

Evolution in Complex Systems

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What features characterize complex system dynamics? Power laws and scale invariance of fluctuations are often taken as the hallmarks of complexity, drawing on analogies with equilibrium critical phenomena. Here we argue that slow, directed dynamics, during which the system's properties change significantly, is fundamental. The underlying dynamics is related to a slow, decelerating but spasmodic release of an intrinsic strain or tension. Time series of a number of appropriate observables can be analyzed to confirm this effect. The strain arises from local frustration. As the strain is released through "quakes," some system variable undergoes record statistics with accompanying log-Poisson statistics for the quake event times. We demonstrate these phenomena via two very different systems: a model of magnetic relaxation in type II superconductors and the Tangled Nature model of evolutionary ecology and show how quantitative indications of aging can be found. © 2004 Wiley Periodicals, Inc. Complexity 10: 49–56, 2004

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Many macroscopic systems evolve through periods of relative quiescence separated by brief outbursts of hectic activity. We describe the prototype complex dynamics using two specific systems from physics and biology: the magnetic behavior of type II superconductors and biological macroevolution. Each system is metastable when observed on short time scales, whereas at long time scales, each evolves towards greater stability. The models were

introduced and discussed in general terms in Refs. 1–4. Our aim in the present article is to focus on the nature of the long time relaxation associated with the intermittent activity. This intermittent dynamics is in itself important and has attracted much interest [5, 6]. Even more crucial is the often neglected fact that the punctuated dynamics of complex systems may lead to substantial changes in global properties, induced by the system following a distinct directed evolutionary path [7]. Descriptions borrowed from equilibrium and/or stationary systems are thus of limited value and will be unable to catch the essential time dependence of the dynamics. The long time effect of complex dynamics is evident in biological macroevolution, for example, in the form of a slowly decreasing extinction rate [8]. Similar effects are one of the main characteristics of spin glasses [9],

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and has been suggested to be relevant to the long time behavior of geological faults [10, 11] (though this nonstationary aspect of fault dynamics is often excluded in simple models [12]) and in economics [13].

Why do the properties of a complex system change as a result of the intrinsic system dynamics and how can this change be described quantitatively? In general terms, this is because complex systems consist of many components coupled together through a network of interactions. Because it is unlikely that the first, random configuration fully or even partly optimizes all interactions, a complex system will initially be in a state of high frustration and strain, i.e., in a state unable to locally fulfil all constraints imposed by the mutual interactions in a many component system.

The ensuing dynamics will act to release this strain and thus relax or optimize the system, resulting in a more stable configuration. A many component system needs to find combined dynamical moves that collectively improve the distribution of interactions. Most of the dynamics of the individual degrees of freedom will not add up in a coherent and constructive way but will give rise to fluctuations about some metastable configuration. However, the inbuilt strain of the initial configuration does exert a directed push on all the components and will once in a while lead to coherent rearrangements of parts of the system. These essential events will be like snow avalanches or earthquakes in a geological fault. They induce an irreversible change in the properties of the system. We will call these events *quakes* to stress their dramatic effect on the stability of the system.

It is important to distinguish the dynamics of complex systems presented here from the avalanche scenario outlined for Self-Organized Critical systems (SOC [5, 6]). SOC systems emphasizes that the concept of scale invariance or criticality (as encountered in equilibrium phase transitions) are of generic relevance to complex systems. The reason for this, as was argued in the seminal article by Bak et al. (BTW; [14]), is that large collections of interacting over-damped degrees of freedom that evolve according to dynamics controlled by thresholds will, if they are slowly driven by external actions, self-organize into a state that lacks any characteristic scale except for the one imposed by the finite system size. The widespread observation of power laws in nature is, within the SOC paradigm, considered to be a consequence of the anticipated scale invariance of the *stationary* self-organized critical state. These power laws are the stationary probability distributions describing the SOC response in terms of (generalized) avalanche events. Let us mention a few examples. In the BTW sandpile model [14], sand is sprinkled on to a surface at random. After a while, a stationary state is expected to develop in which the distribution of sand avalanches is described by a power law [15]. In the forest fire model [16], a stationary state is established by randomly growing trees at rate p on the empty sites of a lattice and having lightning ignite a tree at a much smaller

frequency ν . The self-organized stationary state was expected to lead to a scale invariant distribution of the sizes of fires [17].

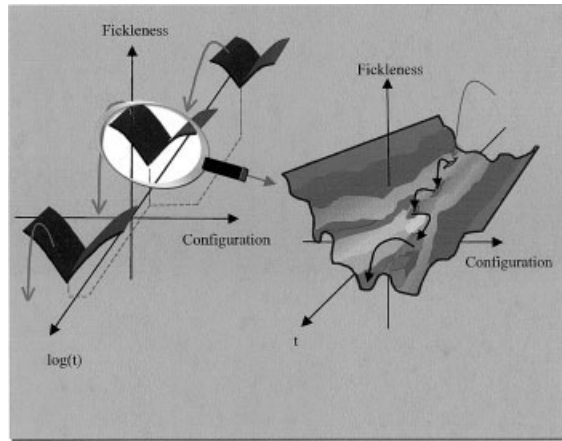
For completeness, we mention that Boettcher and Paczuski [18] have used the term aging in a less restrictive way, than we do in this article, in their study of the Bak-Sneppen model [19]. In this model, the return time of the activity of an individual avalanche depends on the age of that avalanche. This type of aging is unrelated to the phenomena we discuss here because it does not involve any change with time of physical properties.

Whereas the focus in SOC is on the power laws encountered in the stationary state, the dynamics of complex systems we describe here concentrates on the fact that the quakes of complex systems gradually change both the physical and statistical properties of the system. We do not claim that complex systems are necessarily scale invariant and described by power laws, but we stress that the effect of a quake is to take the complex system into a new metastable configuration differing slightly from the previous state. Because the quake released some of the strain or improved a collection of interactions, this new metastable state will tend to be more stable. Hence, ever larger fluctuations are needed to take the system out of consecutive metastable configurations. Figure 1 illustrates the situation schematically. The system will tend to spend more and more time in the metastable states as it searches for a sufficiently large fluctuation that brings about a new and, on average, more stable configuration. This leads to a slowing down of the pace of evolution.

To make this sketch more concrete and to be able to describe a methodology for observing and analyzing the slow but crucial evolution of system properties, we now turn to a discussion of two specific systems. We choose two very different phenomena to illustrate the general nature of the description and analysis: magnetic relaxation in type II superconductors [1, 2] and biological macroevolution [3, 4]. A similar analysis can be carried out for spin glasses [20, 21]. We emphasize that the analysis is nonintrusive and applicable to any system for which the time evolution of an appropriate observable is available.

The qualitative description of the previous paragraph suggests that the dynamics of complex systems tends to on average increase the stability or decrease what we call in Figure 1, fickleness. The transition from one metastable configuration to the next is accompanied by a drift in some measure: flux density for the model superconductor [1, 2], population size for the model ecosystem [3, 4] and energy in the spin glass model [20, 21]. We call this measure the record parameter because its temporal evolution consists of a sequence of ever increasing record values. Each new record is triggered by a quake. On a logarithmic time scale, the quakes are essentially instantaneous and hence the quake number k is well determined by a single time,

FIGURE 1



The dynamics of a complex system can be qualitatively summarized by considering the relation between time, configuration, and fickleness. The smaller the fickleness value (i.e., the lower it is along the z-axis) the more stable the system becomes. The long time dynamics consists of a slow evolution in the form of jumps, or quakes, from one metastable configuration to the next, indicated by the sequence of ever deeper wells, or valleys, at the left of the figure. The quakes are only seen when the system is observed over many decades of time, hence the logarithmic time axis. The dynamics between the quakes is represented by the magnification shown on the right. On a linear (short) time scale, the system undergoes smaller jumps between sub-valleys within a single main valley. Short time dynamics slightly improves the stability of the system as indicated by the decrease of the system's fickleness with time. The quakes have a similar effect on a logarithmic time scale, as indicated by the deepening of the valleys on the left of the figure.

namely, $\ln(t_k)$, the logarithm of the time of the onset of the quake. We now show how the dynamics is characterized by the statistical properties of the logarithmic waiting time between quakes, given by the sequence $\tau_k = \ln(t_k) - \ln(t_{k-1}) = \ln(t_k/t_{k-1})$.

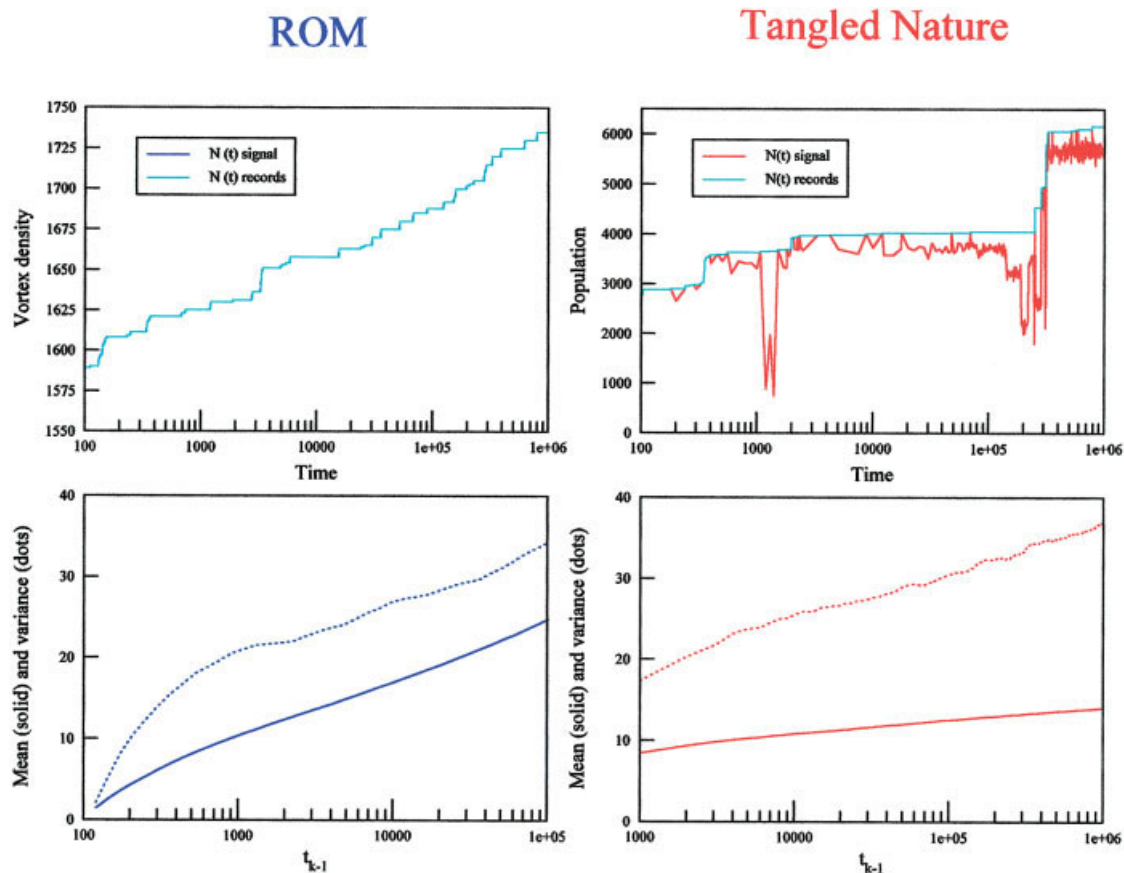
We consider two models. Both are experimentally and observationally accessible [22–24]. First we consider the gradual penetration of the external magnetic field into the bulk of a type II superconductor after an initial ramping of the external magnetic field up to a fixed value. We use the Restricted Occupancy Model, or ROM [1, 2] to simulate this system. The data presented here are for the more realistic three-dimensional layered version of the ROM. We imagine a stack of two-dimensional superconducting planes. Each plane is divided into $L \times L$ squares. Here, $L = 8$ and there are five layers, though similar behavior is seen for different system sizes. A square can contain from zero up to a maximum number of N_{c2} magnetic vortices. The state of the vortex system is specified by the number of vortices on each square. The vortices interact repulsively with the vortices in the nearest neighbor squares of the same plane (because

parallel magnetic line segments repel each other). The only difference between the three- and the two-dimensional version of the ROM model is an attractive interaction between the vortices in squares right above each other in adjacent planes (an attraction similar to that between aligned compass needles). The model is updated using Monte Carlo dynamics [1, 2]. The density of vortices at the boundary of the system represents the external magnetic field and is kept at a constant level. We monitor how the number of vortices inside the system increases with time as they enter the bulk sites of the model.

We obtain the sequence of quake waiting times, τ_k , from a time signal $N(t)$, which denotes the number of flux quanta inside the superconducting sample. As seen in Figure 2, $N(t)$ is primarily a monotonically increasing step function. The characteristic behavior of $N(t)$ can be identified from the observation that $N(t)$ is essentially equal to the record signal derived from it. The record is simply the largest value of $N(t)$ obtained up to time t . The jumps in $N(t)$ define a sequence of quake times t_k at which new flux quanta are able to penetrate into the sample. The intervals between the quake times are spent rearranging the internal flux in search of a configuration that better accommodates the magnetic pressure of the external applied field. The essential feature of the logarithmic slowing down of the evolution of the complex system's dynamics is modeled by a Poisson (or approximately Poisson) distribution of the logarithm of the quake times [20, 21, 25], i.e., a log-Poisson. This implies that the dynamics of the quakes are most naturally observed on a logarithmic time scale. Figure 2 confirms this, because the average and variance of the number of quakes of a set of independent realisations of $N(t)$ increases linearly with the logarithm of time, as expected for a log-Poisson process. The logarithmic time dependence is equivalent to a rate of events, ν , which decays inversely with time: $\nu \propto 1/t$. We note that the internal state at time t is characterized by the highest number of flux quanta achieved up to that time.

As our second example, we use the Tangled Nature model of evolutionary ecology [3, 4] to simulate the macro-evolution of an ecosystem. This is an individual-based model consisting of different interacting genotypes each characterized by a sequence of L numbers that can be $+1$ or -1 . This is to be thought of as the individual's genotype. For the results shown here, $L = 20$, allowing for up to 2^{20} different genotypes, but very similar findings have been obtained for $L = 8$. Reproduction is asexual and the reproduction probability of an individual is determined according to a weight function calculated from the frequency-dependent interactions it has with other genotypes [3, 4]. During reproduction, mutations can occur with a fixed probability. This leads to motion of the population in genotype space. Death consists of individuals being removed from the system with a fixed probability independent of time and genotype.

FIGURE 2

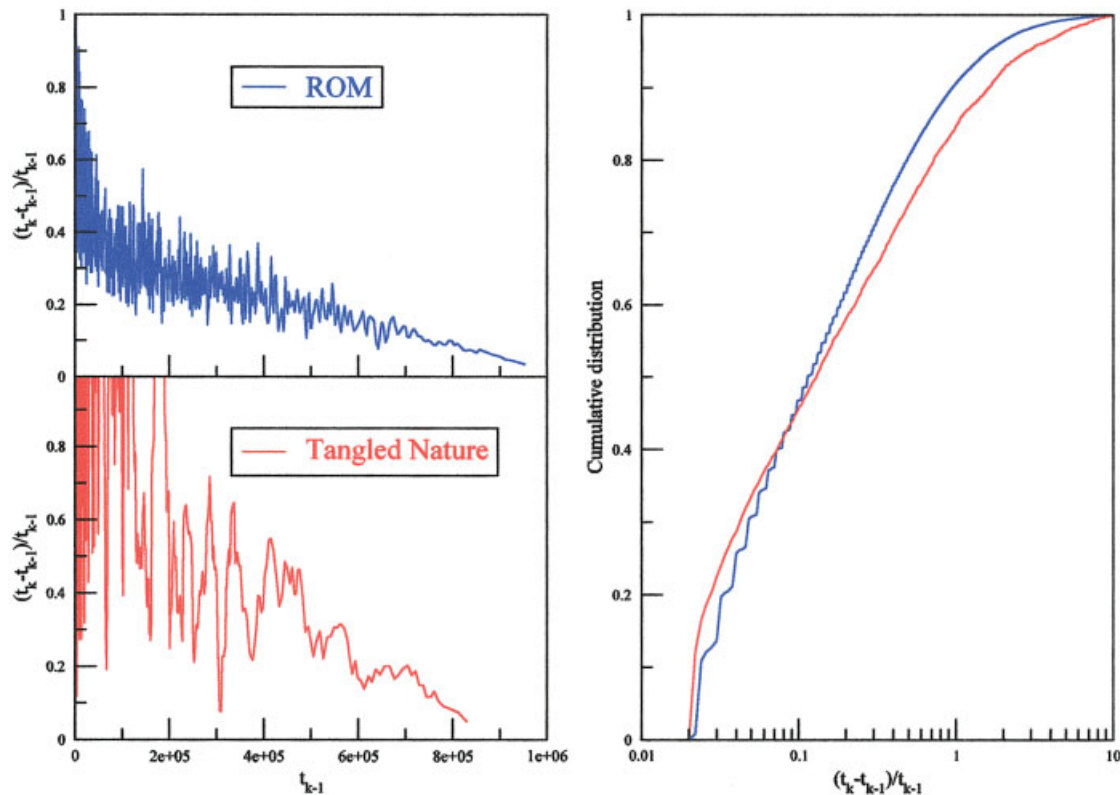


Temporal evolution in the ROM and the Tangled Nature model. The top plots show the value of $N(t)$ (and the corresponding records) for a single realization. The bottom panels show the average behavior obtained from a set of realizations. The solid lines indicate the average number of quakes up to time t . The variance in the number of quakes is shown by the dotted curves. Both quantities exhibit an approximately linear dependence on the logarithm of time, as expected for a log-Poisson distribution.

The time signal $N(t)$ is defined as the size of the entire population of the ecosystem. The choice of measure is not strictly unique. It is obviously important for the experimental or observational verification of the scenario outlined here that more than one choice exists for the signal $N(t)$ in general. For example, the records of the total number of occupied genotypes follow a very similar pattern to the total population. We see in Figure 2 that the population size exhibits an overall increase with time, though fluctuations can make $N(t)$ decrease for short intervals. For a single realisation of $N(t)$, we derive the record signal, i.e., the largest value of $N(t)$ obtained up to time t . The quake times are identified as the record times of $N(t)$. The exact relation between these record times and the transitions between different metastable states of the Tangled Nature model is not clear, but our numerical investigations give us good reason to believe that the statistics of the record times mirrors the statistics of the transition times between meta-

stable states. Averaged over a collection of independent realizations, $N(t)$ is found to increase logarithmically. Thus the record is a good indicator of the long time behavior of $N(t)$. We expect the quakes occurring at these times (corresponding to reorganisation through species extinction and creation) to be responsible for the gradual collective adaptation observed in the model [3, 4]. The quake times in this model are more difficult to identify compared to the ROM because of stronger fluctuations. Figure 2 shows that the records follow the same Poisson statistics on a logarithmic time scale as discussed above for the magnetic flux model. Again we see that the average and variance of the cumulated number of quakes increase linearly with the logarithm of time. The difference in the slopes might be attributed to a certain degree of over-counting. This may happen when a single quake is composed of a rapid succession of micro-quakes, which may be counted as separate quakes, even though strictly they are part of the same quake. The precise

FIGURE 3



Evidence of aging. The plots on the left show the ratio of the waiting time between quakes, $t_k - t_{k-1}$, and the time of the $(k - 1)$ th quake t_{k-1} , as a function of t_{k-1} . For a stationary process, where the average value of the durations $t_k - t_{k-1}$ is a constant, Δt say, this ratio will decay to zero like $\Delta t/t_{k-1}$ with increasing t_{k-1} . For the two models considered here, the ratio remains nearly constant over about six orders of magnitude. The slight decay at late times is due to the finite time T_{obs} of the entire time sequence, which imposes the constraint $t_k - t_{k-1} < T_{\text{obs}} - t_{k-1}$. The panel to the right shows the cumulative distribution for this ratio in the two models. For a stationary process, this function would be a step function equal to one for $(t_k - t_{k-1})/t_{k-1} > 0$.

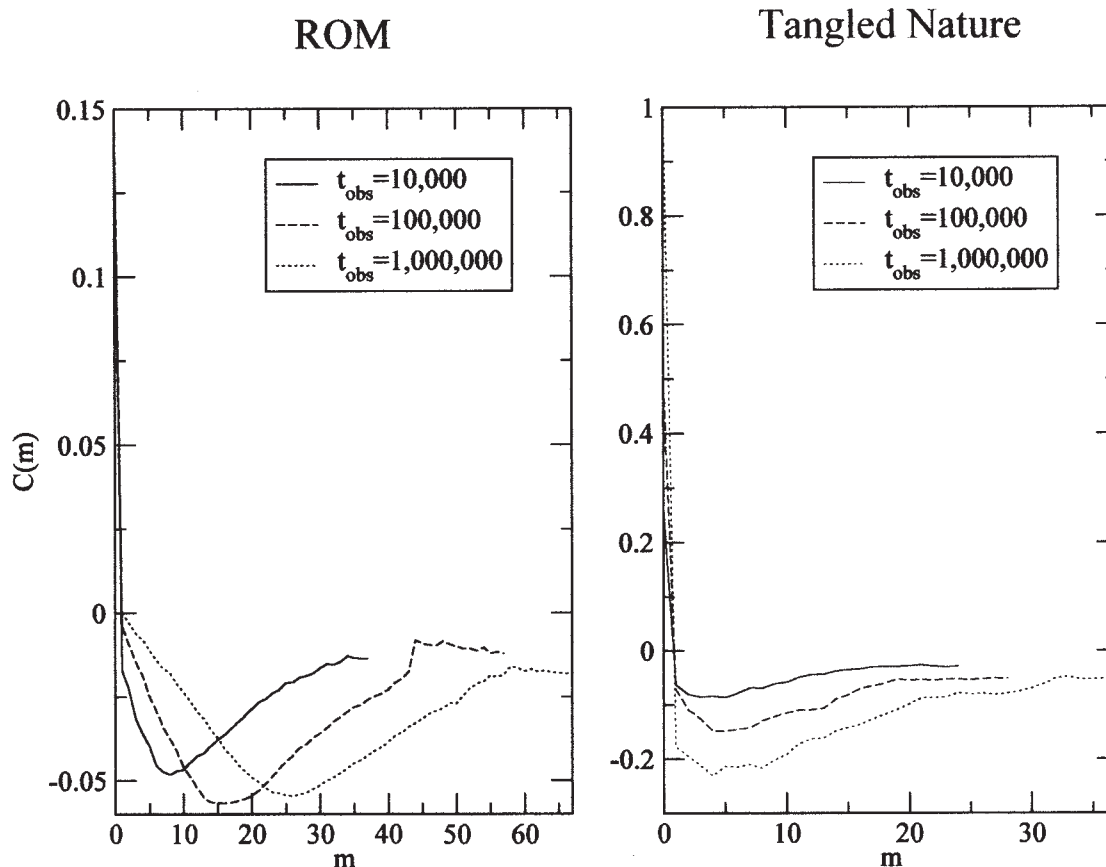
identification of quakes in Tangled Nature is a very difficult problem.

The constant logarithmic rate of quake events implies that the average waiting time between quakes grows linearly with the age of the system. This implies that the variable $X = (t_k - t_{k-1})/t_{k-1}$ should fluctuate about a constant value. This behavior is to be distinguished from an ordinary Poisson process for which the average time between events is independent of time. In this case the variable X will exhibit a rather rapid decay proportional to $1/t_{k-1}$. However, note that for a finite observation time t_{obs} , the ratio X must decrease as t_{k-1} approaches t_{obs} because $t_k - t_{k-1}$ cannot exceed the value $t_{\text{obs}} - t_{k-1}$.

In Figure 3, we show that for both models, X varies no more than one order of magnitude even though t_{k-1} spans six orders of magnitude. Thus, the older the system (equivalent to large values of t_{k-1}) the longer the time between records, $t_k - t_{k-1}$. The correlation functions shown in Figure

4 indicate that correlations between consecutive quakes are negligible. We see that after a fast decay of the correlation function, a degree of negative (anti) correlation occurs before the correlations approach zero. It should also be noticed that these anti-correlations become more pronounced as the observation time t_{obs} increases. (The largest t_{obs} in either case is one million: the number of time steps the simulations ran for.) The anti-correlation is due to the fact that, for a given observation window, a longer-than-average quiescent period will most likely be followed by a shorter one. This is because the activity slows down like $1/t_{k-1}$. It is accordingly impossible to achieve an observation window that is long compared with the longest waiting time, and the numerically estimated correlation function is therefore always influenced by the finite duration of the observation no matter how big this window is taken to be. The effect can be easily mimicked in a standard Poisson process, but is not usually observed because under usual circumstances, the

FIGURE 4



The plots show, as a function of m , the correlation between τ_k and τ_{k+m} where $\tau_k = \ln(t_k) - \ln(t_{k-1}) = \ln(\tau_k/t_{k-1})$ is the difference between the logarithmic times of occurrence of two successive quakes. The rapid decay of $C(m)$ agrees reasonably well with the theoretical form of a log-Poisson process, i.e., a Kronecker delta in m . The modest amount of anti-correlation seen for intermediate m values is due to observational effects; see main text. t_{obs} is the observation time. For example, $t_{\text{obs}} = 10,000$ means that only records that occur before $t = 10,000$ are included in the analysis. Short range anti-correlations are observed for all window sizes.

observation window can always be chosen to be much longer than the average time between the events.

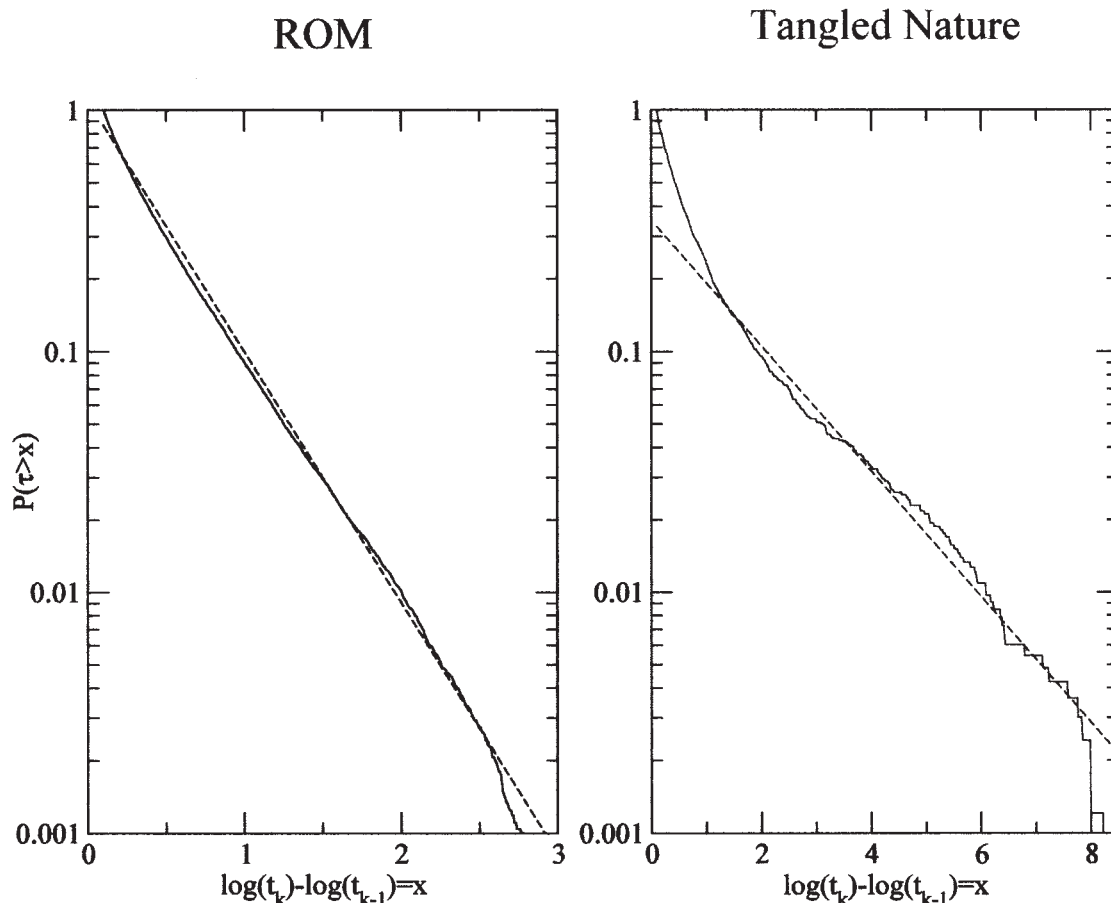
The rapid decay of the correlation functions does not of course imply that the quakes are statistically independent, though it is consistent with assuming independence. If the quakes are independent, the logarithmic time intervals between quakes should be exponentially distributed. In Figure 5 we show that this is the case, to a good approximation. We consider this finding as further indication that consecutive quake waiting times are essentially statistically independent.

It is worth mentioning that it is difficult to precisely identify the metastable states. The transition from metastable state to the next might be best described by the change in some measure characterising the stability of the configurations. This might be an eigenvalue of the stability matrix associated with a set of effective evolution

equations or, as in the simulation study of spin-glass relaxation in Ref. 21, from a very detailed analysis of the internal energy. In general one will have to rely on the measures that are accessible, though perhaps less than optimal. We have demonstrated here that record dynamics in complex systems can be analyzed approximately even if all one can obtain is some simple macroscopic measurable quantity.

Metastable systems of great complexity with huge numbers of interacting heterogeneous components are common throughout nature. It is crucial to realize that they are forever evolving at a decelerating pace toward configurations of greater stability, and so concepts from equilibrium and stationary systems will only be of relevance over relatively short time scales. We have demonstrated above how aging and record dynamics can be detected and described within an analytical framework.

FIGURE 5



As further evidence that the records, or “quakes,” follow the log-Poisson distribution, we include the distribution of the log waiting times for both the ROM and the Tangled Nature model, $P(\tau > x)$. For reference, we have shown an exponential distribution (broken line). If a series of records follows the log-Poisson distribution, the cumulated distribution of the log waiting times should follow an exponential [25]. The fit is good in both cases and provides a strong indication that the data do indeed obey a log-Poisson.

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