Flight Control Design Using Backstepping

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Abstract

Today's prevailing nonlinear design method for aircraft flight control is feedback linearization. This paper presents a new method to deal with the nonlinear aerodynamic forces and moments acting on the aircraft: backstepping. Specifically, we derive backstepping control laws for angle of attack and sideslip control that require less knowledge of the lift and side forces compared to feedback linearization designs. The control laws are made adaptive to errors in the aerodynamic moment coefficients using nonlinear observer techniques.

Keywords: Aircraft control, backstepping, Lyapunov functions, adaptive control

1 Introduction

The development of high performance aircraft operating at high angles of attack and at high angular rates has stimulated the interest in applying nonlinear control techniques to aircraft flight control. Currently, feedback linearization [11], or nonlinear dynamic inversion (NDI), as it is often referred to in the field, seems to be the prevailing nonlinear design method with numerous applications reported, see [9, 4, 10] and the references therein.

Feedback linearization, as the name implies, aims at cancelling the nonlinear system behavior. By using nonlinear feedback, the closed loop system is rendered linear. A drawback with this approach is that for the cancellation to be possible, all the nonlinearities involved must be known exactly. In aircraft flight control, the aerodynamic forces and moments acting on the aircraft are important sources of nonlinearity to be dealt with. In practice these can not be modeled exactly and hence, perfect cancellation is not possible.

In this paper we propose a new approach to robust aircraft flight control. Our main mathematical tool is backstepping [7]. Backstepping offers a more flexible way of dealing with nonlinearities compared to feedback linearization. Nonlinearities that act stabilizing may be kept in the closed loop system while destabilizing nonlinearities may be cancelled or dominated. Using this freedom, we design controllers for angle of attack and sideslip control that do not require a complete description of the lift force and the side force respectively. The key is to rely on the generic characteristics of these forces.

To realize the control laws in terms of control surface deflections, the mapping from these deflections to the resulting aerodynamic moment needs to be inverted. Assuming a general mapping between the two, this control allocation problem is solved using nonlinear optimization. Robustness against uncertainties and model errors in the mapping is achieved by recursively estimating the bias from the nominal model and using the estimate for feedback. This can be seen as an alternative to traditional integral feedback.

The paper is organized as follows. In Section 2, the aircraft model to be used is presented. In Section 3, the control strategy is defined and the general control framework is presented. The actual control design is performed in Section 4 and simulation results are shown in Section 5. Finally, Section 6 contains some concluding remarks.

2 Aircraft Model

In this contribution, the controlled variables are the angle of attack, α, the sideslip, β, and the roll rate about the stability x-axis, p_s. The equations of motion describing the aircraft dynamics in terms of these variables are [2, 12]:

$$\begin{aligned}
\dot{\alpha} &= q - (p\cos\alpha + r\sin\alpha)\tan\beta && \text{(1a)}\\
&\quad + \frac{1}{mV_T\cos\beta}(-L - F_T\sin\alpha + mg_1) \\
\dot{\beta} &= p\sin\alpha - r\cos\alpha && \text{(1b)}\\
&\quad + \frac{1}{mV_T}(Y - F_T\cos\alpha\sin\beta + mg_2) \\
M &= I\dot{\omega} + \omega\times I\omega && \text{(1c)}
\end{aligned}$$

Here, m is the aircraft mass, I is the inertia matrix, and V_T is the true airspeed. L and Y are the lift and side forces respectively, F_T is the engine thrust force, and

$$g_1 = g(\cos\alpha\cos\theta\cos\phi + \sin\alpha\sin\theta)$$
$$g_2 = g(\cos\beta\cos\theta\sin\phi + \sin\beta\cos\alpha\sin\theta - \sin\alpha\sin\beta\cos\theta\cos\phi)$$

represent the force contributions due to gravity. These depend on the orientation of the aircraft, given by the pitch angle, θ, and the roll angle, ϕ. M is the net torque applied to the aircraft and

$$\omega = \begin{pmatrix} p & q & r \end{pmatrix}^T$$

is the angular velocity of the aircraft expressed in the body axes frame.

The stability axes angular velocity

$$\omega_s = \begin{pmatrix} p_s & q_s & r_s \end{pmatrix}^T$$

is related to the body axes angular velocity through the transformation

$$\omega_s = S_\alpha \omega, \quad S_\alpha = \begin{pmatrix} \cos\alpha & 0 & \sin\alpha \\ 0 & 1 & 0 \\ -\sin\alpha & 0 & \cos\alpha \end{pmatrix} \tag{2}$$

The control input, δ, consists of the elevator (δ_e), aileron (δ_a), and rudder (δ_r) deflections.

For backstepping to be applicable, we will assume these control surface deflections only to produce aerodynamic moments, and not forces. We will also neglect the derivatives of the aerodynamic forces with respect to the angular velocity [1]. Essentially, this yields

$$L(\alpha) = \bar{q}SC_L(\alpha)$$
$$Y(\beta) = \bar{q}SC_Y(\beta)$$

where $\bar{q} = \rho V_T^2/2$ is the aerodynamic pressure, ρ is the air density, and S is the wing platform area.

In the control design to come, $\dot{\omega}_s$, the stability axes angular acceleration, will at first be considered the control input. Introducing

$$u = \begin{pmatrix} u_1 & u_2 & u_3 \end{pmatrix}^T = \dot{\omega}_s \tag{3}$$

we can rewrite the aircraft dynamics (1) as

$$\dot{p}_s = u_1 \tag{4}$$
$$\dot{\alpha} = q_s - p_s \tan\beta + \frac{1}{mV_T\cos\beta}(-L(\alpha) - F_T\sin\alpha + mg_1) \tag{5}$$
$$\dot{q}_s = u_2 \tag{6}$$
$$\dot{\beta} = -r_s + \frac{1}{mV_T}(Y(\beta) - F_T\cos\alpha\sin\beta + mg_2) \tag{7}$$
$$\dot{r}_s = u_3 \tag{8}$$

[1] These assumptions are the same as in feedback linearization applications.

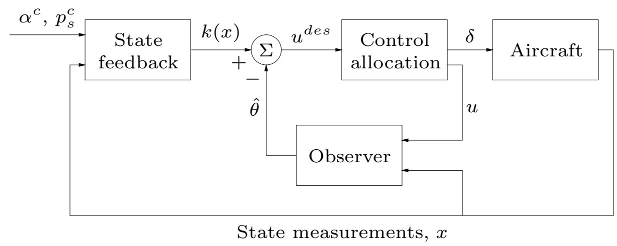

Figure 1: Overview of the control framework.

The relationship between u and the true control input, δ, can be found by combining equations (1c), (2), and (3). Regarding α as a constant while realizing the lateral control demands u_1 and u_3 yields $u = S_\alpha\dot{\omega}$. Inserting this into Equation (1c) gives us

$$u = S_\alpha I^{-1}(M(\delta) - \omega \times I\omega) \tag{9}$$

where we assume M to be a static function of the demanded control surface deflections, δ, ignoring the fast actuator dynamics.

Introducing the state vector $x = \begin{pmatrix} \alpha & \beta & p_s & q_s & r_s \end{pmatrix}^T$ we can use the compact form

$$\begin{aligned} \dot{x} &= f(x) + Bu \\ u &= g(\delta, x) \end{aligned} \tag{10}$$

to describe the uncontrolled aircraft dynamics (4)–(9).

3 Control Preliminaries

3.1 Control strategy

The angle of attack and the stability axis roll rate should follow the pilot commanded values α^c and p_s^c respectively. A stability axis roll, also known as a velocity vector roll, is a roll performed at constant angle of attack and zero sideslip. The sideslip is to be kept zero at all times. Speed control is assumed to be handled separately.

3.2 The general framework

The block diagram in Figure 1 gives an overview of the general framework that will be used for the flight control design.

The first block outputs the desired angular acceleration, $u = k(x)$. If this could be produced exactly by deflecting the control surfaces properly, the closed loop dynamics would be

$$\dot{x} = f(x) + Bk(x) \tag{11}$$

In Sections 4.1 and 4.2 we derive state feedback control laws $k(x)$ such that $p_s = p_s^c$, $\alpha = \alpha^c$, and $\beta = 0$ becomes a globally stable equilibrium of the closed loop system (11).

Realizing $u = k(x)$ requires precise knowledge of which angular acceleration, and in particular which aerodynamic moment, M, is produced for a certain set of control deflections, δ. To allow a model error to be present in this usually quite complex mapping, we remodel the system dynamics as

$$\begin{aligned}\dot{x} &= f(x) + B(u + \theta)\\ u &= g(\delta, x)\end{aligned}$$

where θ is an unknown but constant bias. Using nonlinear observer techniques, an exponentially converging estimate, $\hat{\theta}$, can be produced. This estimate can be used for feedback in a straightforward way:

$$u = k(x) - \hat{\theta}$$

In Section 4.3 we show that closed loop stability is retained using this adaptive control law.

Finally, in the control allocation block, control surface deflections are found such that $u = g(\delta, x)$ is achieved, if possible. This is done using nonlinear optimization as discussed in Section 4.4.

4 Control Design

A general assumption that we will make is that longitudinal and lateral commands are not applied simultaneously. The mathematical effect of this is that when deriving the angle of attack control law, β, p_s, and r_s are considered constant. Vice versa, when designing the sideslip and roll control laws, α and q_s are considered constant. Also, due to the time-scale separation, all other variables, such as the Euler angles, ψ, θ, and ϕ, and the aircraft velocity, V_T, are considered constant.

4.1 Stability axis roll control

Controlling the stability axis roll is straightforward. With

$$\dot{p}_s = u_1 \tag{12}$$

from Equation (4), simply assign

$$u_1 = \frac{1}{\tau}(p_s^c - p_s) \tag{13}$$

where τ is the desired roll time constant.

4.2 Angle of attack and sideslip control

To begin with, we note the structural similarities between (5)–(6) and (7)–(8). Both these second order systems can be written

$$\begin{aligned}\dot{x}_1 &= f(x_1, y) + x_2\\ \dot{x}_2 &= u\end{aligned} \tag{14}$$

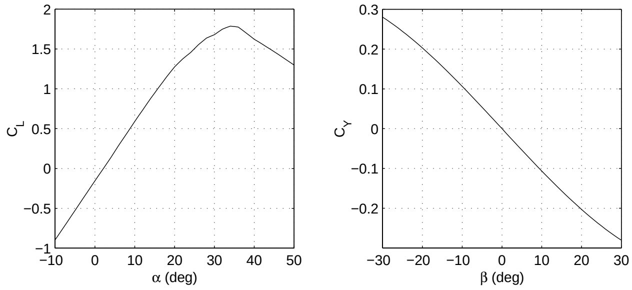

Figure 2: Typical lift force coefficient versus angle of attack and side force coefficient versus sideslip.

where y represents the influence of variables that we regard as constant. The main characteristics of the nonlinear term f in the two cases are decided by $-L(\alpha)$ and $Y(\beta)$ respectively. From Figure 2 we see that the gradients of $-L$ and Y are negative in large parts of the operating range (the only exception is the post-stall behavior of the lift force). The key property of backstepping is that it allows us to benefit from these inherently stabilizing forces and not cancel them.

In a nutshell, this is where our backstepping approach differs from feedback linearization. Feedback linearization renders the closed loop system linear by cancelling f using nonlinear feedback. This requires complete knowledge of $f(x_1, y)$ as well as $\partial f(x_1, y)/\partial x_1$. Our backstepping control law on the other hand will be linear in x_1 and x_2 and only require knowledge of f at the desired equilibrium and an upper bound of the slope of f.

Let us first derive a backstepping control law for the generic system (14) and then apply it to α and β control.

4.2.1 A generic backstepping design

Consider the system (14), and determine a control law that globally stabilizes the system at $x_1 = r$. To see things more clearly, we introduce the deviations from steady state, $\xi_1 = x_1 - r$, $\xi_2 = x_2 + f(r, y)$, and $\varphi(\xi_1) = f(\xi_1 + r, y) - f(r, y)$ (we drop the φ depencence on y and r for notational convenience). This yields

$$\dot{\xi}_1 = \varphi(\xi_1) + \xi_2 \tag{15a}$$
$$\dot{\xi}_2 = u \tag{15b}$$

Now assume that there exists a maximum slope

$$a = \max_{\substack{\xi_1 \in \mathbb{R}\\ r \in \Omega_r\\ y \in \Omega_y}} \frac{\varphi(\xi_1)}{\xi_1} \leq \max_{\substack{x_1 \in \mathbb{R}\\ y \in \Omega_y}} \frac{\partial f(x_1, y)}{\partial x_1} \tag{16}$$

Equality holds if r is not restricted, i.e., when $\Omega_r = \mathbb{R}$. To use this property in the Lyapunov framework of backstepping, we can rewrite it as

$$\xi_1 \varphi(\xi_1) \leq a \xi_1^2 \tag{17}$$

We now turn to the actual control design.

Step 1: Let us for a moment regard ξ_2 as the control input of Equation (15a) and find a desired stabilizing "virtual" control law ξ_2^{des}, using the control Lyapunov function (clf)

$$V_1 = \frac{1}{2}\xi_1^2$$

Differentiating with respect to time, we get

$$\dot{V}_1 = \xi_1(\varphi(\xi_1) + \xi_2) \leq \xi_1(a\xi_1 + \xi_2)$$

using (17). $\dot{V}_1$ is made negative definite by selecting

$$\xi_2^{des} = -c_1\xi_1, \quad c_1 > a$$

The resulting ξ_1 dynamics, $\varphi(\xi_1) - c_1\xi_1$, lie in the second and fourth quadrants only and thus, ξ_1 is stabilized.

Step 2: Continue by introducing the residual

$$\tilde{\xi}_2 = \xi_2 - \xi_2^{des} = \xi_2 + c_1\xi_1$$

and rewrite the system dynamics in terms of ξ_1 and $\tilde{\xi}_2$.

$$\dot{\xi}_1 = \varphi(\xi_1) - c_1\xi_1 + \tilde{\xi}_2 \tag{18a}$$

$$\dot{\tilde{\xi}}_2 = u + c_1(\varphi(\xi_1) - c_1\xi_1 + \tilde{\xi}_2) \tag{18b}$$

In Equation (18b) it is not clear whether the ξ_1 components are beneficial or not. Proceeding in the usual backstepping manner, by adding a $\tilde{\xi}_2^2$ term to the clf, would lead to a control law that cancels these components. The control law would thereby require exact knowledge of φ and, consequently, f, not only at the equilibrium, $x_1 = r$. As we will see, this can be avoided by also adding a general term $F(\xi_1)$ to be decided as an extra degree of freedom. This extension of backstepping is due to [8]. Thus,

$$V_2 = \frac{c_0}{2}\xi_1^2 + F(\xi_1) + \frac{1}{2}\tilde{\xi}_2^2$$

where $F(\xi_1)$ is a positive definite, radially unbounded function, satisfying

$$F'(\xi_1)\xi_1 > 0, \quad \xi_1 \neq 0 \tag{19}$$

where $F'(\xi_1) = \partial F(\xi_1)/\partial \xi_1$.

We now aim at finding a u that will make $\dot{V}_2$ negative definite.

$$\begin{aligned}\dot{V}_2 = &\, c_0\xi_1(\varphi(\xi_1) - c_1\xi_1 + \tilde{\xi}_2) \\ &+ F'(\xi_1)(\varphi(\xi_1) - c_1\xi_1 + \tilde{\xi}_2) \\ &+ \tilde{\xi}_2(u + c_1(\varphi(\xi_1) - c_1\xi_1 + \tilde{\xi}_2))\end{aligned}$$

At this stage it is rewarding to make the split

$$\varphi(\xi_1) = \varphi_-(\xi_1) + a\xi_1$$

where $\varphi_-(\xi_1)$ is guaranteed to just stay inside the second and fourth quadrants. I.e.,

$$\xi_1\varphi_-(\xi_1) \leq 0$$

We note that $\varphi(\xi_1) - c_1\xi_1$ is also restricted to the second and fourth quadrants. Combining this with Equation (19) we have that

$$F'(\xi_1)(\varphi(\xi_1) - c_1\xi_1) \leq 0$$

also holds. Using these relationships we get

$$\begin{aligned}\dot{V}_2 \leq &- c_0(c_1 - a)\xi_1^2 + \tilde{\xi}_2(c_0\xi_1 + F'(\xi_1) \\ &+ u + c_1(\varphi_-(\xi_1) + (a - c_1)\xi_1 + \tilde{\xi}_2))\end{aligned}$$

We can further simplify this expression using out design freedom. The choices

$$\begin{aligned} c_0 &= c_1(c_1 - a) \\ F'(\xi_1) &= -c_1\varphi_-(\xi_1), \quad c_1 > 0,\ F(0) = 0\end{aligned}$$

render the final expression

$$\dot{V}_2 \leq -c_1(c_1 - a)^2\xi_1^2 + \tilde{\xi}_2(u + c_1\tilde{\xi}_2)$$

To make the right hand side negative definite, and the closed loop system globally stable, we select the control law

$$u = -c_2\tilde{\xi}_2, \quad c_2 > c_1 \tag{20}$$

Although f, and consequently V_2, is y dependent, closed loop stability is guaranteed by the LaSalle-Yoshizawa theorem [7] for each constant value of y.

Summary: Let us summarize our results. Despite the nonlinear nature of the system (15), the linear control law (20) is globally stabilizing. In terms of the original state variables from (14) the control law becomes

$$u = -c_2(x_2 + c_1(x_1 - r) + f(r, y)) \tag{21}$$

c_1 and c_2 are design parameters restricted by

$$c_2 > c_1 > \max(a, 0)$$

with a from (16).

4.2.2 Application to α and β control

The control law (21) can be applied to angle of attack control by comparing Equations (5)–(6) with the generic system (14) and substituting

$$x_1 = \alpha, \quad x_2 = q_s, \quad u = u_2$$

With f from (5) we get

$$a = \max \frac{1}{mV_T \cos\beta}(-\frac{\partial L(\alpha)}{\partial \alpha} + mg)$$

using (16) and neglecting the beneficial impact of the thrust force term. a can be readily computed assuming some lower bound on V_T and some upper bound on β.

For the β control case, Equations (7)–(8) determine the open loop dynamics. Again comparing with the generic system (14) we substitute

$$x_1 = \beta, \quad x_2 = -r_s, \quad u = -u_3$$

With f from (7) we get

$$a = \max \frac{1}{mV_T}(\frac{Y(\beta)}{\beta} + mg)$$

with the same remarks as above. Here, we have used the fact that the sideslip command is always zero. Assuming that the side force is zero for zero sideslip yields the control law

$$u_3 = c_2(-r_s + c_1\beta + \frac{g}{V}\cos\theta\sin\phi)$$

Note that u_3 does not depend explicitly on the side force.

4.3 Adapting to input uncertainties

In the previous section, we derived globally stabilizing flight control laws regarding $u = \dot{\omega}_s$ as the control input.

In Section 3.2 we outlined an intuitively appealing idea of how to make these control laws adaptive to a static model error in the mapping (9) from the actual control input, δ, to $\dot{\omega}_s$. Writing this relationship as

$$\dot{\omega}_s = u + \theta$$

where θ is an unknown but constant bias vector, the uncontrolled dynamics become

$$\begin{aligned}\dot{x} &= f(x) + B(u + \theta)\\ \dot{\theta} &= 0\end{aligned}$$

We note that the nonlinearity f depends only on measurable entities. This means that an exponentially converging estimate, $\hat{\theta}$, can be computed using observer techniques along the lines of [6].

Adjusting the state feedback laws to also incorporate $\hat{\theta}$ with the aim of cancelling θ,

$$u = k(x) - \hat{\theta} \tag{22}$$

yields the closed loop dynamics

$$\dot{x} = f(x) + B(k(x) + \theta - \hat{\theta})$$

Is global stability retained with this adaptive strategy? This issue was investigated in [5] for the single input case. It was shown that global stability is retained given that the control law is augmented by a term

$$l(x) = -\lambda\partial V/\partial x_n, \quad \lambda > 0$$

where $V(x)$ is a Lyapunov function for the nominal closed loop system, $\dot{x} = f(x) + Bk(x)$, and x_n is the variable whose derivative is directly affected by the input.

In the generic backstepping case, $l = -\lambda\tilde{\xi}_2$. This can be seen as a part of the control law (20) already by performing the split

$$u = -c_2\tilde{\xi}_2 = -\tilde{c}_2\tilde{\xi}_2 - \lambda\tilde{\xi}_2, \quad \tilde{c}_2 > c_1,\ \lambda > 0$$

This also holds for the roll control case using, e.g., $V = (p_s - p_s^c)^2$.

Thus, the desired equilibrium is globally stabile using the adaptive control law (22).

4.4 Realizing the control laws

In this section, we propose how to realize (22), i.e., how to find control deflections, δ, such that (9) is satisfied. Solving for the net moment to be produced yields

$$M(\delta) = IS_\alpha^T u + \omega \times I\omega$$

This can be converted into the desired aerodynamic moment coefficients, $C^{des} = (C_l^{des}, C_m^{des}, C_n^{des})$. The control allocation problem can now be expressed as minimizing the deviation between the desired aerodynamic coefficients and the actual ones, i.e., to minimize

$$J(\delta) = |C(\delta, x) - C^{des}|^2$$

Using an optimization framework instead of solving $C(\delta, x) = C^0$ as a system of equations allows us to handle cases where there exists no feasible solution to $J(\delta) = 0$, e.g., due to saturating control surfaces.

Nonlinear optimization is a complex matter and in general no guarantee can be given that the global optimum is found. However, since in our case the optimization is performed at each sample time, we can use the solution at one time step as the initial guess at the next time step. This strategy of each time starting very close to the optimal solution yields very good performance.

In the implementation, the Broyden-Fletcher-Goldfarb-Shanno (BFGS) variable metric method [3], a quasi-Newton method, was chosen. This takes $J(\delta)$ and the gradient $\nabla J(\delta)$ as arguments, where the latter can be numerically computed. It recursively estimates the inverse Hessian of J and performs a line search in this direction to update δ. Reusing the inverse Hessian at the next time step significantly improves the performance.

5 Simulations

We evaluate the control laws using Admire, a Matlab/Simulink environment for the Generic Aerodata

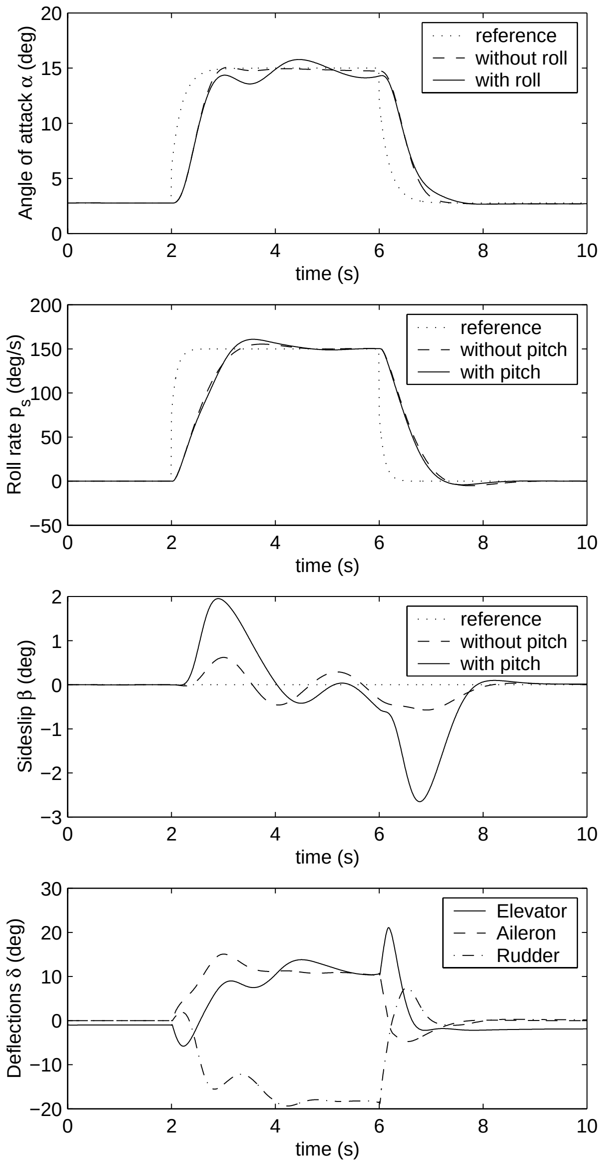

Figure 3: Simulated aircraft responses to pitch and stability axis roll commands.

Model (GAM), a small generic fighter aircraft, developed by Saab AB, Sweden [1]. The simulations are performed at an initial speed of 0.5 Mach at an altitude of 1000 m. The following control laws parameters were used. For α control, $c_1 = 2$, $c_2 = 5$, for β control, $c_1 = 3$, $c_2 = 5$, and for roll control, $\tau = 0.5$. The poles of the θ observers were placed in $-8 \pm i$.

Figure 3 shows the responses to a sole angle of attack command, a sole stability axis roll command, and the combination of the two. Control surface deflections for the combined manoeuvre are shown in the bottom picture.

6 Conclusions

In this paper we have demonstrated the potential of using backstepping techniques to design flight control laws. In comparison to feedback linearization, the control laws can be made computationally simpler by not cancelling the beneficial nonlinear parts of the lift and side forces. We have also proposed the use of nonlinear observers as an alternative to traditional integral feedback for reaching the desired steady state in the presence of additional, unknown inputs.

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