

Disjunctive Programming

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 Springer

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ISBN 978-3-030-00147-6 ISBN 978-3-030-00148-3 (eBook)
<https://doi.org/10.1007/978-3-030-00148-3>

Library of Congress Control Number: 2018957148

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Preface

This book is meant for students and practitioners of optimization, mainly integer and nonconvex optimization, as an introduction to, and review of, the recently developed discipline of disjunctive programming. It should be of interest to all those who are trying to overcome the limits set by convexity to our problem-solving capability. Disjunctive programming is optimization over disjunctive sets, the first large class of nonconvex sets shown to be convexifiable in polynomial time.

There are no prerequisites for the understanding of this book, other than some knowledge of the basics of linear and integer optimization. Clarity of exposition was a main objective in writing it. Where background material is required, it is indicated through references.

Habent sua fata libelli, goes the Latin saying: *books have their own fate*. The basic document on Disjunctive Programming is the July 1974 technical report “Disjunctive Programming: Properties of the convex hull of feasible points. MSRR #348”, which however has not appeared in print until 24 years later, when it was published as an invited paper with a remarkable foreword by Gerard Cornuejols and Bill Pulleyblank [7] (for a history of this episode see the introduction to [11]). The new theory, which was laid out without implementation of its cutting planes, let alone computational experience, stirred little if any enthusiasm at the time of its inception. But 15 years later, when Sebastian Ceria, Gerard Cornuejols and myself recast essentially the same results in a new framework which we called lift-and-project [19], the reaction was quite different. This time our work was focused on algorithmic aspects, with the cutting planes generated in rounds and embedded into an enumerative (branch and cut) framework, and was accompanied by an efficient computer code, MIPO, that was able to solve many problem instances that had been impervious to solution by branch-and-bound alone. This has led to a general effort on the part of software builders to incorporate cutting planes into a new generation of integer programming solvers, with the outcome known as the revolution in the state of the art in mixed integer programming that took place roughly in the period 1990–2005.

From a broader perspective, disjunctive programming is one of the early bridges between convex and nonconvex programming.

Several friends and colleagues have contributed to the improvement of this book through their comments and observations. Their list includes David Bernal, Dan Bienstock, Gérard Cornuéjols, Ignacio Grossmann, Michael Juenger, Alex Kazachkov, Tamas Kis, Andrea Qualizza, Thiago Serra, and an anonymous editor of the Springer Series Algorithms and Combinatorics.

The research underlying the results reported on in this book was supported generously throughout the last four decades by the National Science Foundation and the US Office of Naval Research. The writing of the book itself was supported by NSF Grant 1560828 and ONR Contract N000141512082.

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Contents

1	Disjunctive Programming and Its Relation to Integer Programming	1
1.1	Introduction	1
1.2	Intersection Cuts	2
1.3	Inequality Systems with Logical Connectives	6
1.4	Valid Inequalities for Disjunctive Sets	9
1.5	Duality for Disjunctive Programs	12
2	The Convex Hull of a Disjunctive Set	17
2.1	The Convex Hull Via Lifting and Projection	17
2.1.1	Tightness of the Lifted Representation	22
2.1.2	From the Convex Hull to the Union Itself	23
2.2	Some Facts About Projecting Polyhedra	26
2.2.1	Well Known Special Cases	28
2.2.2	Dimensional Aspects of Projection	29
2.2.3	When Is the Projection of a Facet a Facet of the Projection?	29
2.3	Projection with a Minimal System of Inequalities	31
2.4	The Convex Hull Via Polarity	31
3	Sequential Convexification of Disjunctive Sets	41
3.1	Faciality as a Sufficient Condition	42
3.2	A Necessary Condition for Sequential Convexifiability	46
4	Moving Between Conjunctive and Disjunctive Normal Forms	49
4.1	The Regular Form and Basic Steps	49
4.2	The Hull Relaxation and the Associated Hierarchy	51
4.3	When to Convexify a Subset	54
4.4	Parsimonious MIP Representation of Disjunctive Sets	59

4.5	An Illustration: Machine Sequencing Via Disjunctive Graphs	61
4.5.1	A Disjunctive Programming Formulation	64
4.5.2	A Tighter Disjunctive Programming Formulation	65
4.6	Disjunctive Programs with Trigger Variables	67
5	Disjunctive Programming and Extended Formulations	69
5.1	Comparing the Strength of Different Formulations	70
5.1.1	The Traveling Salesman Problem	71
5.1.2	The Set Covering Problem	72
5.1.3	Nonlinear 0-1 Programming	73
5.2	Proving the Integrality of Polyhedra	74
5.2.1	Perfectly Matchable Subgraphs of a Bipartite Graph	74
5.2.2	Assignable Subgraphs of a Digraph	76
5.2.3	Path Decomposable Subgraphs of an Acyclic Digraph ...	76
5.2.4	Perfectly Matchable Subgraphs of an Arbitrary Graph ...	77
6	Lift-and-Project Cuts for Mixed 0-1 Programs	79
6.1	Disjunctive Rank	80
6.2	Fractionality of Intermediate Points	81
6.3	Generating Cuts	83
6.4	Cut Lifting	83
6.5	Cut Strengthening	86
6.6	Impact on the State of the Art in Integer Programming	87
7	Nonlinear Higher-Dimensional Representations	91
7.1	Another Derivation of Lift-and-Project Cuts	91
7.2	The Lovász-Schrijver Construction	93
7.3	The Sherali-Adams Construction	94
7.4	Lasserre's Construction	95
7.5	The Bienstock-Zuckerberg Lift Operator	95
8	The Correspondence Between Lift-and-Project Cuts and Simple Disjunctive Cuts	97
8.1	Feasible Bases of the CGLP Versus (Feasible or Infeasible) Bases of the LP	98
8.2	The Correspondence Between the Strengthened Cuts	103
8.3	Bounds on the Number of Essential Cuts	103
8.4	The Rank of P with Respect to Different Cuts	104
9	Solving (CGLP)$_k$ on the LP Simplex Tableau	107
9.1	Computing Reduced Costs of (CGLP) $_k$ Columns for the LP Rows	109
9.2	Computing Evaluation Functions for the LP Columns	113
9.3	Generating Lift-and-Project Cuts by Pivoting in the LP Tableau	116
9.4	Using Lift-and-Project to Choose the Best Mixed Integer Gomory Cut	117

- 10 Implementation and Testing of Variants** 121
 - 10.1 Pivots in the LP Tableau Versus Block Pivots in the CGLP Tableau 122
 - 10.2 Most Violated Cut Selection Rule 126
 - 10.3 Iterative Disjunctive Modularization 127
 - 10.4 Computational Results 129
 - 10.5 Testing Alternative Normalizations 135
 - 10.6 The Interplay of Normalization and the Objective Function 136
 - 10.7 Bonami’s Membership LP 139
 - 10.8 The Split Closure 141
- 11 Cuts from General Disjunctions** 145
 - 11.1 Intersection Cuts from Multiple Rows 145
 - 11.2 Standard Versus Restricted Intersection Cuts 149
 - 11.3 Generalized Intersection Cuts 152
 - 11.4 Generalized Intersection Cuts and Lift-and-Project Cuts 153
 - 11.5 Standard Intersection Cuts and Lift-and-Project Cuts 155
 - 11.6 The Significance of Irregular Cuts 163
 - 11.7 A Numerical Example 166
 - 11.8 Strengthening Cuts from General Disjunctions 169
 - 11.9 Stronger Cuts from Weaker Disjunctions 179
 - 11.9.1 Simple Split Disjunction 183
 - 11.9.2 Multiple Term Disjunctions 187
 - 11.9.3 Strictly Weaker Disjunctions 192
- 12 Disjunctive Cuts from the V -Polyhedral Representation** 195
 - 12.1 V -Polyhedral Cut Generator Constructed Iteratively 197
 - 12.1.1 Generating Adjacent Extreme Points 199
 - 12.1.2 How to Generate a Facet of $\text{conv } F$ in n Iterations 201
 - 12.1.3 Cut Lifting 203
 - 12.1.4 Computational Testing 204
 - 12.2 Relaxation-Based V -Polyhedral Cut Generators 205
 - 12.3 Harnessing Branch-and-Bound Information for Cut Generation 208
- 13 Unions of Polytopes in Different Spaces** 215
 - 13.1 Dominants of Polytopes and Upper Separation 215
 - 13.2 The Dominant and Upper Separation for Polytopes in $[0, 1]^n$ 219
 - 13.2.1 Unions of Polytopes in Disjoint Spaces 220
 - 13.3 The Upper Monotone Case 221
 - 13.3.1 $\text{conv}(Z)$: The General Case 222
 - 13.4 Application 1: Monotone Set Functions and Matroids 223

13.5	Application 2: Logical Inference	225
13.6	More Complex Logical Constraints	227
13.7	Unions of Upper Monotone Polytopes in the Same Space	228
13.8	Unions of Polymatroids	230
References	233