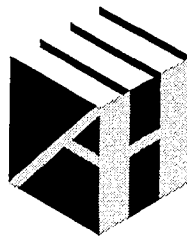


Computational Electrodynamics

The Finite-Difference Time-Domain Method

Third Edition

Allen Taflove
Susan C. Hagness



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