

Distributed Space-Time Coded Protocols for Exploiting Cooperative Diversity in Wireless Networks

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Abstract— We develop and analyze space-time coded cooperative diversity protocols for combating multipath fading across multiple protocol layers in a wireless network. The protocols exploit spatial diversity available among a collection of distributed terminals that relay messages for one another in such a manner that the destination terminal can average the fading, even though it is unknown *a priori* which terminals will be involved. In particular, a source initiates transmission to its destination, and many relays potentially receive the transmission. Those terminals that can fully decode the transmission utilize a space-time code to cooperatively relay to the destination. We demonstrate that these protocols achieve full spatial diversity in the number of cooperating terminals, not just the number of decoding relays, and can be used effectively for higher spectral efficiencies than repetition-based schemes. We discuss issues related to space-time code design for these protocols, emphasizing codes that readily allow for appealing distributed versions.

I. INTRODUCTION

In wireless networks, signal fading arising from multipath propagation is a particularly severe form of interference that can be mitigated through the use of *diversity*—transmission of redundant signals over essentially independent channel realizations in conjunction with suitable receiver combining to average the channel effects. Space, or multi-antenna, diversity techniques are particularly attractive as they can be readily combined with other forms of diversity, *e.g.*, time and frequency diversity, and still offer dramatic performance gains when other forms of diversity are unavailable. In contrast to the more conventional forms of single-user space diversity with physical arrays, this work builds upon the classical relay channel model [1] and examines the problem of creating and exploiting space diversity using a collection of distributed antennas belonging to multiple terminals, each with its own information to transmit. We refer to this form of space diversity as *cooperative diversity* (*cf.* *user cooperation diversity* of [2]) because the terminals share their antennas and other resources to create a “virtual array” through distributed transmission and signal processing.

Cooperative diversity between two cooperating terminals is examined in [3], [4], and a variety of repetition-based proto-

cols are developed and analyzed. For example, a relay either amplifies what it receives, or fully decodes, re-encodes, and repeats the source message. It is shown in [4] that these simple protocols can be extended to more than two terminals to provide *full spatial diversity*: if m is the number of cooperating terminals, each with a single transmit antenna, system performance can behave as if each terminal employs m transmit antennas. For example, the outage probability performance of repetition-based cooperative diversity decays asymptotically proportional to $1/\text{SNR}^{m(1-R_{\text{norm}})}$, where SNR corresponds to the average signal-to-noise ratio (SNR) between terminals, and $0 < R_{\text{norm}} < 1/m$ corresponds to a suitably-normalized spectral efficiency of the protocol [4]. In this context, full diversity refers to the fact that, as $R_{\text{norm}} \rightarrow 0$, the outage probability decays as $1/\text{SNR}^m$. By contrast, the outage probability performance of non-cooperative transmission decays asymptotically as $1/\text{SNR}^{(1-R_{\text{norm}})}$, where $0 < R_{\text{norm}} < 1$ is allowed, and as $1/\text{SNR}$ as $R_{\text{norm}} \rightarrow 0$. Thus, while the outage probability performance of cooperative diversity can decay faster, it does so only for small R_{norm} , in particular, for $R_{\text{norm}} < 1/(m+1)$.

Of course, there are more general forms of decode-and-forward transmission, just as there are more general forms of space-time codes. Indeed, we will see in this paper that, once we introduce a few variations on the decode-and-forward theme laid out in [4], the vast array of space-time coding literature can be brought to bear in the context of cooperative diversity, leading to a class of protocols that we call *space-time coded cooperative diversity*. Essentially, our new protocols consist of the following: all relays that can decode the original transmission re-transmit in the same subchannel using a suitably designed space-time code. Fig. 1 illustrates the two phases of the protocol.

Space-time coded cooperative diversity leads to schemes whose outage probability performance decays asymptotically proportional to roughly $1/\text{SNR}^{m(1-2R_{\text{norm}})}$. Thus, they (a) achieve full spatial diversity order m as $R_{\text{norm}} \rightarrow 0$, (b) have larger diversity order than repetition-based algorithms for all R_{norm} , and (c) are preferable to non-cooperative transmission if $R_{\text{norm}} < (m-1)/(2m-1)$. Moreover, we will see that these protocols may be readily implemented in a distributed fashion, because they only require the relays to estimate the SNR of their received signals, decode them if the SNR is sufficiently high, re-

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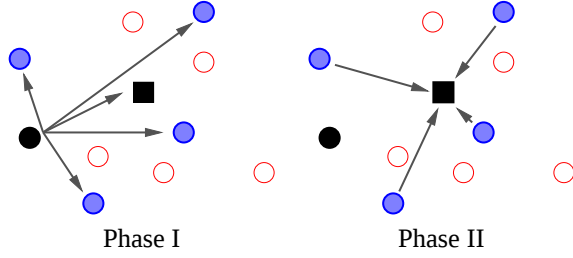


Fig. 1. Illustration of the two-phases of space-time coded cooperative diversity protocols. In the first phase, the source broadcasts to the destination as well as potential relays. Decoding relays are shaded. In the second phase, the decoding relays use a space-time code to transmit to the destination.

encode with the appropriate waveform from a space-time code, and re-transmit in the same subchannel.

In broader context, cooperative diversity can be viewed as a form of *network coding*, in this case designed to exploit spatial diversity in the network. There is a growing body of work focused on network coding for enhancing performance of wireless and other communication networks [5], [6], [7].

II. SYSTEM MODEL

In our model for the wireless channel in Fig. 1, narrowband transmissions suffer the effects of frequency nonselective fading and additive noise. Our analysis in Section III focuses on the case of slow fading, and measures performance by outage probability, to isolate the benefits of space diversity. While our protocols can be naturally extended to the kinds of wideband and highly mobile scenarios in which frequency- and time-selective fading, respectively, are encountered, the potential impact of our protocols becomes less substantial as other forms of diversity can be exploited in the system.

A. Medium Access

As in many current wireless networks, we divide the available bandwidth into orthogonal channels and allocate these channels to the transmitting terminals. The medium-access control (MAC) sublayer typically performs this function. For example, the MAC in many cellular networks seeks to allocate orthogonal channels, *e.g.*, frequency-division, time-division, or code-division, to the terminals in a cell for communicating to the basestation of that cell. As another example, the MAC in the IEEE 802.11 wireless LAN standard uses similar structures for LANs controlled by an access point, or a distributed contention-resolution/collision avoidance algorithm which facilitates random time-division.

For our cooperative diversity protocols described in Section III, transmitting terminals must also process their received signals; however, current limitations in radio implementation preclude the terminals from transmitting and receiving at the same time in the same frequency band. Because of severe signal attenuation over the wireless channel, and insufficient electrical isolation between the transmit and receive circuitry, a terminal's

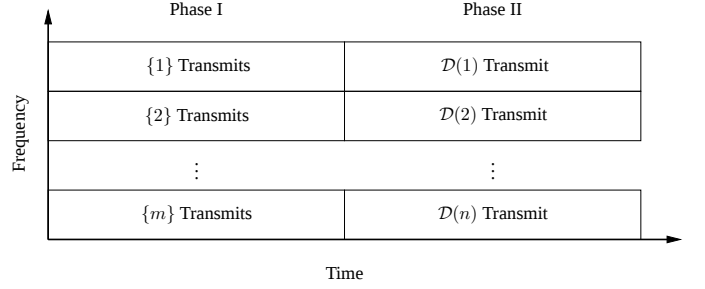


Fig. 2. Example channel allocations across frequency and time. $\mathcal{M} = \{1, 2, \dots, m\}$ denotes the set of cooperating terminals in the network. For source s , $\mathcal{D}(s)$ denotes the set of decoding relays participating in a space-time code during the second phase.

transmitted signal drowns out the signals of other terminals at its receiver input. Thus, we further divide each channel into orthogonal subchannels. Fig. 2 illustrates an example channel allocation satisfying these constraints.

B. Equivalent Channel Models

Under the above orthogonality constraints, we can now conveniently, and without loss of generality, characterize our channel models. Let $\mathcal{M} = \{1, 2, \dots, m\}$ be the set of cooperating terminals in the network. Due to the symmetry of the channel allocations, we focus on the message of the source $s \in \mathcal{M}$ in transmitting to its destination $d(s)$, potentially using terminals $\mathcal{M} - \{s\}$ as relays. Thus there are m cooperating terminals communicating to $d(s)$. We utilize a baseband-equivalent, discrete-time channel model for the continuous-time channel, and we consider N consecutive uses of the channel, where N is a large integer.

During the first phase, each potential relay $r \in \mathcal{M} - \{s\}$ receives

$$y_r[n] = a_{s,r} x_s[n] + z_r[n], \quad (1)$$

for, say, $n = 1, \dots, N/2$, where $x_s[n]$ is the source transmitted signal and $y_r[n]$ is the received signal at r . If the SNR is sufficiently large for r to decode this transmission, then r serves as a decoding relay for the source s , so that $r \in \mathcal{D}(s)$. We characterize the set $\mathcal{D}(s)$ more specifically in Section III, but for now it is sufficient to define it qualitatively and bear in mind that it is a *random* set.

The destination receives signals during both phases. During the first phase, we model the received signal at $d(s)$ as

$$y_{d(s)}[n] = a_{s,d(s)} x_s[n] + z_{d(s)}[n], \quad (2)$$

for $n = 1, \dots, N/2$. During the second phase, we model the received signal at $d(s)$ as

$$y_{d(s)} = \sum_{r \in \mathcal{D}(s)} a_{r,d(s)} x_r[n] + z_{d(s)}[n], \quad (3)$$

for $n = N/2 + 1, \dots, N$, where $x_r[n]$ is the transmitted signal of relay r . It is during this second phase that the decoding relays

employ an appropriately designed space-time code, allowing $d(s)$ to separate, weight, and combine the signals even though they are transmitted in the same subchannel.

In (1)-(2), $a_{i,j}$ captures the effects of path-loss, shadowing, and frequency nonselective fading, and $z_j[n]$ captures the effects of receiver noise and other forms of interference in the system. We consider the scenario in which the fading coefficients are known to, *i.e.*, accurately measured by, the appropriate receivers, but not fully known to (or not exploited by) the transmitters. Statistically, we model $a_{i,j}$ as zero-mean, independent, circularly-symmetric complex Gaussian random variables with variances $1/\lambda_{i,j}$, so that the magnitudes $|a_{i,j}|$ are Rayleigh distributed ($|a_{i,j}|^2$ are exponentially distributed with parameter $\lambda_{i,j}$) and the phases $\angle a_{i,j}$ are uniformly distributed on $[0, 2\pi)$. Furthermore, we model $z_j[n]$ as zero-mean mutually independent, circularly-symmetric, complex Gaussian random sequences with variance N_0 .

C. Parameterizations

As in [3], [4], two important parameters of the system are the transmit signal-to-noise ratio SNR and the spectral efficiency R . We now define these parameters in terms of standard parameters in the continuous-time channel. For a continuous-time channel with bandwidth W Hz available for transmission, the discrete-time model contains W two-dimensional symbols per second (2D/s).

If the transmitting terminals have an average power constraint in the continuous-time channel model of P_c Joules/s, we see that this translates into a discrete-time power constraint of $P = 2P_c/W$ Joules/2D since each terminal transmits in a fraction $1/2$ of the available degrees of freedom (*cf.* Fig. 1). Thus, the channel model is parameterized by the SNR random variables $\text{SNR} |a_{i,j}|^2$, where

$$\text{SNR} = \frac{2P_c}{N_0 W} = \frac{P}{N_0} \quad (4)$$

is the SNR without fading.

In addition to SNR, transmission schemes are further parameterized by the spectral efficiency R b/s/Hz attempted by the transmitting terminals. Note that throughout the paper R is the transmission rate normalized by the number of degrees of freedom utilized by each terminal, not by the total number of degrees of freedom in the channel.

Nominally, one could parameterize the system by the pair (SNR, R) ; however, our results lend more insight, and are substantially more compact, when we parameterize the system by $(\text{SNR}, R_{\text{norm}})$, where¹

$$R_{\text{norm}} = R / \log(1 + \text{SNR}) . \quad (5)$$

III. SPACE-TIME CODED COOPERATIVE DIVERSITY

We now develop and analyze a decode-and-forward based class of cooperative diversity protocols that we call *space-time*

coded cooperative diversity. As we alluded in Section I, such protocols consist of the source broadcasting its transmission to its destination and potential relays. Potential relays that can decode the transmission become decoding relays and participate in the second phase of the protocol. Although the set of decoding relays $\mathcal{D}(s)$ is a random set, we will see that protocols of this form offer full spatial diversity in the number of cooperating terminals, not just the number of decoding relays participating in the second phase. Interestingly, potential relays that cannot decode contribute as much to the performance of the protocol as the decoding relays.

A. Mutual Information and Outage Probability

Since the channel average mutual information I is a function of, *e.g.*, the coding scheme, the rule for including potential relays into the decoding set $\mathcal{D}(s)$, and the fading coefficients of the channel, it too is a random variable. The event $I < R$ that this mutual information random variable falls below some fixed spectral efficiency R is referred to as an *outage event*, because reliable communication is not possible for realizations in this event. The probability of an outage event, $\Pr[I < R]$, is referred to as the *outage probability* of the channel.

Since $\mathcal{D}(s)$ is a random set, we first use the total probability law to write

$$\Pr[I < R] = \sum_{\mathcal{D}(s)} \Pr[\mathcal{D}(s)] \Pr[I < R | \mathcal{D}(s)] , \quad (6)$$

and we examine each term in the summation.

1) *Outage Conditional on Decoding Set:* Conditioned on $\mathcal{D}(s)$ being the decoding set, the mutual information between s and $d(s)$ for random codebooks generated i.i.d. circularly-symmetric, complex Gaussian at the source and all potential relays can be shown to be

$$\frac{1}{2} \log(1 + \text{SNR} |a_{s,d(s)}|^2) + \frac{1}{2} \log(1 + \text{SNR} \sum_{r \in \mathcal{D}(s)} |a_{r,d(s)}|^2) , \quad (7)$$

the sum of the mutual informations for two “parallel” channels, one from the source to the destination, and one from the set of decoding relays to the destination. Thus $\Pr[I < R | \mathcal{D}(s)]$ involves $|\mathcal{D}(s)| + 1$ independent fading coefficients, so we might expect it to decay asymptotically proportional to $1/\text{SNR}^{|\mathcal{D}(s)|+1}$. Indeed, while we leave out the details due to space considerations, [4] develops the high SNR approximation²

$$\begin{aligned} \Pr[I < R | \mathcal{D}(s)] &\sim \left[\frac{2^{2R} - 1}{\text{SNR}} \right]^{|\mathcal{D}(s)|+1} \\ &\times \lambda_{s,d(s)} \prod_{r \in \mathcal{D}(s)} \lambda_{r,d(s)} \\ &\times A_{|\mathcal{D}(s)|} (2^{2R} - 1) , \end{aligned} \quad (8)$$

²The approximation $f(\text{SNR}) \sim g(\text{SNR})$ is in the sense of $f(\text{SNR})/g(\text{SNR}) \rightarrow 1$ as $\text{SNR} \rightarrow \infty$.

¹Unless otherwise indicated, logarithms in this paper are taken to base 2.

where

$$A_n(t) = \frac{1}{(n-1)!} \int_0^1 \frac{w^{(n-1)}(1-w)}{(1+tw)} dw, \quad (9)$$

for $n > 0$, and $A_0(t) = 1$. Note that we have expressed (8) in such a way that the first term captures the dependence upon SNR and the second term captures the dependence upon $\{\lambda_{i,j}\}$.

2) *Decoding Set Probability*: Next, we consider the term $\Pr[\mathcal{D}(s)]$, the probability of a particular decoding set. As one rule for selecting from the potential relays, we can require that a potential relay fully decode the source message in order to participate in the second phase. Indeed, full decoding is required in order for the mutual information expression (7) to be correct; however, nothing prevents us from imposing additional restrictions on the members of the set $\mathcal{D}(s)$. For example, we might require that a potential relay fully decode *and* see a realized SNR some factor larger than its average.

Since the realized mutual information between s and r for i.i.d. complex Gaussian codebooks is given by

$$\frac{1}{2} \log(1 + \text{SNR} |a_{s,r}|^2),$$

we have under this rule

$$\begin{aligned} \Pr[r \in \mathcal{D}(s)] &= \Pr[|a_{s,r}|^2 > (2^{2R} - 1)/\text{SNR}] \\ &= \exp[-\lambda_{s,r}(2^{2R} - 1)/\text{SNR}]. \end{aligned}$$

Moreover, since each potential relay makes this decision independently, and the fading coefficients are independent under our model, we have

$$\begin{aligned} \Pr[\mathcal{D}(s)] &= \prod_{r \in \mathcal{D}(s)} \exp[-\lambda_{s,r}(2^{2R} - 1)/\text{SNR}] \\ &\quad \times \prod_{r \notin \mathcal{D}(s)} (1 - \exp[-\lambda_{s,r}(2^{2R} - 1)/\text{SNR}]) \\ &\sim \left[\frac{2^{2R} - 1}{\text{SNR}} \right]^{m - |\mathcal{D}(s)| - 1} \times \prod_{r \notin \mathcal{D}(s)} \lambda_{s,r}. \end{aligned} \quad (10)$$

Note that, any selection means through which $\Pr[r \in \mathcal{D}(s)] \sim 1$ and $(1 - \Pr[r \in \mathcal{D}(s)]) \propto 1/\text{SNR}$, for SNR large, independently for each r , will result in similar behavior for $\Pr[\mathcal{D}(s)]$.

Combining (8) and (10) in to (6), we obtain

$$\begin{aligned} \Pr[I < R] &\sim \left[\frac{2^{2R} - 1}{\text{SNR}} \right]^m \\ &\quad \times \sum_{\mathcal{D}(s)} \lambda_{s,d(s)} \prod_{r \in \mathcal{D}(s)} \lambda_{r,d(s)} \prod_{r \notin \mathcal{D}(s)} \lambda_{s,r} \\ &\quad \times A_{|\mathcal{D}(s)|}(2^{2R} - 1). \end{aligned} \quad (11)$$

B. Convenient Bounds

While the approximation given in (11) is quite general and can be numerically evaluated to determine performance, it is

not very convenient for further analysis. There are two factors contributing to its complexity: dependence upon $\{\lambda_{i,j}\}$, and the unwieldy closed-form expression for $A_n(t)$ as n grows. In this section, we developed upper and lower bounds for (11) that we exploit in the sequel.

Our objective is simplify the summation in (11). To this end, we note that for a given decoding set $\mathcal{D}(s)$, either $r \in \mathcal{D}(s)$, in which case $\lambda_{r,d(s)}$ appears in the corresponding term in (11), or $r \notin \mathcal{D}(s)$, in which case $\lambda_{s,d(s)}$ appears in the corresponding term in (11). We therefore define

$$\underline{\lambda}_r = \min\{\lambda_{r,d(s)}, \lambda_{s,r}\}, \quad \bar{\lambda}_r = \max\{\lambda_{r,d(s)}, \lambda_{s,r}\}, \quad (12)$$

and $\bar{\lambda}_s = \underline{\lambda}_s = \lambda_{s,d(s)}$. Then the product dependent upon $\{\lambda_{i,j}\}$ is bounded by

$$\underline{\lambda}^m \leq \lambda_{s,d(s)} \prod_{r \in \mathcal{D}(s)} \lambda_{r,d(s)} \prod_{r \notin \mathcal{D}(s)} \lambda_{s,r} \leq \bar{\lambda}^m, \quad (13)$$

where $\underline{\lambda}$ is the geometric mean of the $\underline{\lambda}_i$ and $\bar{\lambda}$ is the geometric mean of the $\bar{\lambda}_i$, for $i \in \mathcal{M}$. We note that the upper and lower bounds in (13) are independent of $\mathcal{D}(s)$. We also note that the bounds in (13) coincide, i.e., $\bar{\lambda} = \underline{\lambda}$, if (though not only if) $\bar{\lambda}_i = \underline{\lambda}_i$ for all $i \in \mathcal{M}$. Viewing $\lambda_{i,j}$ as a measure of distance between terminals i and j , the class of network geometries in two dimensions that satisfy this condition are those in which all the relays lie with arbitrary spacing along the perpendicular bisector between the source and destination. A more general study of the effects of such network geometry on performance is beyond the scope of this paper.

To avoid dealing with (9), we exploit the bounds

$$\frac{1}{(n+1)!(1+t)} \leq A_n(t) \leq \frac{1}{n!}. \quad (14)$$

Combining (13) and (14) into (11), we arrive at the following simplified bounds for outage probability

$$\Pr[I < R] \geq \left[\frac{2^{2R} - 1}{\text{SNR}/\underline{\lambda}} \right]^m 2^{-2R} \sum_{\mathcal{D}(s)} \frac{1}{(|\mathcal{D}(s)| + 1)!} \quad (15)$$

$$\Pr[I < R] \leq \left[\frac{2^{2R} - 1}{\text{SNR}/\bar{\lambda}} \right]^m \sum_{\mathcal{D}(s)} \frac{1}{|\mathcal{D}(s)|!}. \quad (16)$$

C. Diversity-Multiplexing Tradeoff

An interesting tradeoff between diversity and multiplexing arises when we parameterize our results in terms of $(\text{SNR}, R_{\text{norm}})$, with R_{norm} given in (5). Specifically, when we approximate $\Pr[I < R] \doteq \text{SNR}^{-\Delta(R_{\text{norm}})}$, in the sense of equality to first-order in the exponent, i.e.,

$$\Delta(R_{\text{norm}}) = \lim_{\text{SNR} \rightarrow \infty} -\frac{\log(\Pr[I < R])}{\log(\text{SNR})}, \quad (17)$$

we find that increasing R_{norm} reduces Δ . This tradeoff was originally observed in the context of multi-antenna systems [8],

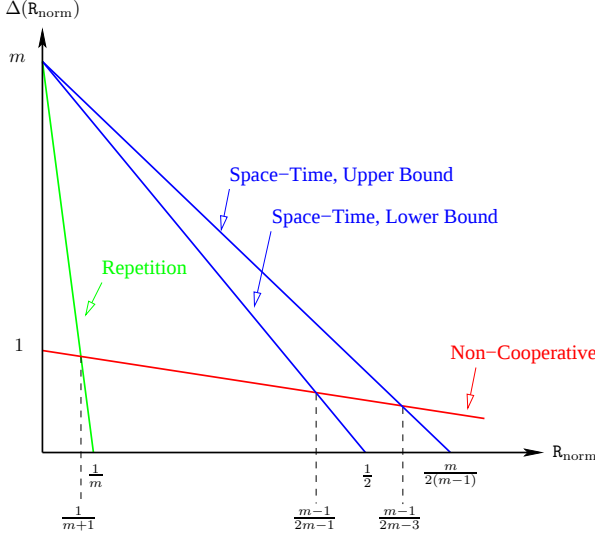


Fig. 3. Diversity order $\Delta(R_{\text{norm}})$ for non-cooperative transmission (red), repetition-coded cooperative diversity (green), and space-time coded cooperative diversity (blue, bounds from (18)). As $R_{\text{norm}} \rightarrow 0$, all cooperative diversity protocols provide full spatial diversity order m , the number of cooperating terminals. Relative to direct transmission, space-time coded cooperative diversity can be effectively utilized for a much broader range of R_{norm} than repetition-coded cooperative diversity, especially as m becomes large.

so it is not surprising that it also arise in the context of cooperative diversity [4].

Utilizing our lower and upper bounds (15)-(16) in (17) yields upper and lower bounds, respectively, on the diversity order

$$m(1 - 2R_{\text{norm}}) \leq \Delta(R_{\text{norm}}) \leq m \left(1 - \left\lfloor \frac{m-1}{m} \right\rfloor 2R_{\text{norm}} \right) \quad (18)$$

Fig. 3 compares these bounds, along with the corresponding tradeoffs for non-cooperative transmission, $\Delta(R_{\text{norm}}) = 1 - R_{\text{norm}}$, and repetition-based cooperative diversity $\Delta(R_{\text{norm}}) = m(1 - mR_{\text{norm}})$. Clearly, space-time coded cooperative diversity offers larger diversity order than repetition-based algorithms and can be effectively utilized for higher spectral efficiencies than repetition-based schemes.

IV. PRACTICAL ISSUES

A. Space-Time Code Design

The outage analysis in Section III relies on a random coding argument, and demonstrates that full spatial diversity can be achieved using such a rich set of codes. In practice, one may wonder whether or not there exist space-time codes for which the number of participating antennas is not known *a priori* and yet full diversity can be achieved. More specifically, if we design a space-time code for a maximum of N transmit antennas, but only a randomly selected subset of n of those antennas actually transmit, can the space-time code offer diversity n ? It turns out that the class of space-time block codes based upon orthogonal designs have this property [9]. Essentially, these codes have orthogonal waveforms emitted from each antenna,

corresponding to columns in a code matrix. Absence of an antenna corresponds to deletion of a column in the matrix, but the columns remain orthogonal, allowing the code to maintain its diversity benefits. Thus space-time coded cooperative diversity protocols may be readily deployed in practice using these codes.

B. Distributed Implementation

Given a suitably designed space-time code, space-time coded cooperative diversity reduces to a simple, distributed network protocol. When each terminal transmits its message, the other terminals receive and potentially decode, requiring only an SNR measurement. If a relay can decode, it transmits the information in the second phase using its column from the space-time code matrix. Because the destination receiver can measure the fading, it can determine which relays are involved in the second phase and adapt its decoding rule appropriately. Although certainly the terminals could exchange more information in order to adapt power to the network geometry, for example, such overhead is not required in order to obtain full diversity.

One of the key challenges to implementing such a protocol could be block and symbol synchronization of the cooperating terminals. Such synchronization might be obtained through periodic transmission of known synchronization prefixes, as proposed in current wireless LAN standards. A detailed study of issues involved with synchronization is beyond the scope of the present paper.

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