{L_d}$ is the element-wise mapping from data $\mathbf{s}_k$ to the transmit symbol $\mathbf{d}_k$, such that the symbol is used to compute the target function through the MIMO-AirComp. Thus, the input symbol vector to the MIMO-AirComp with the preprocessing becomes

$$\mathbf{d}_k = \eta(\phi_k(\mathbf{y}_k)). \quad (3)$$

- MIMO-AirComp: Let us denote $\mathbf{d} = \sum_{k=1}^K \mathbf{d}_k$ as the desired vector of the MIMO-AirComp for ease of exposition. The overall operation of the MIMO-AirComp is to apply the transmit beamforming matrix $\boldsymbol{\mu}_k \in \mathbb{C}^{N_{\text{tx}} \times L_d}$ to each preprocessed signal $\mathbf{d}_k$ and the receive beamforming matrix $\boldsymbol{\nu} \in \mathbb{C}^{N_{\text{rx}} \times L_d}$ to the received signals [15]. The fading channel model consists of the MIMO channel matrix $\mathbf{H}_k \in \mathbb{C}^{N_{\text{rx}} \times N_{\text{tx}}}$ for the link from node k to the FC and the additive white Gaussian noise vector with independent identically distributed elements $\mathbf{n} \in \mathcal{CN}(0, \sigma_{\text{cn}}^2 \mathbf{I})$. Each MIMO channel is subject to independent identically distributed Rayleigh fading, modeled as independent identically distributed complex Gaussian random variables with zero mean and unit variance. Then, the received symbol vector by the FC is

$$\hat{\mathbf{d}} = \boldsymbol{\nu}^H \left(\sum_{k=1}^K \mathbf{H}_k \boldsymbol{\mu}_k \mathbf{d}_k + \mathbf{n} \right), \quad (4)$$

which is the superimposed received signal sent by each node over the fading channel. Thus, the MIMO-AirComp essentially converts the set of the transmitted symbol of all nodes $\{\mathbf{d}_k\}_{k=1}^K$ to $\hat{\mathbf{d}} \in \mathbb{R}^{L_d}$.

- Postprocessing at the FC: The postprocessing function of the FC estimates the target vector $\mathbf{f} \in \mathbb{R}^{L_f}$ by using the aggregated received vector $\hat{\mathbf{d}} \in \mathbb{R}^{L_d}$. It first applies the source decoder $\boldsymbol{\eta}^{-1}(\cdot)$ in order to extract $\hat{\mathbf{s}} \approx \sum_{k=1}^K \mathbf{s}_k$, where $\hat{\mathbf{s}}$ is the vector required at the application layer. Then, the application-dependent function $\boldsymbol{\psi}(\cdot)$ is applied to $\hat{\mathbf{s}} \in \mathbb{R}^{L_d}$ to obtain the estimated target vector $\hat{\mathbf{f}} \in \mathbb{R}^{L_f}$. Thus, the postprocessing function of the FC is

$$\hat{\mathbf{f}} = \boldsymbol{\psi}(\boldsymbol{\eta}^{-1}(\hat{\mathbf{d}})), \quad (5)$$

where $\boldsymbol{\psi} : \mathbb{R}^{L_d} \rightarrow \mathbb{R}^{L_f}$ can be a highly nonlinear function dependent on the application.

The computation task of processing K signals is divided into $(K + 1)$ small tasks $\boldsymbol{\phi}_1(\cdot), \dots, \boldsymbol{\phi}_K(\cdot), \boldsymbol{\psi}(\cdot)$ for the nodes and the FC, respectively. It greatly reduces the computation complexity of the FC, especially when K is large.

While similar system models are presented in [15,16], the core components of the pre-processing and postprocessing functions are not well defined for practical applications. Our proposed system model decomposes the preprocessing function (the postprocessing function, respectively) into the application-dependent preprocessing function (the application-dependent postprocessing function, respectively) and the source encoder (source decoder, respectively).

3.1. Source Encoder/Decoder

Much of the existing AirComp research assumes that the source data of the nodes follow the standard normal distribution with zero mean and unit variance, $\mathbf{d}_k \in \mathcal{N}(0, \mathbf{I})$ [15, 17,18]. One of the simplest source encoders is to convert the source data with the standard normal distribution. Thus, the source encoder $\boldsymbol{\eta}(\cdot)$ normalizes the random variable $\mathbf{s}_k$ as

$$\mathbf{d}_k = (\mathbf{s}_k - \bar{\mathbf{s}}) \oslash \sqrt{\sigma_{\text{ed}}^2 + \epsilon}, \quad (6)$$

where $\bar{\mathbf{s}} \in \mathbb{R}^{L_d}$ and $\sigma_{\text{ed}}^2 \in \mathbb{R}^{L_d}$ are the element-wise mean and variance of $\mathbf{s}_k$ of all nodes, respectively, ϵ is the small constant vector, and $\oslash$ is the element-wise division. The constant vector ϵ prevents the numeric problem when the variance converges to 0, such as the consensus problem [10,24]. Note that the state estimation application computes the mean

$\bar{\mathbf{s}}$ and variance σ_{ed}^2 of $\mathbf{s}_k$ based on the specification or experimental measurements. By considering Equation (6), the source decoder η^{-1} then becomes

$$\hat{\mathbf{s}} = \sqrt{\sigma_{\text{ed}}^2 + \epsilon} \circ \hat{\mathbf{d}} + \mathbf{v}^H \left(\sum_{k=1}^K \mathbf{H}_k \mu_k \bar{\mathbf{s}} \right), \quad (7)$$

where $\circ$ is the element-wise product.

3.2. Optimization

In Equation (4), the transmit and receive beamforming attempts to reduce the distortion of the received vector with respect to the desired vector $\mathbf{d}$ by compensating the fading MIMO channel with additive noise. Note that the receive beamforming is applied to both the aggregated received K symbol vectors and the noise. The mean square error (MSE) is commonly used as the performance metric to evaluate the distortion of the received vector compared to the desired one. Thus, the joint beamforming optimization problem of the MIMO-AirComp has the objective of minimizing the MSE, subject to the constraints in the transmit and receive beamforming matrix; namely,

$$\min_{\mu, \nu} \quad \|\mathbf{d} - \hat{\mathbf{d}}\|_2^2 \quad (8)$$

$$\mu \in \tilde{\mu}, \nu \in \tilde{\nu}, \quad (9)$$

where $\tilde{\mu}$ and $\tilde{\nu}$ are the feasible sets of the transmit and receive beamforming matrices, respectively. Several recent works have considered the similar optimization problem of the transmit and receive beamforming gains [15,17–19].

4. Distributed Estimation using MIMO-AirComp

This section discusses the integration of the proposed MIMO-AirComp framework to estimate the system state from spatially distributed sensor readings [12,21].

We consider multiple sensors that measure a time-invariant variable and transmit it to the FC. The FC is responsible for processing the sensors' data to extract an interesting parameter. We assume a random variable $\mathbf{x} \in \mathbb{R}^{L_x}$ being observed by K sensors that generate measurements $\mathbf{y}_k \in \mathbb{R}^{L_y}$ of the form

$$\mathbf{y}_k = \mathbf{C}_k \mathbf{x} + \mathbf{v}_k, \quad \forall k = 1, \dots, K, \quad (10)$$

where $\mathbf{C}_k \in \mathbb{R}^{L_y \times L_x}$ is the observation matrix, and $\mathbf{v}_k \in \mathcal{N}(0, \mathbf{R}_{v,k})$ is a white Gaussian noise with covariance matrix $\mathbf{R}_{v,k} \in \mathbb{R}^{L_y \times L_y}$ of node k .

The objective of the state estimation problem is to obtain an estimate of $\mathbf{x}$ given all the measurements $\mathbf{y} = [\mathbf{y}_1^\top, \dots, \mathbf{y}_K^\top]^\top \in \mathbb{R}^{KL_y}$. Note that the length of the desired vector is $L_d = L_x$. Let us denote the estimate of $\mathbf{x}$ based on all measurements $\{\mathbf{y}_k\}_{k=1}^K$ (respectively, only the measurement $\mathbf{y}_k$) by $\hat{\mathbf{x}}$ (respectively, $\hat{\mathbf{x}}_k$). The conventional minimum MSE estimation problem uses the linear estimator $\hat{\mathbf{x}} = \mathbf{G}\mathbf{y}$, where the estimator is linear with the state measurements [21]. The linear estimation of $\mathbf{x}$ becomes

$$\mathbf{P}^{-1} \hat{\mathbf{x}} = \sum_{k=1}^K \mathbf{P}_k^{-1} \hat{\mathbf{x}}_k, \quad (11)$$

where $\mathbf{P} \in \mathbb{R}^{L_x \times L_x}$ (resp. $\mathbf{P}_k \in \mathbb{R}^{L_x \times L_x}$) is the error covariance corresponding to $\hat{\mathbf{x}}$ (resp. $\hat{\mathbf{x}}_k$). The error covariance satisfies

$$\mathbf{P}^{-1} = \sum_{k=1}^K \mathbf{P}_k^{-1} - (K-1) \mathbf{R}_x^{-1}, \quad (12)$$

where $\mathbf{R}_x \in \mathbb{R}^{L_x \times L_x}$ is the covariance of $\mathbf{x}$ [22]. In Equation (11), each estimate $\hat{\mathbf{x}}_k$ is weighted by the inverse of the error covariance matrix. The higher the confidence we have in a particular sensor, the higher the trust we place in its measurement.

In Equation (11), the weighted estimation $\mathbf{P}_k^{-1}\hat{\mathbf{x}}_k$ of node k is

$$\mathbf{P}_k^{-1}\hat{\mathbf{x}}_k = \mathbf{C}_k^\top \mathbf{R}_{v,k}^{-1} \mathbf{y}_k, \quad (13)$$

where $\hat{\mathbf{x}}_k$ only depends on the local parameters of $\mathbf{P}_k$, $\mathbf{C}_k$, and $\mathbf{R}_{v,k}$ and the local measurement $\mathbf{y}_k$ [22]. Thus, the form of the global estimator of Equation (11) follows the linear form with the measurement $\mathbf{y}_k$.

The estimation problem is solved if the FC has access to all local measurements of sensors at every time step. However, this conventional separated communication and computation approach is not the preferred implementation due to the high communication and computation loads. In particular, the FC needs to handle matrix operations whose size grows in proportion to the number of sensors in the global estimator of Equation (11). Thus, we attempt to transmit the sensing reading after some local processing rather than transmitting raw measurements to reduce the computational burden.

Equation (11) essentially shows that what we need at the FC is a weighted mean of the local estimates $\mathbf{P}_k^{-1}\hat{\mathbf{x}}_k$. By inputting Equation (13) into Equation (11), the target estimated state $\hat{\mathbf{x}}$ forms the nomographic function. Thus, the proposed framework computes the linear estimation $\hat{\mathbf{x}}$ over the MIMO channel as

$$\hat{\mathbf{x}} = \underbrace{\mathbf{P}}_{\psi}^{-1} \left(\mathbf{v}^H \left(\sum_{k=1}^K \mathbf{H}_k \mu_k \underbrace{\eta(\mathbf{C}_k^\top \mathbf{R}_{v,k}^{-1} \mathbf{y}_k)}_{\phi_k} + \mathbf{n} \right) \right), \quad (14)$$

where $\mathbf{d}_k = \eta(\mathbf{C}_k^\top \mathbf{R}_{v,k}^{-1} \mathbf{y}_k) \in \mathbb{R}^{L_x}$ is the input multimodal symbol vector after the local preprocessing, and $\psi = \mathbf{P}$ is the application-dependent postprocessing function.

The proposed distributed estimator using the MIMO-AirComp consists of a three-step procedure:

1. First, all nodes generate the input multimodal symbol vector $\mathbf{d}_k$ through the local preprocessing function $\eta(\mathbf{C}_k^\top \mathbf{R}_{v,k}^{-1} \mathbf{y}_k)$.
2. Then, the MIMO-AirComp applies the appropriate transmit beamforming matrix μ_k of node k and the receive beamforming matrix $\mathbf{v}$ of the FC to compute $\hat{\mathbf{d}}$ while compensating for the channel fading and noise.
3. Finally, the FC estimates $\hat{\mathbf{x}}$ after the postprocessing operation $\mathbf{P}\eta^{-1}(\cdot)$ to the aggregated received vector $\hat{\mathbf{d}}$.

The proposed MIMO-AirComp framework reduces the computation complexity of the FC by delegating some computational load to the nodes. Each node runs the matrix multiplication $\mathbf{W}_k \mathbf{y}_k$, where $\mathbf{W}_k = \mathbf{C}_k^\top \mathbf{R}_{v,k}^{-1} \in \mathbb{R}^{L_x \times L_y}$ is the precomputed weight matrix. Then, the source encoder applies the element-wise division to $\mathbf{s}_k \in \mathbb{R}^{L_d}$. Since each node only needs to compute a linear combination of its sensing vector as the local preprocessing function, this low-complexity algorithm can be implemented on typical wireless embedded systems with low energy consumption [25]. Furthermore, the wireless channel enables the automatic matrix summations involved in the global estimation due to its superposition property. In addition, the communication delay is also very small, as all nodes simultaneously transmit their scaled state measurements. The result with the star topology can be generalized for the arbitrary graphs with the lines of the average consensus algorithms (see, e.g., [26]).

5. Performance Evaluation

This section investigates how the proposed MIMO-AirComp framework performed for distributed estimation. For numerical experiments, we considered the conventional localization scenarios of mobile vehicles, where the target vehicle determined its position by measuring the direction and/or range to nearby beacon nodes whose positions were known [27]. The target vehicle estimated the position using the received signal strength of the wireless signals transmitted by K beacon nodes [28]. We placed the target vehicle at $\mathbf{x} = [20 \ 10]^\top$, whereas the K beacon nodes were randomly deployed in the range of $100 \text{ m} \times 100 \text{ m}$ around the target position. The linear observation model of the position estimation of beacon k is

$$\tilde{\mathbf{y}}_k = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \mathbf{x} + \mathbf{b}_k + \mathbf{v}_k, \quad \forall k = 1, \dots, K, \quad (15)$$

where $\mathbf{b}_k \in \mathbb{R}^2$ is the position vector of beacon k , and $\mathbf{v}_k \in \mathbb{R}^2$ is the measurement noise. By defining the debiased measurement state $\mathbf{y}_k = \tilde{\mathbf{y}}_k - \mathbf{b}_k$, Equation (15) has the form comparable to Equation (10). Each beacon only multiplied the (2×2) matrix with the (2×1) vector as the local preprocessing and ran the element-wise division of the (2×1) vector as the source encoder. In addition, a sliding window average was used to reduce the impact of the measurement noise on the position estimation.

To minimize the computation error, the differential geometry was used to derive the close-to-optimal transmit and receive beamforming gains of the MIMO-AirComp [15]. We used the solution of the weighted centroid of points on a Grassmann manifold as the joint beamforming matrix of the MIMO-AirComp framework.

We compared the proposed approach to a conventional separated communication and computation approach, where the global estimation of Equation (11) uses all individually received measurements through the Time Division Multiple Access (TDMA) scheme. The link reliability of the TDMA scheme was set to the block error rate of the 5G physical uplink control channel [29]. We note that this scheme provided extremely reliable communication performance for our considered range of network parameters. Furthermore, the measurement collection delay using the TDMA scheme was proportional to the number of nodes. For a fair comparison, the TDMA scheme allowed updating the position states when it received more than three beacon signals by considering the previously received information.

For the numerical analysis, we defined the performance metric called the service delay based on the settling time of a dynamical system [30]. The service delay T_s is the elapsed time for the system curve to reach and remain within a specified error norm bound δ to the ideal value $\mathbf{r}$:

$$T_s = \min\{t > 0 : \|\mathbf{x}(t') - \mathbf{r}\| \leq \delta \|\mathbf{r}\|, \forall t' \geq t\}. \quad (16)$$

The inverse of the service delay $1/T_s$ is analogue to the computation throughput. The network parameters such as the reliability and node density heavily affect the service delay. Furthermore, we showed the performance of the state estimation in terms of the root mean square error (RMSE) of the estimated state to the target position after 5 s.

We analyzed the service delay and the RMSE of the proposed framework for the various number of nodes, antenna configurations, channel noise, and measurement noise. We considered various channel noises with variance $\sigma_{\text{cn}}^2 = 1, 0.5$, which corresponded to the average SNR between 0dB and 3dB with the maximum transmit power. We also considered various measurement noises with covariance matrix $\mathbf{R}_{v,k} = \sigma_{\text{sn}}^2 \mathbf{I}$. For simplicity, we considered the block fading channels, where the wireless channels remained unchanged within each time slot but may change from one slot to another. We assumed that the channel coefficients over different time slots were generated from a stationary and ergodic stochastic process. In each time slot, the channel coefficients were generated following Rayleigh fading with unit variance. We set the maximum beamforming gain $\mu \leq 1$, the

MIMO configuration $N_{tx} = N_{rx} = 3$, and the error norm bound $\delta = 0.01$, unless otherwise stated. We calculated the service delay and the RMSE using 1000 Monte Carlo simulation results, where each simulation ran 10 s with 10 ms unit time.

Figure 2 plots the estimated position states $\hat{x}_1$ and $\hat{x}_2$ by using MIMO-AirComp and TDMA with different numbers of beacon nodes $K = 4, 20$ over time. We set $\sigma_{cn}^2 = 1$ and $\sigma_{sn}^2 = 0.9$. The solid bold lines present the actual position in two-dimensional space. Both schemes stabilized in all scenarios but at different rates, heavily depending on the node density.

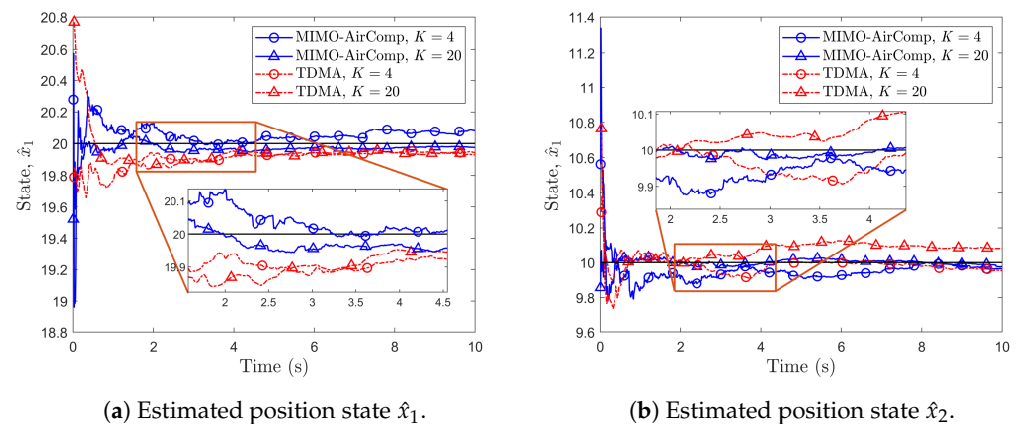
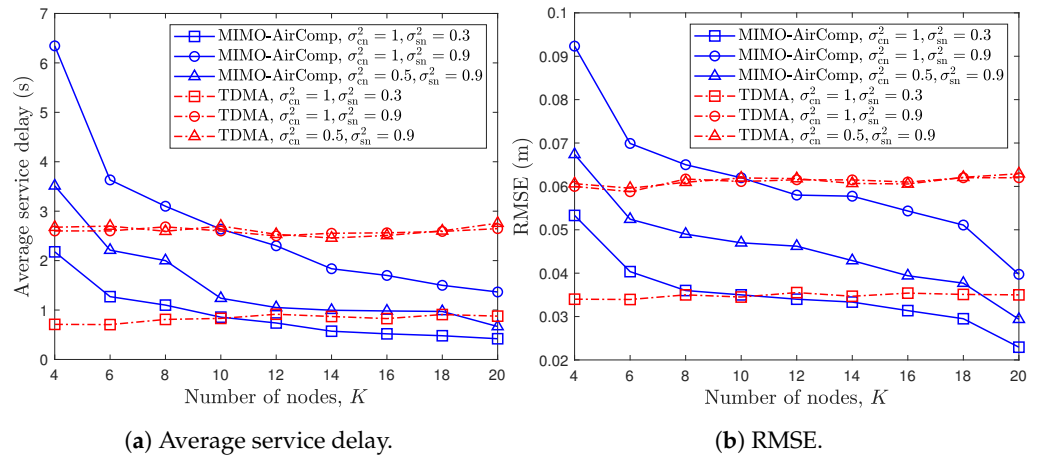


Figure 2. Estimated position states $\hat{x}_1$ and $\hat{x}_2$ of the MIMO-AirComp and TDMA with different numbers of beacon nodes $K = 4, 20$ over time.

For the small number of nodes $K = 4$, the overall estimated states of the TDMA were slightly better than those using the MIMO-AirComp. On the other hand, the MIMO-AirComp provided a faster convergence of the estimated states compared to that of the TDMA scheme when the number of nodes $K = 20$ was large. Moreover, the MIMO-AirComp considerably reduced the stationary error without having significant oscillation. In each time step, the MIMO-AirComp efficiently aggregated the sensing data of many beacon nodes. The TDMA, on the other hand, was almost independent of the node population, since it updated the position states in each received beacon information. The MIMO-AirComp showed a significant improvement in the state estimation performance with $K = 20$. We analyzed further details of the static estimation performance on the service delay and RMSE by considering the impact of the channel and measurement noise, the node population, and the antenna configuration.

Figure 3 shows the average service delay and RMSE using MIMO-AirComp and TDMA with different parameters $\sigma_{cn}^2 = 0.5, 1$ and $\sigma_{sn}^2 = 0.3, 0.9$ for the various numbers of nodes $K = 4, \dots, 20$. One interesting observation was that the MIMO-AirComp and TDMA provided considerably different behavior as a function of different node populations. While the MIMO-AirComp improved the service delay and the RMSE to meet the error bounds as K increased, the TDMA scheme showed an approximately constant service delay and RMSE, even after adding more nodes. This was because the MIMO-AirComp with a large K essentially suppressed the noise variance due to the wireless channel and the measurement in each time slot. On the other hand, the TDMA scheme mainly reduced the channel noise at the cost of the time delay.



The higher amount of sensing noise $\sigma_{sn}^2 = 0.9$ naturally degraded the service delay and the RMSE of the MIMO-AirComp and the TDMA more than with the lower amount of sensing noise $\sigma_{sn}^2 = 0.3$. Under both types of sensing noise $\sigma_{sn}^2 = 0.3, 0.9$, the TDMA scheme outperformed the MIMO-AirComp for the small number of nodes $K \leq 10$, while the MIMO-AirComp performed better for a large number of nodes $K > 10$.

The TDMA scheme provided a similar service delay and RSME even with different amounts of channel noise $\sigma_{cn}^2 = 0.5, 1$, since it achieved extremely reliable link performance under the harsh channel condition $\sigma_{cn}^2 = 1$. On the other hand, the low amount of channel noise $\sigma_{cn}^2 = 0.5$ considerably reduced the service delay and the RMSE of the MIMO-AirComp compared to that with the higher amount of channel noise $\sigma_{cn}^2 = 1$. In particular, the MIMO-AirComp remarkably outperformed the TDMA scheme when the sensing noise $\sigma_{sn}^2 = 0.9$ was high but the channel noise was low, $\sigma_{cn}^2 = 0.5$ for $K \geq 6$. On the other hand, under the harsh channel condition $\sigma_{cn}^2 = 1$, both the average service delay and the RMSE of TDMA were prominent compared to that using the MIMO-AirComp for the limited number of nodes $K \leq 10$.

The TDMA reliably delivered the sensing data over the harsh wireless channel at the cost of the time delay to collect each measurement dependent on the node population. While the global estimation of Equation (11) reduced the impact of the sensing noise, the time delay still heavily affected the overall service delay and the RMSE. On the other hand, the MIMO-AirComp compensated for both the channel and measurement noise by aggregating the sensing data each time, while it was still vulnerable to the channel noise. The noise suppression gain of the MIMO-AirComp was strong under the favorable wireless condition (i.e., low amount of channel noise) and weak under the hostile one (i.e., high amount of channel noise). The adverse effects of the different wireless conditions diminished with the increased node population. Thus, the performance gain of the MIMO-AirComp compared to the TDMA increased when the network consisted of a large number of noisy measurements under the fair channel condition. Our framework efficiently computed the target multifunction relying on massive IoT networks, where the densely deployed wireless nodes are typically built on low-cost sensors.

Next, we investigated how the MIMO configuration affected the service delay and the RMSE of both the MIMO-AirComp and TDMA for the limited number of beacon nodes. Figure 4 presents the average service delay and the RMSE using MIMO-AirComp and TDMA with different $N_{tx} = N_{rx} = 3, 4, 5$ for various amounts of channel noise $\sigma_{cn}^2 = 0.1, \dots, 1$. We set $\sigma_{sn}^2 = 0.6$ and $K = 6$. The TDMA scheme provided an almost flat service delay and RMSE independent of the channel noise and the MIMO configuration, since it already achieves extremely reliable performance even with $N_{tx} = N_{rx} = 3$. On the other hand, the average service delay and the RMSE using the MIMO-AirComp increased linearly with the channel noise. Adding more MIMO antennas $N_{tx} = N_{rx} = 4, 5$ considerably

reduced both the average service delay and the RMSE of the MIMO-AirComp by effectively compensating for the harsh channel condition.

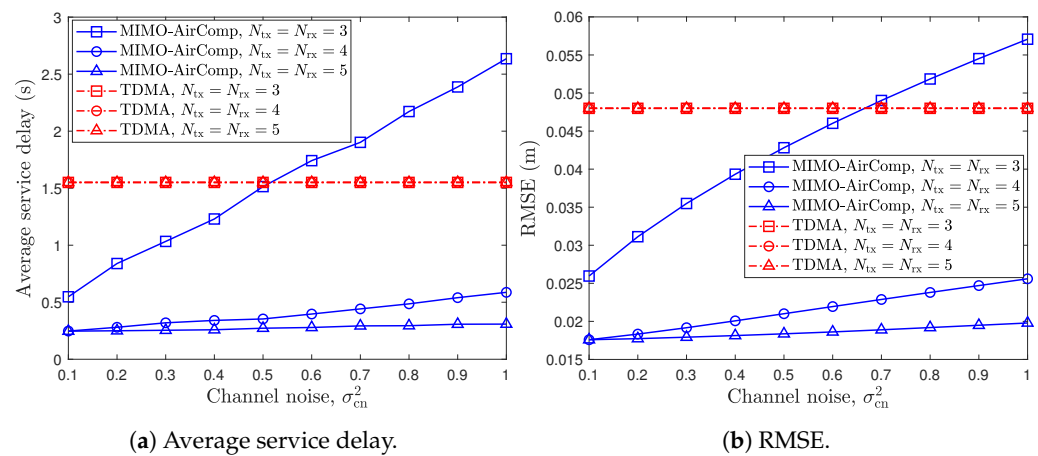


Figure 4. The average service delay and the RMSE of the MIMO-AirComp and TDMA with different $N_{tx} = N_{rx} = 3, 4, 5$ for various amounts of channel noise $\sigma_{cn}^2 = 0.1, \dots, 1$.

6. Conclusions

This article presented the MIMO-AirComp framework to efficiently compute the target multifunction of various applications while reducing the spectrum utilization and delay of wireless networks. The representative target function of the distributed estimation was explicitly converted to the nomographic function, which was matched to the structure of the multiple access channel with proper processing functions. The proposed framework efficiently computed the nomographic function by collaborating with the local preprocessing at each node, the signal aggregation of the wireless channel, and the postprocessing at the fusion center. The MIMO-AirComp framework with a large number of nodes suppressed the noise impact of both the wireless channel and the measurement, while the conventionally separated communication and computation approach mainly reduced the impact of the channel noise at the cost of the time delay to collect measurements. Specifically, the proposed approach achieved a high computation throughput compared to the conventional approach when the network relied on a large number of noisy measurements obtained by distributed sensors under the fair channel condition.

Author Contributions: Conceptualization, P.P.; Investigation, P.P., H.S. and P.D.M.; Methodology, P.P.; Software, P.D.M.; Validation, H.S.; Writing—original draft, P.P.; Writing—review & editing, P.P. and P.D.M. All authors have read and agreed to the published version of the manuscript.

Funding: The work of Pangun Park was supported in part by the Basic Research Laboratory of the National Research Foundation under Grant NRF-2020R1A4A2002021 and in part by Regional Innovation Strategy funded by the Ministry of Education (2021RIS-004). The work of Piergiuseppe Di Marco was supported by the Centre of EXcellence on Connected, Geo-Localized and Cybersecure Vehicles (EX-Emerge), funded by the Italian Government under Grant CIPE 70/2017 (7 August 2017).

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Data Availability Statement: Not applicable.

Conflicts of Interest: The authors declare no conflicts of interest.

References

1. Fang, J.; Ma, A. IoT Application Modules Placement and Dynamic Task Processing in Edge-Cloud Computing. *IEEE Internet Things J.* **2021**, *8*, 12771–12781.
2. Park, P.; Ergen, S.C.; Fischione, C.; Lu, C.; Johansson, K.H. Wireless Network Design for Control Systems: A Survey. *IEEE Commun. Surv. Tutorials* **2018**, *20*, 978–1013.

3. Wang, X.; Han, Y.; Leung, V.C.M.; Niyato, D.; Yan, X.; Chen, X. Convergence of Edge Computing and Deep Learning: A Comprehensive Survey. *IEEE Commun. Surv. Tutorials* **2020**, *22*, 869–904.
4. Letaief, K. B.; Chen, W.; Shi, Y.; Zhang, J.; Zhang, Y.-J. A. The Roadmap to 6G: AI Empowered Wireless Networks. *IEEE Commun. Mag.* **2019**, *57*, 84–90.
5. Zhu, G.; Liu, D.; Du, Y.; You, C.; Zhang, J.; Huang, K. Toward an Intelligent Edge: Wireless Communication Meets Machine Learning. *IEEE Commun. Mag.* **2020**, *58*, 19–25.
6. Park, P.; Marco, P.D.; Fischione, C. Optimized Over-the-Air Computation for Wireless Control Systems. *IEEE Commun. Lett.* **2022**, *26*, 424–428.
7. Pang, Z.; Luvisotto, M.; Dzung, D. Wireless High-Performance Communications: The Challenges and Opportunities of a New Target. *IEEE Ind. Electron. Mag.* **2017**, *11*, 20–25.
8. Goldenbaum, M.; Stanczak, S. Robust Analog Function Computation via Wireless Multiple-Access Channels. *IEEE Trans. Commun.* **2013**, *61*, 3863–3877.
9. Park, P.; Marco, P.D.; Fischione, C. Wireless for Control: Over-the-Air Controller. *IEEE Commun. Lett.* **2021**, *25*, 3437–3441.
10. Park, J.; Samarakoon, S.; Bennis, M.; Debbah, M. Wireless Network Intelligence at the Edge. *Proc. IEEE* **2019**, *107*, 2204–2239.
11. Tezergil, B.; Onur, E. Wireless Backhaul in 5G and Beyond: Issues, Challenges and Opportunities. *IEEE Commun. Surv. Tutorials* **2022**, *24*, 2579–2632.
12. Zhu, G.; Xu, J.; Huang, K.; Cui, S. Over-the-Air Computing for Wireless Data Aggregation in Massive IoT. *IEEE Wirel. Commun.* **2021**, *28*, 57–65.
13. Nazer, B.; Gastpar, M. Computation Over Multiple-Access Channels. *IEEE Trans. Inf. Theory* **2007**, *53*, 3498–3516.
14. Abari, O.; Rahul, H.; Katabi, D. Over-the-air function computation in sensor networks. *arXiv* **2016**, arxiv:1612.02307
15. Zhu, G.; Huang, K. MIMO Over-the-Air Computation for High-Mobility Multimodal Sensing. *IEEE Internet Things J.* **2019**, *6*, 6089–6103.
16. Wen, D.; Zhu, G.; Huang, K. Reduced-Dimension Design of MIMO Over-the-Air Computing for Data Aggregation in Clustered IoT Networks. *IEEE Trans. Wirel. Commun.* **2019**, *18*, 5255–5268.
17. Li, X.; Zhu, G.; Gong, Y.; Huang, K. Wirelessly Powered Data Aggregation for IoT via Over-the-Air Function Computation: Beamforming and Power Control. *IEEE Trans. Wirel. Commun.* **2019**, *18*, 3437–3452.
18. Liu, W.; Zang, X.; Li, Y.; Vucetic, B. Over-the-Air Computation Systems: Optimization, Analysis and Scaling Laws. *IEEE Trans. Wirel. Commun.* **2020**, *19*, 5488–5502.
19. Cao, X.; Zhu, G.; Xu, J.; Huang, K. Optimized Power Control for Over-the-Air Computation in Fading Channels. *IEEE Trans. Wirel. Commun.* **2020**, *19*, 7498–7513.
20. Wang, Z.; Zhao, Y.; Zhou, Y.; Shi, Y.; Jiang, C.; Letaief, K. B. Over-the-Air Computation: Foundations, Technologies, and Applications. *arXiv* **2022**, arxiv:2210.10524
21. Il, M. L.; Hall, D.; Llinas, J. *Handbook of Multisensor Data Fusion: Theory and Practice*; CRC Press: Boca Raton, FL, USA, 2017.
22. Duan, P.; Qian, J.; Wang, Q.; Duan, Z.; Shi, L. Distributed State Estimation for Continuous-Time Linear Systems With Correlated Measurement Noise. *IEEE Trans. Autom. Control* **2022**, *67*, 4614–4628.
23. Buck, R. Approximate complexity and functional representation. *J. Math. Anal. Appl.* **1979**, *70*, 280–298.
24. Dinh, T.Q.; La, Q.D.; Quek, T.Q.S.; Shin, H. Learning for Computation Offloading in Mobile Edge Computing. *IEEE Trans. Commun.* **2018**, *66*, 6353–6367.
25. Park, P.; Marco, P.D.; Nah, J.; Fischione, C. Wireless Avionics Intracommunications: A Survey of Benefits, Challenges, and Solutions. *IEEE Internet Things J.* **2021**, *8*, 7745–7767.
26. Yi, J.-W.; Chai, L.; Zhang, J. Average Consensus by Graph Filtering: New Approach, Explicit Convergence Rate, and Optimal Design. *IEEE Trans. Autom. Control* **2020**, *65*, 191–206.
27. Park, P.; Marco, P.D.; Jung, M.; Santucci, F.; Sung, T. Multi-directional Differential RSS Technique for Indoor Vehicle Navigation. *IEEE Internet Things J.* **2022**, *1*, 241–253.
28. Hostettler, R.; Sarkka, S. Basics of Sensor Fusion. Uppsala University and Aalto University. 2020, Available online: https://users.aalto.fi/~ssarkka/pub/basics_of_sensor_fusion_2020.pdf (accessed on 1 July 2022).
29. 5G; NR; Physical Channels and Modulation. 2020. Available online: https://www.etsi.org/deliver/etsi_ts/138200_138299/138211/15.02.00_60/ts_138211v150200p.pdf (accessed on 1 July 2022).
30. Goodwin, G.C.; Graebe, S.F.; Salgado, M.E. *Control System Design*; Prentice Hall: Hoboken, NJ, USA, 2000.

Disclaimer/Publisher’s Note: The statements, opinions and data contained in all publications are solely those of the individual author(s) and contributor(s) and not of MDPI and/or the editor(s). MDPI and/or the editor(s) disclaim responsibility for any injury to people or property resulting from any ideas, methods, instructions or products referred to in the content.