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COMPUTATIONALLY EFFICIENT SUBSPACE-BASED METHOD FOR DIRECTION ESTIMATION AND TRACKING IN ARRAY PROCESSING

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ABSTRACT

In this paper, we propose a new computationally efficient subspace-based method without eigendecomposition (SUMWE) for the direction-of-arrival (DOA) estimation of narrowband signals impinging on a uniform linear array (ULA) by exploiting the array geometry and its shift invariance property. Further an adaptive implementation of the SUMWE is presented for tracking the time-varying directions of slowly moving (relative to the sampling rate) signals. The effectiveness of the proposed algorithm is verified through numerical examples, and it is shown that the proposed algorithm is computationally simple and has a good tracking performance.

1. INTRODUCTION

The directions-of-arrival (DOAs) estimation of signals impinging on an array of sensors is a fundamental problem in array processing, and a computationally simple direction estimation method with good statistical performance is much attractive in most practical applications. Although subspace-based methods have received widely attention because of their relatively high resolution and computational simplicity (e.g. [1], [2]), most of these methods require an eigenvalue decomposition (EVD) or singular value decomposition (SVD) to estimate the signal or noise (null) subspace. Unfortunately, the eigendecomposition is computationally intensive and time-consuming [3], especially when the number of array sensors is large. Therefore, these methods are usually limited in many practical situations where we need to track the DOAs of moving signals, because they require repeated EVD/SVD to update the signal/noise subspace with the acquisition of new data and the deletion of the old data.

For alleviating the difficulty of subspace-based methods, some computationally simple subspace-based direction estimation methods without eigendecomposition have been developed [4]-[7]. In linear operation based methods such as the BEWE [4], OPM [5], and SWEDE [6], the signal or noise (null) subspace is easily obtained from the array data relying on a partition of array response matrix, and then the directions are estimated in a manner similar to that of the MUSIC [2]. However, their accuracy is generally poorer than that of the conventional subspace-based methods (e.g. MUSIC) from the statistical viewpoint [8], [6], [5]. Although the WSF-E [7] achieves the asymptotic efficiency when either the number of snapshots or the signal-to-noise ratio (SNR) is large, it is computationally much more complicated than linear operation based algorithms. Furthermore, most of these computationally simple subspace-based methods suffer serious degradation when the incident signals are coherent (i.e. fully correlated) in some practical scenarios due to multipath propagation. Even the WSF-E and a variant of BEWE can resolve the coherent signals, their performance degrades severely at low SNR and with a small number of snapshots.

Therefore in this paper, we propose a new computationally efficient subspace-based method without eigendecomposition (SUMWE) for the DOA estimation of coherent narrowband signals impinging on a uniform linear array (ULA) by exploiting the array geometry and its shift invariance property. The SUMWE does not require the computationally cumbersome eigendecomposition and the evaluation of all correlations of the array data, and the effect of additive noise is eliminated. Further the SUMWE has a remarkable insensitivity to the correlation between the incident signals, and it can be extended to the spatially correlated noise by choosing appropriate subarrays (i.e. cross-correlations of array data). Moreover, an adaptive implementation of the SUMWE is presented for tracking the directions of slowly moving (relative to the sampling rate) signals. The performance of the presented method is verified through numerical examples, and the simulation results show that the proposed algorithm is computationally simple and has a good tracking performance.

2. DATA MODEL AND BASIC ASSUMPTIONS

Consider a ULA of M identical and omnidirectional sensors with spacing d, and suppose that p narrowband signals $\{s_k(n)\}$ with the centre frequency f_0 are in the field far from the array and impinge on the array from distinct directions $\{\theta_k(n)\}$. Under the narrowband assumption, the received noisy signal $y_i(n)$ at the i th sensor can be expressed as [1], [2], [4]-[10]

$$y_i(n) = x_i(n) + w_i(n) \tag{1}$$

$$x_i(n) = \sum_{k=1}^{p} s_k(n) e^{j\omega_0 (i-1)\tau(\theta_k(n))} \tag{2}$$

where $x_i(n)$ is the noiseless received signal, $w_i(n)$ is the additive noise, $\omega_0 \triangleq 2\pi f_0$, $\tau(\theta_k(n)) \triangleq (d/c)\sin\theta_k(n)$, c is the propagation speed, and $\{\theta_k(n)\}$ are measured relative to the normal of array. The received signals can be reexpressed more compactly as

$$\boldsymbol{y}(n) = \boldsymbol{A}(\theta)\boldsymbol{s}(n) + \boldsymbol{w}(n) \tag{3}$$

where $\boldsymbol{y}(n)$, $\boldsymbol{s}(n)$, and $\boldsymbol{w}(n)$ are the vectors of the received signals, the incident signals, and the additive noise, $\boldsymbol{A}(\theta) \triangleq [\boldsymbol{a}(\theta_1(n)), \boldsymbol{a}(\theta_2(n)), \cdots, \boldsymbol{a}(\theta_p(n))]$ with $\boldsymbol{a}(\theta_k(n)) \triangleq [1, e^{j\omega_0\tau(\theta_k(n))}, \cdots, e^{j\omega_0(M-1)\tau(\theta_k(n))}]^T$, and $(\cdot)^T$ denotes the transpose.

In this paper, we make the following basic assumptions.

- The array is calibrated and the array response matrix $A(\theta)$ is unambiguous. Equivalently $A(\theta)$ has full rank.
- Without loss of generality, the signals $\{s_k(n)\}$ are all coherent. Under the flat-fading multipath propagation, they can be expressed as [9], [10]

$$s_k(n) = \beta_k s_1(n) \quad (4)$$

for $k = 1, 2, \cdots, p$, where β_k is the complex attenuation coefficient with $\beta_k \neq 0$ and $\beta_1 = 1$.

- The incident signal $s_1(n)$ is a temporally complex white Gaussian random process with zero-mean and the variance given by

$$E\{s_1(n)s_1^*(t)\} = r_s\, \delta_{n,t}, \quad E\{s_1(n)s_1(t)\} = 0 \quad (5)$$

where $E\{\cdot\}$, $(\cdot)^*$, and $\delta_{n,t}$ denote the expectation, the complex conjugate, and Kronecker delta.

- The additive noise $\{w_i(n)\}$ is a temporally and spatially complex white Guassian random process with zero-mean and the following covariance matrix

$$E\{\boldsymbol{w}(n)\boldsymbol{w}^H(t)\} = \sigma^2 \boldsymbol{I}_M\, \delta_{n,t}, \quad E\{\boldsymbol{w}(n)\boldsymbol{w}^T(t)\} = \boldsymbol{O}_{M\times M} \quad (6)$$

where $\boldsymbol{I}_m$, $\boldsymbol{O}_{m\times q}$, and $(\cdot)^H$ indicate the $m\times m$ identity matrix, the $m \times q$ null matrix, and Hermitian transpose. And the noise is uncorrelated with the incident signals.

- The number of incident signals p is known or estimated by some proposed techniques (e.g. [10] and references therein), and it satisfies the inequality that $p < M/2$ for an array of M sensors.

3. SUBSPACE-BASED METHOD WITHOUT EIGENDECOMPOSITION — SUMWE

3.1 Derivation of SUMWE

In this section, we consider the estimation of constant directions of coherent signals, where $\theta_k(n) = \theta_k$. From (3), we have the array covariance matrix $\boldsymbol{R}$ as

$$\boldsymbol{R} \triangleq E\{\boldsymbol{y}(n)\boldsymbol{y}^H(n)\} = \boldsymbol{A}(\theta)\boldsymbol{R}_s\boldsymbol{A}^H(\theta) + \sigma^2\boldsymbol{I}_M \quad (7)$$

where $\boldsymbol{R}_s \triangleq E\{\boldsymbol{s}(n)\boldsymbol{s}^H(n)\}$. By defining the correlation r_{ik} between the signals $y_i(n)$ and $y_k(n)$ as $r_{ik} \triangleq E\{y_i(n) \cdot y_k^*(n)\}$, where $r_{ik} = r_{ki}^*$, we find that the diagonal elements $\{r_{kk}\}$ of $\boldsymbol{R}$ are affected by the noise variance σ^2.

Now by dividing the full array into L overlapping subarrays with p sensors in the forward and backward directions [9], [11], where $L = M - p + 1$, the signals in the l th forward subarray and the conjugate signals in the l th backward subarray can be expressed compactly [10], [12]

$$\boldsymbol{y}_{fl}(n) \triangleq [y_l(n), \cdots, y_{l+p-1}(n)]^T = \boldsymbol{A}_1\boldsymbol{D}^{l-1}\boldsymbol{s}(n) + \boldsymbol{w}_{fl}(n) \quad (8)$$

$$\boldsymbol{y}_{bl}(n) \triangleq [y_{M-l+1}(n), \cdots, y_{L-l+1}(n)]^H = \boldsymbol{A}_1\boldsymbol{D}^{-(M-l)}\boldsymbol{s}^*(n) + \boldsymbol{w}_{bl}(n) \quad (9)$$

for $l = 1, 2, \cdots, L$, where $\boldsymbol{w}_{fl}(n) \triangleq [w_l(n), w_{l+1}(n), \cdots, w_{l+p-1}(n)]^T$, $\boldsymbol{w}_{bl}(n) \triangleq [w_{M-l+1}(n), w_{M-l}(n), \cdots, w_{L-l+1}(n)]^H$, $\boldsymbol{D} \triangleq \mathrm{diag}(e^{j\omega_0\tau(\theta_1)}, e^{j\omega_0\tau(\theta_2)}, \cdots, e^{j\omega_0\tau(\theta_p)})$, and $\boldsymbol{A}_1$ is the submatrix of $A(\theta)$ in (3) consisting of the first p rows with the column $\boldsymbol{a}_1(\theta_k) \triangleq [1, e^{j\omega_0\tau(\theta_k)}, \cdots, e^{j\omega_0(p-1)\tau(\theta_k)}]^T$. By defining four correlation vectors as $\varphi_{fl} \triangleq E\{\boldsymbol{y}_{fl}(n)y_M^*(n)\}$, $\bar{\varphi}_{fl} \triangleq E\{\boldsymbol{y}_{fl}(n)y_1^*(n)\}$, $\varphi_{bl} \triangleq E\{y_1(n) \cdot \boldsymbol{y}_{bl}(n)\}$, and $\bar{\varphi}_{bl} \triangleq E\{\boldsymbol{y}_{bl}(n)y_M(n)\}$, we obtain four Hankel correlation matrices by some algebraic manipulations [12]

$$\boldsymbol{\Phi}_f \triangleq [\varphi_{f1}, \varphi_{f2}, \cdots, \varphi_{fL-1}]^T = \rho_M\, r_s \bar{\boldsymbol{A}}\boldsymbol{B}\boldsymbol{A}_1^T \quad (10)$$

$$\bar{\boldsymbol{\Phi}}_f \triangleq [\bar{\varphi}_{f2}, \bar{\varphi}_{f3}, \cdots, \bar{\varphi}_{fL}]^T = \rho_1\, r_s \bar{\boldsymbol{A}}\boldsymbol{B}\boldsymbol{D}\boldsymbol{A}_1^T \quad (11)$$

$$\boldsymbol{\Phi}_b \triangleq [\varphi_{b1}, \varphi_{b2}, \cdots, \varphi_{bL-1}]^T = \rho_1^*\, r_s \bar{\boldsymbol{A}}\boldsymbol{B}^*\boldsymbol{D}^{-(M-1)}\boldsymbol{A}_1^T \quad (12)$$

$$\bar{\boldsymbol{\Phi}}_b \triangleq [\bar{\varphi}_{b2}, \bar{\varphi}_{b3}, \cdots, \bar{\varphi}_{bL}]^T = \rho_M^*\, r_s \bar{\boldsymbol{A}}\boldsymbol{B}^*\boldsymbol{D}^{-(M-2)}\boldsymbol{A}_1^T \quad (13)$$

where $\bar{\boldsymbol{A}}$ is the $(M-p)\times p$ submatrix of the matrix $\boldsymbol{A}$ in (3) consisting of its first $L-1$ rows with the column $\bar{\boldsymbol{a}}(\theta_k) = [1, e^{j\omega_0\tau(\theta_k)}, \cdots, e^{j\omega_0(L-2)\tau(\theta_k)}]^T$, $\boldsymbol{B} \triangleq \mathrm{diag}(\beta_1, \beta_2, \cdots, \beta_p)$, $\rho_i \triangleq \boldsymbol{\beta}^H\boldsymbol{b}_i^*(\theta)$, $\boldsymbol{\beta} \triangleq [\beta_1, \beta_2, \cdots, \beta_p]^T$, and $\boldsymbol{b}_i(\theta) \triangleq [e^{j\omega_0(i-1)\tau(\theta_1)}, e^{j\omega_0(i-1)\tau(\theta_2)}, \cdots, e^{j\omega_0(i-1)\tau(\theta_p)}]^T$.

Clearly the Hankel correlation matrices in (10)-(13) are not affected by the additive noise, and $\boldsymbol{\Phi}_b = \boldsymbol{J}_{M-p}\bar{\boldsymbol{\Phi}}_f^*\boldsymbol{J}_p$ and $\bar{\boldsymbol{\Phi}}_b = \boldsymbol{J}_{M-p}\boldsymbol{\Phi}_f^*\boldsymbol{J}_p$, where $\boldsymbol{J}_m$ is an $m\times m$ counteridentity matrix. Further these matrices can be just formed from the elements $\{r_{i1}\}$ and $\{r_{iM}\}$ in the 1st and M th columns of array covariance matrix $\boldsymbol{R}$ in (7) except for the auto-correlations r_{11} and r_{MM}, which contain the noise variance σ^2. From the assumptions, we can find that the ranks of theses $(M-p)\times p$ Hankel correlation matrices equal p, i.e. the dimension of their signal subspace equals to the number of coherent signals.

Because it is assumed that $M > 2p$ (i.e. $L-1 > p$), from the definition of the matrices $\boldsymbol{A}_1$ and $\bar{\boldsymbol{A}}$, we can partition the $(M-p)\times p$ matrix $\bar{\boldsymbol{A}}$ and hence the correlation matrices in (10)-(13) into two submatrices as

$$\bar{\boldsymbol{A}} \triangleq \begin{bmatrix} \boldsymbol{A}_1 \\ \boldsymbol{A}_2 \end{bmatrix} \begin{matrix} \}p \\ \}M-2p \end{matrix}, \quad \boldsymbol{\Phi}_f \triangleq \begin{bmatrix} \boldsymbol{\Phi}_{f1} \\ \boldsymbol{\Phi}_{f2} \end{bmatrix} \begin{matrix} \}p \\ \}M-2p \end{matrix} \quad (14)$$

$$\bar{\boldsymbol{\Phi}}_f \triangleq \begin{bmatrix} \bar{\boldsymbol{\Phi}}_{f1} \\ \bar{\boldsymbol{\Phi}}_{f2} \end{bmatrix} \begin{matrix} \}p \\ \}M-2p \end{matrix} \quad \boldsymbol{\Phi}_b \triangleq \begin{bmatrix} \boldsymbol{\Phi}_{b1} \\ \boldsymbol{\Phi}_{b2} \end{bmatrix} \begin{matrix} \}p \\ \}M-2p \end{matrix}, \quad \bar{\boldsymbol{\Phi}}_b \triangleq \begin{bmatrix} \bar{\boldsymbol{\Phi}}_{b1} \\ \bar{\boldsymbol{\Phi}}_{b2} \end{bmatrix} \begin{matrix} \}p \\ \}M-2p \end{matrix} \quad (15)$$

Under the model assumptions, we can find that $\bar{\boldsymbol{A}}$ and $\boldsymbol{A}_1$ are of full rank and the rows of $\boldsymbol{A}_2$ can be expressed as a linear combination of linearly independent rows of $\boldsymbol{A}_1$; i.e. there is a linear operator $\boldsymbol{P}$ between $\boldsymbol{A}_1$ and $\boldsymbol{A}_2$ [5]

$$\boldsymbol{P}^H\boldsymbol{A}_1 = \boldsymbol{A}_2. \quad (16)$$

Consequently from (10)-(15), the relation between $\boldsymbol{A}_1$ and $\boldsymbol{A}_2$ can be expressed as one between the submatrices of $\boldsymbol{\Phi}_f$, $\bar{\boldsymbol{\Phi}}_f$, $\boldsymbol{\Phi}_b$, and $\bar{\boldsymbol{\Phi}}_b$ as

$$\boldsymbol{P}^H\boldsymbol{\Phi}_1 = \boldsymbol{\Phi}_2, \quad \text{i.e.} \quad \boldsymbol{Q}^H\bar{\boldsymbol{A}} = \boldsymbol{O}_{(M-2p)\times p} \quad (17)$$

where $\boldsymbol{\Phi}_1 \triangleq [\boldsymbol{\Phi}_{f1}, \bar{\boldsymbol{\Phi}}_{f1}, \boldsymbol{\Phi}_{b1}, \bar{\boldsymbol{\Phi}}_{b1}]$, $\boldsymbol{\Phi}_2 \triangleq [\boldsymbol{\Phi}_{f2}, \bar{\boldsymbol{\Phi}}_{f2}, \boldsymbol{\Phi}_{b2}, \bar{\boldsymbol{\Phi}}_{b2}]$, $\boldsymbol{Q} \triangleq [\boldsymbol{P}^T, -\boldsymbol{I}_{M-2p}]^T$, and $\boldsymbol{P} = \boldsymbol{A}_1^{-H}(\theta)\boldsymbol{A}_2^H(\theta) = (\boldsymbol{\Phi}_1\boldsymbol{\Phi}_1^H)^{-1}\boldsymbol{\Phi}_1\boldsymbol{\Phi}_2^H$. Obviously the columns of $\boldsymbol{Q}$ in fact form the basis for the null space $\mathcal{N}(\bar{\boldsymbol{A}}^H(\theta))$ of $\bar{\boldsymbol{A}}^H(\theta)$.

Therefore when the finite array data are available, the directions $\{\theta_k\}$ can be estimated without any EVD/SVD by minimizing the following cost function

$$f(\theta) = \bar{\boldsymbol{a}}^H(\theta)\boldsymbol{\Pi}_{\hat{Q}}\bar{\boldsymbol{a}}(\theta) \quad (18)$$

where $\bar{\boldsymbol{a}}(\theta) \triangleq [1, e^{j\omega_0\tau(\theta)}, \cdots, e^{j\omega_0(L-2)\tau(\theta)}]^T$, $\boldsymbol{\Pi}_{\hat{Q}} = \hat{\boldsymbol{Q}}(\hat{\boldsymbol{Q}}^H\hat{\boldsymbol{Q}})^{-1}\hat{\boldsymbol{Q}}^H$, and $\hat{\boldsymbol{P}} = (\hat{\boldsymbol{\Phi}}_1\hat{\boldsymbol{\Phi}}_1^H)^{-1}\hat{\boldsymbol{\Phi}}_1\hat{\boldsymbol{\Phi}}_2^H$.

Remark 1: Although the incident signals are assumed to be fully coherent, the proposed SUMWE algorithm can be extended to the case of partly coherent or incoherent signals. Further the SUMWE can accommodate a more general noise model of the spatially correlated noise if we choose the signal vectors $\boldsymbol{y}_{fl}(n)$ and $\boldsymbol{y}_{bl}(n)$ used to form the matrices $\boldsymbol{\Phi}_f$, $\bar{\boldsymbol{\Phi}}_f$, $\boldsymbol{\Phi}_b$, and $\bar{\boldsymbol{\Phi}}_b$ appropriately [12]. □

3.2 Batch-Implementation of SUMWE

The implementation of the SUMWE for estimating the constant directions of incident signals with the finite array data $\{\boldsymbol{y}(n)\}_{n=1}^{N}$ is summarized as follows:

a): Calculate the correlation vector $\hat{\varphi}$ between $\boldsymbol{y}(n)$ and $y_M^*(n)$ and that $\hat{\bar{\varphi}}$ between $\boldsymbol{y}(n)$ and $y_1^*(n)$ as

$$\hat{\varphi} = \sum_{n=1}^{N} \boldsymbol{y}(n) y_M^*(n) \Big/ N , \quad \hat{\bar{\varphi}} = \sum_{n=1}^{N} \boldsymbol{y}(n) y_1^*(n) \Big/ N \tag{19}$$

where $\hat{\varphi} = [\hat{r}_{1M}, \hat{r}_{2M}, \cdots, \hat{r}_{MM}]^T$, and $\hat{\bar{\varphi}} = [\hat{r}_{11}, \hat{r}_{21}, \cdots, \hat{r}_{M1}]^T$.

b): Form the estimated correlation matrices $\hat{\boldsymbol{\Phi}}_f$, $\hat{\bar{\boldsymbol{\Phi}}}_f$, $\hat{\boldsymbol{\Phi}}_b$, and $\hat{\bar{\boldsymbol{\Phi}}}_b$ from $\hat{\varphi}$ and $\hat{\bar{\varphi}}$ by using (10)-(13).

c): Estimate the linear operator $\boldsymbol{P}$ as

$$\hat{\boldsymbol{P}} = (\hat{\boldsymbol{\Phi}}_1 \hat{\boldsymbol{\Phi}}_1^H)^{-1} \hat{\boldsymbol{\Phi}}_1 \hat{\boldsymbol{\Phi}}_2^H \tag{20}$$

and calculate the orthogonal projector $\boldsymbol{\Pi}_{\hat{Q}}$ as

$$\boldsymbol{\Pi}_{\hat{Q}} = \hat{\boldsymbol{Q}}(\boldsymbol{I}_{M-2p} - \hat{\boldsymbol{P}}^H (\hat{\boldsymbol{P}}\hat{\boldsymbol{P}}^H + \boldsymbol{I}_p)^{-1} \hat{\boldsymbol{P}}) \hat{\boldsymbol{Q}}^H \tag{21}$$

d): Estimate the directions $\{\theta_k\}$ by searching the p highest peaks of the spatial spectrum $P(\theta)$ or by finding the phases of the p zeros of the polynomial $p(z)$ closest to the unit circle in the z-plane, where $P(\theta) \triangleq 1 / \bar{\boldsymbol{a}}^H(\theta) \boldsymbol{\Pi}_{\hat{Q}} \bar{\boldsymbol{a}}(\theta)$, $p(z) \triangleq z^{L-2} \boldsymbol{p}^H(z) \boldsymbol{\Pi}_{\hat{Q}} \boldsymbol{p}(z)$, $\boldsymbol{p}(z) \triangleq [1, z, \cdots, z^{L-2}]^T$, and $z \triangleq e^{j\omega_0 \tau(\theta)}$.

Remark 2: The number of MATLAB flops required by the SUMWE algorithm is nearly $16NM + 16M(M-p)^2$, when $N \gg M \gg p$ [12]. Further the statistical analysis of the SUMWE is studied, and the asymptotic mean-squared-error (MSE) expression is given explicitly in [12]. □

4. ADAPTIVE ALGORITHM FOR DIRECTION TRACKING

4.1 Updating of Null Space and Direction

Now we consider the real-time implementation of the SUMWE for tracking the slowly time-varying (relative to the sampling rate [6]) directions of moving signals.

First the estimation of linear operator $\boldsymbol{P}(n)$ at the time n can be reduced to the minimization of the instantaneous cost function $J(n)$ given by

$$J(n) \triangleq \left\| \boldsymbol{E}(n) \right\|^2 \tag{22}$$

where $\boldsymbol{E}(n)$ is the estimation error given by $\boldsymbol{E}(n) \triangleq \boldsymbol{\Phi}_2^H(n) - \boldsymbol{\Phi}_1^H(n)\boldsymbol{P}(n-1)$, $\boldsymbol{\Phi}_1(n)$ and $\boldsymbol{\Phi}_2(n)$ are the instantaneous correlation matrices, and $\|\cdot\|^2$ denotes the square of the Frobenius norm. Thus we easily have the normalized least-mean-square (NLMS) algorithm for updating the linear operator $\boldsymbol{P}(n)$ [13]

$$\boldsymbol{P}(n) = \boldsymbol{P}(n-1) + \frac{\mu \boldsymbol{\Phi}_1(n) \boldsymbol{E}(n)}{\mathrm{tr}\{\boldsymbol{\Phi}_1(n)\boldsymbol{\Phi}_1^H(n)\}} \tag{23}$$

where μ is the step-size ($0 < \mu < 2$). By performing the QR decomposition with the Householder transformation on the matrix $\boldsymbol{P}(n)\boldsymbol{P}^H(n) + \boldsymbol{I}_p$ as

$$\boldsymbol{P}(n)\boldsymbol{P}^H(n) + \boldsymbol{I}_p \triangleq \bar{\boldsymbol{P}} = \bar{\boldsymbol{Q}}\bar{\boldsymbol{R}} \tag{24}$$

where $\bar{\boldsymbol{Q}}$ is a $p \times p$ unitary matrix, and $\bar{\boldsymbol{R}}$ is a $p \times p$ upper-triangular matrix, then the instantaneous orthogonal projector $\boldsymbol{\Pi}(n)$ can be obtained

$$\boldsymbol{\Pi}(n) = \boldsymbol{Q}(n)(\boldsymbol{I}_{M-2p} - \boldsymbol{P}^H(n)\bar{\boldsymbol{R}}^{-1}\bar{\boldsymbol{Q}}^H \boldsymbol{P}(n))\boldsymbol{Q}^H(n) \tag{25}$$

where $\boldsymbol{Q}(n) = [\boldsymbol{P}^T(n), -\boldsymbol{I}_{M-2p}]^T$. Note that the inversion $\bar{\boldsymbol{R}}^{-1}$ is easily got by a simple back-substitution, because the matrix $\bar{\boldsymbol{R}}$ is upper- triangular matrix.

Next as the estimate $\hat{\theta}_k$ is obtained by minimizing $f(\theta)$ in (18), based on the second-order Taylor series expansion of $f(\theta)$, we can get the approximate Newton's iteration method for direction estimation as [13], [12], [6]

$$\hat{\theta}_k(n) = \hat{\theta}_k(n-1) - \left. \frac{\mathrm{Re}\{\bar{\boldsymbol{a}}^H(\theta)\boldsymbol{\Pi}(n)\boldsymbol{d}(\theta)\}}{\boldsymbol{d}^H(\theta)\boldsymbol{\Pi}(n)\boldsymbol{d}(\theta)} \right|_{\theta = \hat{\theta}_k(n-1)} \tag{26}$$

4.2 On-line Algorithm for Direction Tracking

Based on the above analysis, the real-time algorithm for tracking the time-varying directions is given as follows:

1): Calculate the instantaneous correlation vector $\varphi(n)$ between $\boldsymbol{y}(n)$ and $y_M^*(n)$ and that $\bar{\varphi}(n)$ between $\boldsymbol{y}(n)$ and $y_1^*(n)$ as

$$\varphi(n) = \boldsymbol{y}(n) y_M^*(n) , \quad \bar{\varphi}(n) = \boldsymbol{y}(n) y_1^*(n) \tag{27}$$

where $\varphi(n) = [\hat{r}_{1M}(n), \hat{r}_{2M}(n), \cdots, \hat{r}_{MM}(n)]^T$, and $\bar{\varphi}(n) = [\hat{r}_{11}(n), \hat{r}_{21}(n), \cdots, \hat{r}_{M1}(n)]^T$.

2): Form the instantaneous estimates of Hankel correlation matrices $\boldsymbol{\Phi}_f(n)$, $\bar{\boldsymbol{\Phi}}_f(n)$, $\boldsymbol{\Phi}_b(n)$, and $\bar{\boldsymbol{\Phi}}_b(n)$ from $\varphi(n)$ and $\bar{\varphi}(n)$ by using (10)-(13).

3): Update the linear operator $\boldsymbol{P}(n)$ by using (23).

4): Calculate an auxiliary matrix as $\bar{\boldsymbol{P}} = \boldsymbol{P}(n)\boldsymbol{P}^H(n) + \boldsymbol{I}_p$, and perform the QR factorization of $\bar{\boldsymbol{P}}$ based on Householder transformation as (24).

5): Form the projector as $\boldsymbol{Q}(n) = [\boldsymbol{P}^T(n), -\boldsymbol{I}_{M-2p}]^T$, and calculate the orthogonal projector $\boldsymbol{\Pi}(n)$ as (25).

6): Update the estimates of directions $\{\hat{\theta}_k(n)\}$ by using the approximate Newton's iteration method as (26).

In addition, the NLMS algorithm is initialized by $\boldsymbol{P}(0) = \boldsymbol{O}_{p\times(M-2p)}$, and the first the first $K_0 = 2M$ snapshots of the received data are accumulated for an off-line SUMWE to provide the initial values of directions $\{\hat{\theta}_k(n)\}$ for the Newton's iteration method.

5. NUMERICAL EXAMPLES

The ULA with M sensors is separated by a half-wavelength, and two signals with equal power come from angles θ_1 and θ_2. The SNR is defined as the ratio of the power of the signals to that of the additive noise at each sensor. The results are based on 1000 independent trials.

Example 1: Performance of SUMWE versus SNR

The directions of two coherent signals are $\theta_1 = 5^{\circ}$ and $\theta_2 = 12^{\circ}$, and their SNR is varied from -10 to 25 dB. The number of sensors is $M = 10$, and the number of snapshots is $N = 128$. Additionally the subarray size is set as $m = 7$ for the spatial smoothing (SS) based algorithms. The empirical root-MSEs (RMSEs) of $\hat{\theta}_1$ and $\hat{\theta}_2$ are shown in Fig. 1, where the theoretical RMSEs of the SUMWE [12] and the stochastic Cramér-Rao lower bounds (CRBs) [14] are also plotted. Because the maximum possible number of subarrays and working array aperture are exploited and the effect of additive noise is eliminated by appropriately choosing the used subarrays, the SUMWE method generally outperforms the SS-based root-MUSIC [2], [9] and the methods without EVD such as the BEWE (variant for coherent case) [4] and forward-backward SS (FBSS) based SWEDE (variant G) [6], [9], and it is superior to the FBSS-based OPM [6], [9] at low SNRs. And it performs well than the WSF-E [7] at low to moderate SNRs. In addition, the empirical RMSEs of the SUMWE are very close to the theoretical ones and the difference between the theoretical RMSEs and the CRBs is small.

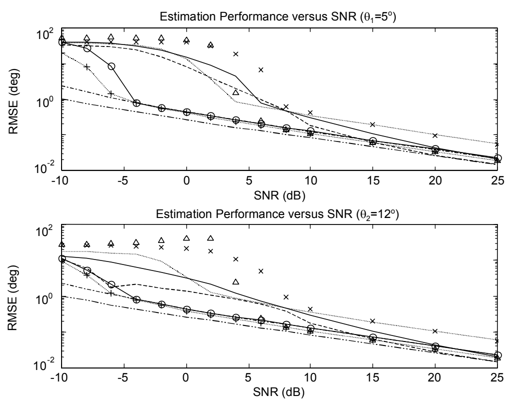

Fig. 1 RMSEs of the estimates versus the SNR (dotted line: SS-based root-MUSIC; dotted line with "+": FBSS-based root-MUSIC; "Δ": FBSS-based OPM; "x": BEWE; solid line: FBSS-based SWEDE; dashed line: WSF-E; solid line with "o": SUMWE; dash-dot line: theoretical RMSE of SUMWE; and dash-dots line: CRB).

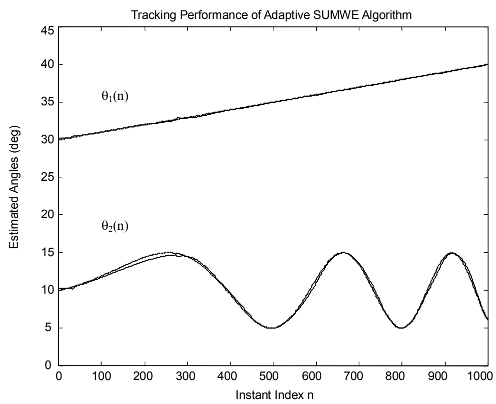

Fig. 2 Averaged estimates of time-varying directions coherent signals (dotted line: actual value, and solid line: proposed algorithm).

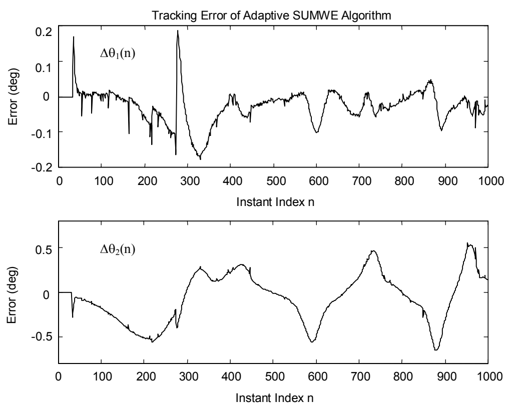

Fig. 3 Averaged estimation errors of estimated directions.

Example 2: Tracking of Time-Varying Directions

Here two coherent signals with equal power come from $\theta_1(n) = 30^\circ + 0.01^\circ(n-1)$ and $\theta_2(n) = 10^\circ + 5^\circ \sin(2\pi(4\times 10^{-4}n + 2.25\times 10^{-6}n^2))$, where $n = 1,2,\cdots,1000$, and the SNR is $20\,\text{dB}$. The number of sensors is $M = 16$, and the step-size of NLMS algorithm is set as $\mu = 1$. The proposed on-line algorithm is carried out, and the averaged estimates and estimation errors are shown in Figs. 2 and 3. Even though the SS and EVD/SVD processes are not used, the proposed algorithm can promptly track the variation in the desired directions of coherent signals with less estimation errors.

6. CONCLUSION

A new computationally efficient subspace-based method called SUMWE was proposed for direction estimation of narrowband signals impinging on a ULA by exploiting the array geometry and its shift invariance property, and its adaptive implementation was presented for tracking the directions of moving signals. The effectiveness of the proposed SUMWE and its adaptive implementation were verified through numerical examples. It was shown that the proposed adaptive algorithm has the advantages of the computational simplicity and good tracking adaptation in a slowly time-varying environment.

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