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BINARY AND MILLISECOND PULSARS

E. S. Phinney

Theoretical Astrophysics, 130-33, California Institute of Technology,
Pasadena, California 91125

S. R. Kulkarni

Department of Astronomy, 105-24, California Institute of Technology,
Pasadena, California 91125

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1. INTRODUCTION

Most of the ~ 600 known pulsars are single and located in the disk of our Galaxy. There is circumstantial evidence that the pulsars in this majority are created in supernova (SN) explosions, by the collapse of the cores of massive stars (initial mass $M_i \gtrsim M_{\text{cr}} \simeq 8 M_{\odot}$). One is created roughly every 100 y in the Galaxy.

Figure 1 is a plot of the pulse period (P) versus the dipole field strength (B , inferred from the observed P and $\dot{P}$, and assuming a vacuum dipole model) for the 545 Galactic pulsars for which such measurements are available (cf Taylor et al 1993). Like the color-magnitude diagram for stars, this B - P diagram offers a convenient graphic representation on which to trace the evolution of pulsars. Young pulsars—those associated with supernova remnants (SNRs)—appear to be born with reasonably small periods, $P \lesssim 0.1$ s, and strong magnetic field strengths, $1 \lesssim B_{12} \lesssim 10$ where $B = 10^{12} B_{12}$ G is the inferred dipole field strength. Pulsars slow down as they age and thus move to the right in this diagram and cease emitting in the radio band as they approach the so-called death line (Figure 1).

The scale height of pulsars is much larger than that of their progenitors, the massive stars. Direct interferometric measurements have established that young

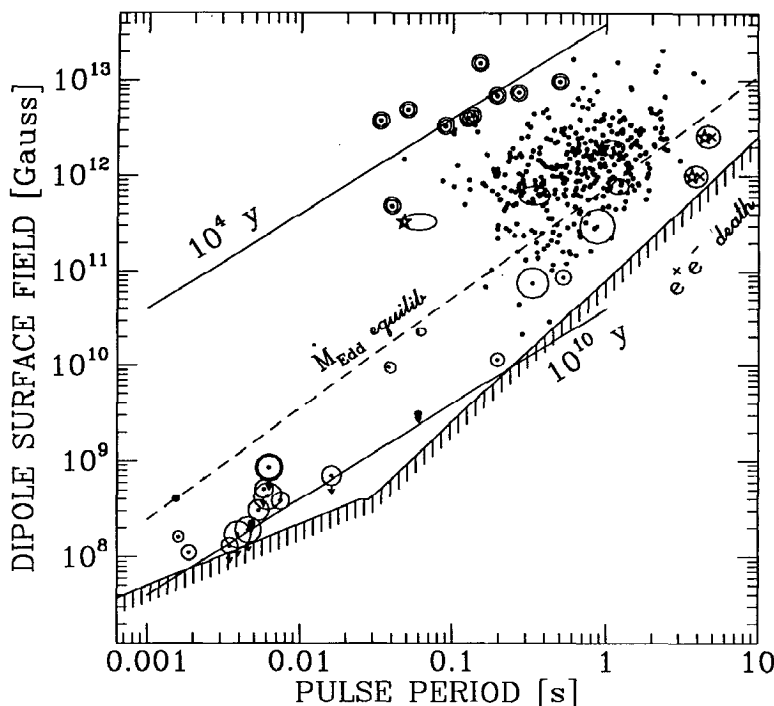


Figure 1 Plot of pulse period vs dipole field strength, for Galactic and Magellanic Cloud radio pulsars. Dipole field estimated by assuming energy loss from a vacuum magnetic dipole, $B^2 = 10^{39} P \dot{P}$ (P in s, B in G). Small points are single disk pulsars. Those surrounded by double circles are in supernova remnants. Binary pulsars lie at the left focus of ellipses with the orbital eccentricity, and semimajor axis proportional to $\log(P_b/0.01 \text{ d})$. The left focus is marked with a dot when the pulsar's companion is a white dwarf or neutron star, and by a 5-pointed star when it is an optically detected B-type star. The solid lines show where the characteristic age $\tau_c = P/2\dot{P}$ has the indicated values; the lower one shows an age equal to that of the Galaxy. Pulsars born with short period and evolving with constant dipole field must lie to the left of the line, and if there were no luminosity evolution, a majority would lie close to the line. The dashed line is the standard Eddington "Rebirth" line (cf Ghosh & Lamb 1992), specified by Equation (2.1). The shaded boundary line is the "death line" discussed in Section 6.2. Note the absence of radio pulsars to the right of the death line. The two binaries near the death line, with $\times$ s at the right focus, are not radio pulsars, but the two accreting X-ray pulsars with known orbital periods and magnetic fields determined from X-ray cyclotron harmonics, X0331+53 and X0115+634. If their B-star companions had lower rates of mass loss, these objects would not have spun down, and would have been radio pulsars. The location of the rebirth and death lines depends on the assumed magnetic topology, and is also subject to some physical uncertainty (see text).

single pulsars have large spatial motion with a median 3-dimensional velocity of $\sim 400 \text{ km s}^{-1}$. The large velocity combined with the finite lifetime set by the death line (Figure 1), offer a first-order explanation for the scale height of pulsars. Considerable circumstantial evidence indicates that pulsars acquire a velocity kick of order $\sim 100\text{--}600 \text{ km s}^{-1}$ at their birth (Section 4).

Pulsars with short spin periods and low magnetic field strengths form a distinct group (Figure 1 and Table 1). The high abundance of binaries in this group indicates that binarity has played an important role in its formation. It is now being appreciated that this group of pulsars has a steady state population approximately *equal* to that of active ordinary radio pulsars (Section 3). The precision of their rotational clocks, and their close companions, allow them to be used for many remarkable experiments ranging from fundamental physics (nuclear equations of state, general relativity), to applied physics (Raman scattering of high-power microwaves in plasma), to astrophysics (planet formation, neutron star magnetospheres and winds, dynamical evolution of globular clusters).

Recently, a large number of pulsars with characteristics similar to the Galactic millisecond and binary pulsars have been discovered in globular clusters. Space limitations prevent us from including here any discussion of these objects, their formation mechanisms, and the remarkable inferences about globular cluster dynamics and evolution that they have made possible. The reader is referred to Phinney (1992, 1993), Manchester (1992), and Phinney & Kulkarni (1994) for a review.

We therefore concentrate on the Galactic millisecond and binary pulsars. Our division of labor assigned Sections 2–5 to SRK, and Sections 6–11, tables, and figures to ESP. Other useful review articles include those by Srinivasan (1989), Verbunt (1990), Bhattacharya & van den Heuvel (1991), and Lamb (1992), and those collected in Lewin et al (1994). Related recent Annual Review articles are by Verbunt (1993), Chanmugam (1992), and Canal et al (1990). Useful conference proceedings have been edited by Ögelman & van den Heuvel (1989) and van den Heuvel & Rappaport (1992).

2. BACKGROUND AND FRAMEWORK

The binary pulsars in Table 1 are broadly divided into two categories: high mass binary pulsars (HMBPs; the upper two groups in the table) and low mass binary pulsars (LMBPs; the lower four groups). The few isolated pulsars present in Table 1 have been classified as follows: $P < 30 \text{ ms}$, LMBPs; HMBPs, otherwise. The rationale for this is explained below. We now summarize our current understanding of the origin of these systems. The reader is referred to Bhattacharya & van den Heuvel (1991), Verbunt (1993), and van den Heuvel & Rappaport (1992) for more extensive reviews.

Table 1 Binary and millisecond pulsars in the Galaxy[†]

Pulsar	P (ms)	P_b (d)	e^a	$f(M)^b$ ($M_\odot$)	M_2^c ($M_\odot$)	$\log(B)^d$ (G)	$P/(2\dot{P})^e$ (y)	Ref
J0045-7319	926.3	51	0.808	2.169	~ 10	12.3	3×10^6	1
J259-63	47.8	1237 ^E	0.870	1.53	~ 10	11.5	3×10^5	2
1820-11	279.8	358	0.794	0.068	(0.8)	11.8	3×10^6	3
1534+12	37.9	0.42	0.274	0.315	1.34	10.0	2×10^8	4
1913+16	59.0	0.32	0.617	0.132	1.39	10.4	1×10^8	5
2303+46	1066.4	12.3	0.658	0.246	1.4	11.9	3×10^7	6
J2145-0750	16.0	6.8	0.000021	0.0241	(0.51) ^O	< 8.9	$> 8 \times 10^9$	7
0655+64	195.7	1.03	7×10^{-6}	0.071	(0.8) ^O	10.1	5×10^9	8
0820+02	864.8	1232	0.0119	0.0030	(0.23) ^O	11.5	1×10^8	9
J1803-2712	334	407	0.00051	0.0013	(0.17)	10.9	3×10^8	10
1953+29	6.1	117	0.00033	0.0024	(0.21)	8.6	3×10^9	11
J2019+2425	3.9	76.5	0.000111	0.0107	(0.37)	8.3	(1×10^{10})	12
J1713+0747	4.6	67.8	0.000075	0.0079	(0.33) ^O	8.3	(9×10^9)	13
1855+09	5.4	12.3	0.000022	0.0056	0.26 ^O	8.5	5×10^9	14
J0437-4715	5.8	5.7	0.000018	0.0012	(0.17) ^O	8.7	(2×10^9)	15
J1045-4509	7.5	4.1	0.000019	0.00177	(0.19)	8.6	6×10^9	7
J2317+1439	3.4	2.46	< 0.000002	0.0022	(0.21)	8.1	(1×10^{10})	16
J0034-0534	1.9	1.6	< 0.0001	0.0012	(0.17)	8.0	4×10^9	7
J0751+18	3.5	0.26	< 0.01	(0.15)				17
1718-19	1004	0.26 ^E	< 0.005	0.00071	(0.14)	12.2	1×10^7	18
1831-00	520.9	1.8	< 0.004	0.00012	(0.07)	10.9	6×10^8	9
1957+20	1.6	0.38 ^E	$< 4 \times 10^{-5}$	5×10^{-6}	0.02 ^O	8.1	2×10^9	19
1257+12	6.2	67,98	0.02,0.02	$5, 3 \times 10^{-16}$	$4, 3 M_\oplus$	8.9	(8×10^8)	20
1937+21	1.6	single				8.6	2×10^8	21
J2235+1506	59.8	single				9.5	(6×10^9)	16
J2322+2057	4.8	single				8.3	(1×10^{10})	12

^aOrbital eccentricity.

^bMass function $f(M_{\text{psr}}, M_2) = (M_2 \sin i)^3 (M_2 + M_{\text{psr}})^{-2}$.

^cMass of pulsar's companion (when in parentheses, tabulated value of M_2 is estimated from $f(M)$, assuming a pulsar mass of $1.4 M_\odot$ and inclination $i = 60^\circ$, the median for randomly oriented binaries).

^dThe value of B given is the dipole surface field, calculated as if the pulsar were an orthogonal vacuum rotator. Higher multipoles could be much stronger.

^eCharacteristic age enclosed in () when $\dot{P}/P$ is dominated by $V_\perp^2/(cD)$ centrifugal acceleration contribution (see Equation 8.1).

^EPulsar eclipsed when behind companion (or its wind).

^OOptical radiation from the white dwarf companion is observed; its temperature combined with the theory of white dwarf cooling roughly confirm the age estimated from $P/(2\dot{P})$ (except for 1957+20's companion, which is heated by the pulsar's relativistic wind).

References: 1. Kaspi et al 1994a, 2. Johnston et al 1994, 3. Lyne & McKenna 1989, 4. Wolszczan 1991, 5. Damour & Taylor 1991, 6. Thorsett et al 1993b, 7. Bailes et al 1994,

2.1 HMBPs

HMBPs are believed to originate from massive binary systems, in which the primary first forms a neutron star. As the secondary evolves, its stellar wind or Roche lobe overflow feeds matter to the neutron star. The strong magnetic field of the young neutron star funnels the accreted matter to the polar cap, which gives rise to pulsed X-ray emission. This phase is identified with Massive X-ray Binaries (MXRBs). If the mass of the secondary is above M_{cr} , it will explode as a supernova. This will generally unbind the system, but a few combinations of recoil velocities and masses can leave two neutron stars bound in an eccentric orbit. Lower-mass secondaries evolve to a white dwarf in a circular orbit around a spun-up pulsar.

Matter that flows to the neutron star settles into an accretion disk, owing to its angular momentum. The accretion disk is terminated at the Alfvén radius—the radius at which the inward accretion ram pressure is balanced by magnetic pressure $B^2/8\pi$. The neutron star gets spun up until its rotation rate equals that of the Keplerian rotation rate at the inner edge of the accretion disk. This

Table 1 (continued)

8. Jones & Lyne 1988, 9. Taylor & Dewey 1988, 10. Taylor et al 1993, 11. Rawley et al 1988, 12. Nice et al 1993, 13. Foster et al 1993, 14. Kaspi et al 1994, 15. Johnston et al 1992, 16. Camilo et al 1993, 17. Lundgren 1994, 18. Lyne et al 1993, 19. Ryba & Taylor 1991, 20. Wolszczan & Frail 1992, 21. Thorsett & Phillips 1992.

†The horizontal lines divide the pulsars into six groups, depending on their probable evolutionary history. The *first* group of pulsars are antediluvian in the sense defined by Phinney & Verbunt (1991): The neutron stars have probably not yet accreted from their companions, and the orbits are eccentric. PSRs 1259–63 and J0045–7319 (in the Small Magellanic Cloud) have visible B stars as companions; 1820–11's companion may be lower main-sequence (Phinney & Verbunt 1991). The remaining pulsars are postdiluvian. For the *second* group, limits on the masses and sizes of the companions suggest that the companions are neutron stars. The short orbital periods and high eccentricities suggest that the pulsar and companion spiraled together in a common envelope, after which a second supernova created the second neutron star. Both stars were initially massive $>8 M_{\odot}$. Pulsars in the *third* group have high-mass white dwarf companions, which must have formed in a massive red giant much larger than the current orbit, which implies that the neutron star spiraled into, and ejected the giant's envelope during unstable mass transfer. The remnant core was not massive enough to create a neutron star (as in the second group of pulsars), so a massive white dwarf was left. The *fourth* group has low-mass white dwarf companions in circular orbits, which could have formed in stable accretion from low-mass companion stars filling their Roche lobes. The residual masses and eccentricities are discussed in Section 10. The *fifth* group consists of systems with companions less massive than the core mass ($\sim 0.16 M_{\odot}$) of the least massive star to have evolved off the main sequence in the age of the universe. Their mass transfer must have been driven by something other than nuclear evolution of the companion, e.g. gravitational radiation, or loss of angular momentum in a magnetic wind. PSR 1718 – 19 is located 2.3' (about 1/4 of the tidal radius) from the core collapsed globular cluster NGC 6342, and might be a cluster pulsar, not a Galactic pulsar. In the *sixth* group, 1257+12 seems to have a planetary system; the other pulsars are single. These pulsars seem to have destroyed the stellar companions that provided the angular momentum to spin them up.

gives rise to the rebirth or “spin-up” line (Figure 1; see Ghosh & Lamb 1992 for subtleties):

$$P_{\text{eq}} = 1.3 \left(\frac{B}{10^{12} \text{ G}} \right)^{6/7} \left(\frac{\dot{M}}{\dot{M}_{\text{Edd}}} \right)^{-3/7} \text{ s}, \quad (2.1)$$

where $\dot{M}$ is the accretion rate and $\dot{M}_{\text{Edd}} \sim 2 \times 10^{-8} M_{\odot} \text{ y}^{-1}$ is the Eddington accretion rate. The pulse periods of X-ray pulsars range from ~ 1 s to 10^3 s (White et al 1994). Detection of cyclotron line features in several of these sources directly confirm the existence of high magnetic field strengths, $0.5 \times 10^{12} \lesssim B \lesssim 5 \times 10^{12}$ G (see Figure 1).

2.2 LMBPs

There is considerable similarity between LMBPs and cataclysmic variables (CVs). Both contain a degenerate object whose progenitor at one time occupied a large volume, yet many CVs and LMBPs have orbital separations that are less than a fraction of the radius of the progenitor giant star. As with CVs, we believe that LMBPs underwent a common envelope (CE) phase during which the secondary was engulfed by the bloated primary and the two stars quickly spiraled closer, ejecting the primary’s envelope in the process.

A stellar system forming an LMBP must overcome two obstacles that are not faced by systems forming CVs. First, the formation of a neutron star is accompanied by copious mass loss, potentially from $\gtrsim M_{\text{cr}}$ down to $M_{\text{n}} \sim 1.4 M_{\odot}$, the measured mass of neutron stars. Thus the system can be expected to unbind if the mass lost suddenly exceeds half the total pre-SN mass of the system. Second, natal velocity kicks (Section 4.2) add a further tendency to disrupt the binary. The magnetic field strengths and spin periods of the neutron stars in LMBPs are anomalously low compared to ordinary pulsars and HMBPs. Some mechanism is needed to reduce the magnetic field strength. Three scenarios which have been suggested (see Webbink 1992) are summarized below.

2.2.1 THE RECYCLED PULSAR MODEL In this model (see Verbunt 1993, Bailes 1989), the progenitors of LMBPs are binaries with a primary mass (M_1) above M_{cr} and a secondary mass below M_{cr} , $M_2 \lesssim 1 M_{\odot}$. The primary evolves and expands; unstable mass transfer then leads to spiral-in of the secondary. A compact binary is thus formed, consisting of the secondary and the evolved He core of the primary which eventually explodes and leaves behind a neutron star. In some cases, a second spiral-in may occur during the late stages of the secondary’s evolution, thus producing even tighter binaries. During the evolution of the secondary, mass is transferred from the secondary to the primary (a neutron star), which is spun up to millisecond periods. Mass transfer from the companion may be driven either by nuclear evolution (for orbital period,

$P_b \gtrsim 1$ d) or gravitational radiation aided by magnetic braking (for $P_b \lesssim 1$ d). This transfer phase circularizes the orbit, and is identified with the Low Mass X-ray Binaries (LMXBs), of which there are $\lesssim 10^2$ in the Galaxy. Unlike the MXRBs, LMXBs do not exhibit pulsed X-ray emission. This suggests that their neutron stars are weakly magnetized, and makes them plausible progenitors of the millisecond pulsars [cf Equation (2.1) and the “rebirth line” in Figure 1].

This scenario can explain the Population I origin of two compact X-ray pulsars with very low mass companions: 1E 2259 + 586 and 4U 1626 – 67. In particular, the model gives a satisfactory explanation for the location of 1E 2259 + 586 in an SNR located in a star forming region (Iwasawa et al 1992). In binaries, the primary star could have lost its H envelope to the companion; we are then left with a He core of much smaller mass. The minimum mass of a He core that will still undergo collapse is still controversial but is expected to be in the range $2.2\text{--}4 M_\odot$ (Habets 1985, Bhattacharya & van den Heuvel 1991), precariously close to our limit. The SN mass loss is symmetric in the frame of the exploding star but not in the center-of-mass frame. Thus assuming that the system survives the SN explosion, the binary system acquires a systemic speed that is a sizable fraction of the orbital speed. As pointed out by Bailes (1989), suitably directed natal kicks can actually *stabilize* the system even if the mass loss exceeds half the total initial mass. A consequence is that this model predicts significant velocities for both LMXBs and LMBPs. This effect may have already been observed (Section 4.4).

There are two potential problems with the recycling model. 1. One must assume that magnetic fields do not decay unless there is accretion (see Section 5). This hypothesis remains to be proven. 2. The expected millisecond pulsations from LMXBs have not been detected. Statistical studies of the birthrates of LMXBs and LMBPs suggest a possible discrepancy (Section 3), which suggests that the observed LMXBs are not the only progenitors of LMBPs.

2.2.2 TRIPLE STAR EVOLUTION In spirit, triple star evolution is similar to the recycled model except it does not appeal to velocity kicks to stabilize the system. Instead, a massive compact binary with a distant low mass tertiary is invoked (Eggleton & Verbunt 1986). Following the first SN and the evolution of the secondary, the neutron star spirals into the evolved secondary and sinks to the center. The binary is transformed into a red supergiant with a neutron star at the core—a Thorne-Żytkow object (Thorne & Żytkow 1977, Biehle 1994). The tertiary then undergoes a common envelope evolution to form an LMXB. From this point on, the evolutionary path is similar to the recycled model.

2.2.3 ACCRETION INDUCED COLLAPSE (AIC) In the accretion induced collapse model, one assumes that the progenitor is an accreting white dwarf which is transmuted to a neutron star once its mass exceeds the Chandrashekar limit

(see Canal et al 1990). There are a number of variants: neutron stars may be assumed to be born with or without natal kicks, and with or without low magnetic field strengths. One advantage of AIC without natal kicks is that very little mass is lost during the SNe (essentially the binding energy of the neutron star, $\sim 0.2 M_{\odot}$). Thus all such systems survive the SNe. By the same token, we do not expect to see substantial systemic motion. However, the velocity data (Section 4) indicate that LMBPs have substantial motions, suggesting that neutron stars in LMBPs suffer velocity kicks for one reason or the other, removing the survival advantage of AIC. This leaves us two variants: The neutron star is born as a millisecond pulsar (Michel 1987) or as an ordinary high field pulsar. The latter model suffers from the same two problems as the recycled model and hence we ignore it.

The physics of the AIC mechanism is not well understood. Does the white dwarf explode (a popular model for Type Ia SNe), or implode to form a millisecond pulsar? Assuming implosion, why are the initial B and P values so much smaller than those of neutron stars formed via Type II SNe? Under what conditions does a white dwarf accrete matter as opposed to ejecting it via thermonuclear flashes as appears to be the case with CVs? In view of these rather fundamental uncertainties, our prejudice is to adopt as the standard model the recycling model, whose physics is relatively better understood. Throughout the article we compare the observations with the standard model. Only when the standard model fails should a search for an alternative be seriously considered.

3. SEARCHES AND DEMOGRAPHY

The demography—distribution and birthrates—of millisecond pulsars offers a valuable clue to their origin. In particular, comparison of the birthrates of LMBPs and LMXBs is a particularly important exercise (Kulkarni & Narayan 1988). A number of searches are currently in progress and the essential results are summarized below.

3.1 Pulsar Searches

Pulsar searches are usually conducted at meter wavelengths since pulsars are steep-spectrum objects. Millisecond pulsars appear to have especially steep spectral indices: $\alpha \gtrsim 2$ (Foster et al 1991), where the flux at frequency ν , $S_{\nu} \propto \nu^{-\alpha}$. The conceptual basis of a pulsar search is quite simple. A spectrum with n channels spread over bandwidth B is recorded every Δt . The pulsar signal, owing to dispersive transmission through the interstellar medium, arrives earlier at the higher frequency channels compared to the lower frequency channels. The first step is to undo this dispersion. If the amount of dispersion is not known, as is the case in pulsar searches, one tries a variety of such shifts. Next, a search for a pulsed train is carried out in each of the dedispersed time series.

This is most easily implemented by a Fourier transform followed by a search for a pattern of evenly spaced peaks (the fundamental and harmonics).

Following the discovery of the first millisecond pulsar 1937 + 21 (Backer et al 1982), a number of searches were launched at Arecibo, Jodrell Bank, and Parkes. The search for millisecond pulsars requires enormous computing capacity since the memory and CPU requirements are $\propto P_{\min}^{-2}$, where $P_{\min}$ is the minimum period to which the search is sensitive. For the succeeding eight years, searches proceeded rather slowly, primarily limited by both the recording and computing capacity. Most of these searches, motivated by the low latitude of the first millisecond pulsar 1937 + 21, were directed towards the Galactic plane.

The association of LMXBs with LMBPs meant that LMBPs should also be found away from the plane. This rationale led to a few searches at intermediate Galactic latitudes. However, it was the successful detection of a millisecond pulsar at high latitude by Wolszczan (1991) that demonstrated the importance of all-sky searches. At about the same time, two technological revolutions made it feasible to conduct large searches: the introduction of inexpensive recording media (8-mm Exabyte tapes), and the vast and relatively inexpensive computing power of workstations and supercomputers. The fully completed Parkes search (Bailes et al 1994; see Section 3.2) will have recorded a total of nearly a terabyte of data and used 20 Sparc-II-years or 2×10^{15} flop to fully reduce the data!

Each pulsar search is limited to pulsars with $P > 2\Delta t$ (from sampling theorem considerations). More importantly, the dispersion of the pulsar signal in the interstellar plasma causes smearing of the pulse signal in the time domain across the finite width B/n of the n frequency channels. This greatly reduces the sensitivity to pulsars with $P \lesssim 2\Delta t_D$, where

$$\Delta t_D = 0.5 \left(\frac{B/n}{250 \text{ kHz}} \right) \left(\frac{DM}{20 \text{ cm}^{-3} \text{ pc}} \right) \left(\frac{\nu}{430 \text{ MHz}} \right)^{-3} \text{ ms.} \quad (3.1)$$

Here DM is the dispersion measure, the integrated electron density along the line of sight to the pulsar. Full sensitivity is obtained for pulsars with $P \gtrsim 10 \max(\Delta t, \Delta t_D)$. Searches for HMBPs are not particularly taxing. Since these are relatively slow pulsars, the sky has been better searched for them. There is tremendous interest in obtaining the deathrate of double-neutron star systems like 1913 + 16 which coalesce in the Hubble time. However, the rapidly changing (Doppler-shifted) apparent periods of pulsars in such close binaries means that simple Fourier transforms smear the signal over a range of frequency bins, and lose sensitivity. More computationally intensive acceleration searches are needed to discover close neutron star binaries (Johnston & Kulkarni 1991; see Anderson 1992 for an implementation and the discovery of PSR2127 + 11C in this way). The gravitational waves generated just before coalescence of

these systems are the dominant known source that defines sensitivity goals for on-going gravitational wave observatory efforts (Abramovici et al 1992). In addition, coalescing binary neutron stars are a leading candidate for models of gamma-ray bursts at cosmological distances (Narayan, Paczyński & Piran 1992). From the survey volumes that led to the detection of 1913 + 16 and 1534 + 12, a Galactic coalescence rate of 10^{-6} y^{-1} to 10^{-7} y^{-1} and a Galactic population $\gtrsim 3 \times 10^4$, comparable to the LMBP population, have been estimated (Narayan, Piran & Shemi 1991; Phinney 1991).

3.2 Summary of Searches for LMBPs

Three large searches are now in progress: at Parkes Observatory (all sky, $\delta < 0^\circ$), at Arecibo (a variety of searches by several groups), and at Jodrell Bank ($\delta > 35^\circ$). All these searches are being conducted at a frequency around 430 MHz.

The Arecibo surveys were done with either an $n = 128$, $B = 10$ MHz system or one with $n = 32$, $B = 8$ MHz. One sequence of surveys discovered three LMBPs and one HMBP in 800 square degrees of high latitude sky (R. S. Foster and A. Wolszczan, personal communication). Another sequence of surveys carried out by Princeton astronomers discovered three new LMBPs over 235 square degrees of Galactic plane (Fruchter 1989, Nice et al 1993) and three LMBPs and one HMBP over 464 square degrees of high latitude sky (F. Camilo, D. Nice, J. Taylor, personal communication). After accounting for lack of detections in other Arecibo searches, we conclude that the Arecibo high latitude success rate is one LMBP per 200 square degrees. Because of the small number n of frequency channels, these searches were sensitive only to millisecond pulsars with small dispersion measure, i.e. nearby pulsars.

The Parkes survey was done with an $n = 256$, $B = 32$ MHz system. Nine millisecond pulsars have been discovered in the 10^4 square degree area searched so far (M. Bailes, A. G. Lyne & R. N. Manchester, personal communication). The survey uncovered the nearest millisecond pulsar, J0437 – 4715 (Johnston et al 1993), located a mere 150 pc from Earth. As with the Arecibo sample, most of these pulsars also appear to be nearby ones.

3.3 Population and Birthrates

Because of their small n , the Arecibo searches are approximately volume-limited. They are sensitive to pulsars within a distance $\lesssim 1$ kpc, comparable to the scale height of LMBPs (Section 4). This, and their high sensitivity, make the Arecibo surveys an ideal tool to estimate the local surface density of LMBPs. If we then assume that LMBPs are distributed like other stars, with a surface density $n(R) \propto \exp(-R/R_d) \text{ kpc}^{-2}$ (where R is the Galactocentric radius and $R_d = 3.6$ kpc is the exponential scale length of the Galactic disk), and take

the solar Galactocentric distance to be $R_0 = 8.5$ kpc, we can extrapolate the inferred local surface density to obtain the total number of LMBPs in the Galaxy, $N_{\text{LMBP}} = 5 \times 10^4 / \bar{f} \chi$. Here, $\bar{f}$ is the beaming fraction for millisecond pulsars, usually assumed to be close to unity, and $\chi \geq 1$ is a factor to account for faint pulsars that could have been missed by the Arecibo survey. Lorimer (1994) obtains a similar number for $N_{\text{LMBP}}(L > 2.5 \text{ mJy kpc}^2) \sim 5 \times 10^4$; here, L is the luminosity at 400 MHz defined as the product of the 400 MHz flux (Jy or mJy) and the square of the distance (kpc^2). This estimate was obtained by laying down pulsars according to a model Galactic distribution; calculating, considering the parameters of all the surveys, the probability p_i (roughly the ratio of the detection volume $V_{i,\text{max}}$ to the Galaxy's volume) that a pulsar of luminosity and period equal to that of a detected pulsar i would have been detected in one of the surveys; and estimating the total Galactic luminosity function as $\Sigma_i (p_i)^{-1} \delta(L - L_i)$.

In our crude estimate, we assume a mean age of $\tau_D/2$, where $\tau_D \sim 10^{10}$ y is the age of the disk. Thus the LMBP birthrate in the Galaxy is $B_{\text{LMBP}} = 10^{-5} \text{ y}^{-1}$. Lorimer (1994) obtains a similar birthrate using the precise estimator $\Sigma_i (2t_p p_i)^{-1}$, where $t_p \sim \tau_c$ is the true age of each pulsar. This should be compared to the birthrate of LMXBs, $B_{\text{LMXB}} = N_{\text{LMXB}}/\tau_X$ where $N_{\text{LMXB}} \lesssim 10^2$ is the number of LMXBs in the Galaxy (van Paradijs 1994). Equality of these two rates is a fundamental expectation of the recycling model and would require $\tau_X \sim 10^7$ y. This is a constraint on the progenitor systems and it needs to be demonstrated that a sufficient number of progenitors with suitable mass transfer histories exist.

An independent constraint can be obtained by noting that the mass that must be accreted to spin up a neutron star to period P_i is

$$\Delta M = f 0.08 \left(\frac{P_i}{2 \text{ ms}} \right)^{-4/3} I_{45} M_\odot, \quad (3.2)$$

where $f \gtrsim 1$ is a factor that depends on the details of the accretion; $f \sim 1$ if the star has a constant magnetic moment low enough that accretion just spins the star to its equilibrium period [Ghosh & Lamb (1992) favor $f \simeq 3$]. Further accretion ($f \gg 1$) will not change P_i unless the magnetic moment decays (as in Shibasaki et al 1989). The disk LMXBs (we exclude the bulge population since they are not relevant to the local population) have X-ray luminosities L_X ranging from $0.1 L_{\text{Edd}}$ to $0.01 L_{\text{Edd}}$ (Verbunt et al 1984, Naylor & Podsiadlowski 1993), with $(dN)/(d \ln L_X) \sim L_X^\alpha$ with $\alpha \sim 0$ to -0.5 . Combining these luminosities with accretion efficiencies (Table 3) and Equation (3.2), implies that the observed X-ray sources would require τ_X ranging from $> 7 \times 10^7 (P_i/2 \text{ ms})^{-4/3}$ y to $> 7 \times 10^8 (P_i/2 \text{ ms})^{-4/3}$ y to produce pulsars with initial period P_i .

The birthrate of LMXBs must be estimated from stellar and binary evolution models. Wide-orbital period ($\gtrsim 1$ d) systems such as Cyg X-2 evolve by nuclear evolution of the secondary, whereas accretion in systems with short orbital

periods is believed to be driven by angular momentum losses (magnetic stellar winds and gravitational radiation; Verbunt 1993). As discussed in Section 10.3, the inferred LMBP and LMXB birthrates are in rough agreement for systems of long orbital periods, but the single short-orbital period LMBPs have a birthrate $\gtrsim 10$ larger than the LMXBs believed to be appropriate progenitors. To bring the rates into agreement would require $\tau_X \sim 10^7$ y.

Our model-independent estimates of τ_X for observed X-ray sources exceed 10^7 y by a factor between 7 and $70 \times (P_i/2\text{ms})^{-4/3}$. This highlights the importance of determining the typical initial period of millisecond pulsars. Camilo et al (1994) have argued that some millisecond pulsars are born with periods not very different from their observed periods, $P \sim 4$ ms. This is certainly the case for some pulsars (e.g. PSR J2145–0750; Bailes et al 1994). An independent reason to suspect that P_i is not small ($\ll 2$ ms) is that to spin many of the observed pulsars up to < 1 ms would in most models require $\dot{M} \sim \dot{M}_{\text{Edd}}$ [see Equation (2.1)] which, as discussed above, does not agree with the observed luminosity function of the LMXBs. Smaller $\dot{M}$ results in a larger equilibrium period.

We conclude that the LMBP birthrate is a factor of $\gtrsim 10$ higher than the (model-dependent) inferred birthrate of observed LMXBs. The birthrates of LMBPs with long orbital period systems agree fairly well with those inferred from well-established models of the X-ray binaries. As discussed in Section 10.3, the discrepancy in the total birthrate arises from “black-widow” pulsars and those in systems of short orbital period, for which models of the accretion phase are most controversial. It appears, as in the globular cluster system (Phinney & Kulkarni 1994), that the majority of these ill-understood LMBPs are produced in very short-lived events (not represented among observed LMXBs) of high $\dot{M}$, or that the mass transfer produces little hard X-ray emission. In the former case, the high $\dot{M}$ would remove the difficulty (see above) in producing LMBPs with $P_i \lesssim 2$ ms.

4. VELOCITY AND KINEMATICS

4.1 Observations

The origin of pulsar velocities is a topic of much interest (see Radhakrishnan 1992, Bailes 1989). The ordinary pulsars, as a group, show large spatial motions, as has been demonstrated via interferometric proper motion observations (Lyne et al 1982). Another technique, less reliable but which can be applied to faint pulsars, is observation of interstellar scintillations (ISS) (Cordes 1986). Motion of the pulsar with respect to the intervening screen of interstellar plasma results in temporal changes of the speckle pattern as viewed on Earth. Recognizing that the scale height of the scattering screen is smaller than that of the interstellar electron layer, by almost an order of magnitude, Harrison & Lyne (1993) present a new calibration curve to con-

vert timescales of changes to transverse speed and conclude that with due care, the ISS velocities are probably accurate to a factor of two. Proper motion measurements are also obtainable from timing observations for a few, mainly millisecond, pulsars.

There now exist almost 100 determinations and significant upper limits for pulsar proper motions (Harrison et al 1993). The mean transverse speed is 217 km s^{-1} using the DM-distance conversion of Lyne et al (1985), and 310 km s^{-1} using the DM-distance conversion of Taylor & Cordes (1993). Both these mean speeds include old pulsars which would have left the Galaxy if they had been born with high velocities. The mean transverse velocity of pulsars young enough not to suffer such velocity selection is (using the Taylor & Cordes scale) an astonishing $360 \pm 70 \text{ km s}^{-1}$ (Lyne & Lorimer 1994), giving a mean space velocity of $460 \pm 90 \text{ km s}^{-1}$. These velocities are much larger than the birth velocities of massive stars, the progenitors of pulsars. Thus pulsars acquire such large velocities at or after their birth. The distribution of velocities, however, has not been well determined.

4.2 *Natal Velocity Kicks*

Pulsars born in binary systems can acquire velocity, immediately following the SN explosion. In the simplest, unavoidable model, the SN mass loss is symmetric in the frame of the exploding star. The resulting asymmetry in the frame of the center of mass of the binary system means that the binary center of mass must recoil. If more than half the total mass is lost, then the binary system is unbound and the neutron star will escape with a velocity of order its orbital velocity in the pre-supernova binary. However, this mechanism alone is insufficient to explain the observed velocity spectrum—in particular, pulsars with speeds approaching 10^3 km s^{-1} e.g. PSR 2224 + 65 (Harrison et al 1993, Cordes et al 1993). Several SNRs have been claimed to be associated with even faster moving pulsars (see Caraveo 1993). In addition, Cir X-1, a 16-day orbital period MXRB has a systemic velocity of $\sim 200 \text{ km s}^{-1}$ (Duncan et al 1993). All of these results indicate that pulsars must acquire additional velocities (in the frame of the exploding star) at, or shortly after birth by some other mechanism.

Such velocity kicks have also been invoked to explain the paucity of binary pulsars. Most massive stars are thought to arise in binaries, yet most pulsars are single. Velocity kicks can disrupt the binary system during the second explosion in the system, releasing an old neutron star and a new pulsar (Radhakrishnan & Srinivasan 1982, Backus et al 1982, Bailes 1989). Dewey & Cordes (1987) simulated the binary population and found that velocity kicks of ~ 100 to 200 km s^{-1} were sufficient to explain the observed binary and single pulsar population (though it is unclear if such low velocities would fit the new data discussed above).

The observed velocities require that kicks sometimes exceed 10^3 km s^{-1} . It is intriguing to note that large kicks appear to be associated with pulsars in or close to SNRs (Caraveo 1993). As many as a third of pulsars may be born with such high speeds. This is an astonishing conclusion because we do not find a large number of middle-aged ($\sim 10^6 \text{ y}$) fast moving pulsars. A related and more understandable observation is that the velocity spectrum of pulsars with $\tau_c < 10^7 \text{ y}$ is peaked at 300 km s^{-1} , a factor of 3 larger than that of older pulsars (Cordes 1986, Harrison et al 1993). This is usually explained as a selection effect by which such pulsars drift away from the Galactic plane in 10^7 y . It is interesting that the proper motion of PSR 2224 + 65 ($\tau_c \sim 10^6 \text{ y}$), the pulsar with the highest measured velocity, is along the Galactic plane—a confirmation of the above selection effect. The on-going all-sky and high latitude searches can significantly constrain the birthrate of fast moving pulsars.

4.3 Kinematics of LMBPs

In the recycling model, we expect LMBPs to have large systemic motions (Section 2.2.1), whether or not pulsars receive natal kicks. The systemic speed should be larger for systems of shorter orbital period (Bailes 1989).

Despite the fact that the current sample is heterogeneous, two simple conclusions can be drawn: 1. The scale height of LMBPs is at least about 0.6 kpc, and 2. the median transverse speed is 75 km s^{-1} , significantly smaller than that of the single pulsars. (In arriving at this latter inference we have used the measured proper motion, if available; otherwise, we derived the minimum z -velocity needed to attain their present z -height above the Galactic plane.) These results are in accord with the model discussed above. Velocity kicks between 100 and 200 km s^{-1} are sufficient to explain these trends. With a much larger data set, it should be possible to see a trend of larger systemic motions with smaller orbital periods. Conversely, long orbital period LMBPs should have smaller scale height than the short-period systems (Bailes et al 1994).

Johnston (1992), using the radial velocity data from optical observations of LMXBs, has carried out a similar exercise for LMXBs and arrives at a similar conclusion. Based on a kinematic analysis, Cowley et al (1988) argue that LMXBs have a tangential motion with respect to the Sun of $U \sim -68 \pm 46 \text{ km s}^{-1}$, and that therefore they are kinematically intermediate between Population I and II objects. Given the large errors, one could also conclude that $U = 0 \text{ km s}^{-1}$ i.e. LMXBs are Population I objects, albeit with large random velocity. Naylor & Podsiadlowski (1993) also argue for a Population I origin based on the observed spatial distribution.

5. MAGNETIC FIELD EVOLUTION

The magnetic field strength is the key parameter governing the evolution of neutron stars. High field strength is necessary for radio and X-ray pulsar ac-

tivity. Low field strength is invoked to explain the absence of pulsations in LMXBs. The real mystery about millisecond pulsars is not the origin of their spin periods, but the origin of their distinctly lower magnetic field strengths. Unfortunately, we have no comprehensive theoretical model for either the origin or the evolution of magnetic fields in neutron stars. In view of this, our focus is primarily phenomenological. For reviews from a theoretical point of view, see Chanmugam (1992) and Bhattacharya & Srinivasan (1994).

5.1 *Early Ideas*

The earliest statistical study of pulsars (Ostriker & Gunn 1969) suggested that pulsars are born with large field strengths, $B_i \sim 10^{12}$ G, and that the field decays exponentially with a characteristic timescale, $\tau_B \sim 4 \times 10^6$ y. Most but not all subsequent statistical studies appeared to essentially confirm this picture (e.g. Narayan & Ostriker 1990). A model-independent “proof” that was quoted in defense of field decay emerged from observations of pulsar proper motions (Lyne et al 1982). Comparison of the characteristic age τ_c with the kinematic age $\tau_z = |z|/v_z$, where z is the vertical distance from the Galactic plane and v_z is the vertical speed, showed that τ_c is systematically larger than τ_z , especially for large τ_c . Assuming an exponential field decay, $B(t) = B_i \exp(-t/\tau_B)$, one obtains

$$t_p = \frac{\tau_B}{2} \ln \left[1 + \frac{2\tau_c}{\tau_B} \left(1 - \frac{P_i^2}{P^2} \right) \right], \quad (5.1)$$

where P is the current period, t_p the true age of the pulsar, and P_i the initial period. These timescales need to be compared to the typical time taken by a pulsar, assumed to be born in the Galactic plane, to reach the highest $|z|$, the “turning point.” For v_z between 100 km s^{-1} and 200 km s^{-1} , this is ~ 50 My in the solar neighborhood. Since we believe that most pulsars are born in the Galactic plane and are rarely still luminous when they return to the plane, $\tau_z \sim t_p$. Interpreting the observations in light of Equation 4, Lyne et al (1982) inferred $\tau_B \lesssim 10$ My. This effect persists in the newer and larger proper motion data (Harrison et al 1993) with $\tau_B \lesssim 10$ My.

5.2 *Residual Fields*

A particularly powerful constraint on the secular evolution of the field strength can be obtained by optical observations of binary pulsars containing a white dwarf system. Since the white dwarf is born after the neutron star (Section 10.1), it is clear that the age of the white dwarf τ_{wd} , as deduced from comparison of the white dwarf luminosity with cooling models, must be a lower limit to t_p . White dwarfs have been detected in 0655+64 (Kulkarni 1986),

0820+02 (Kulkarni 1986, Koester et al 1992), and in the nearby millisecond pulsar J0437–4715 (Bailyn 1993, Bell et al 1993, Danziger et al 1993). Deep searches toward several millisecond pulsars, notably 1855+09 (Callanan et al 1989, Kulkarni et al 1991), have not detected the candidate companion; the white dwarf companion is presumed to be too cool to be seen by modern optical detectors. In all these cases, observations are consistent with $\tau_{\text{wd}} \sim \tau_c$. In the context of the ideas discussed in the previous section, Kulkarni (1986) proposed that magnetic fields stop decaying once they reach a “residual” value. The strength of this field appears to be in the range 3×10^8 G (PSR 1855+09) to 3×10^{11} G (PSR 0820+02).

The concept of a residual field has also arisen from other lines of reasoning, albeit more model dependent. Exponential decay of the field would result in radio lifetimes comparable to τ_B . The LMBP birthrate would then far exceed the birthrate of LMXBs (Bhattacharya & Srinivasan 1986, van den Heuvel et al 1986). From a stellar evolutionary point of view, Verbunt et al (1990) have argued that the LMXB 4U 1626–67 and Her X-1 (located at $z = 3$ kpc) are $\gtrsim 10^8$ y old; yet both are X-ray pulsars and have strong magnetic field strengths. It therefore appears that the residual field strength encompasses the entire range of pulsar magnetic field strengths: 10^8 G to $\gtrsim 10^{12}$ G. The simplest conclusion (ignoring the proper motion data), is that magnetic fields do not decay *at any field strength*.

5.3 No Field Decay?

The realization that fields may not decay has prompted reanalyses of the pulsar population. Monte Carlo comparison of the data with simulated pulsar populations, subjected to a detection process with appropriate selection effects, have led to confusing results. Narayan & Ostriker (1990) find evidence for field decay; Bhattacharya et al (1992) and Wakatsuki et al (1992) find evidence for little if any field decay ($\tau_B > 10^8$ y). It is disturbing that these three analyses use essentially the same data but arrive at divergent conclusions. This simply may illustrate the dangers of trying to determine a $\gtrsim 7$ dimensional distribution function (in P , $\dot{P}$, S_{400} , DM , b , μ_b , μ_ℓ) from only a few hundred points. Perhaps the greatest systematic uncertainties in such studies are the treatment of selection effects and kinematics, and the unknown luminosity evolution law, an issue that bedevils such parametric pulsar population studies—see especially the last two paragraphs in Narayan & Ostriker (1990), the introduction in the Lorimer et al (1993) paper, and Michel (1991). Bhattacharya & Srinivasan (1994) in their review article refer to unpublished work which attempts to circumvent the parametric approaches taken in the above studies by analyzing the pulsar birth current as a function of P and B . They conclude that there is little need for field decay.

Despite the above discussion, there are still two observations that are hard to square with the assumption of *no* field decay: 1. the proper motion data (τ_z vs t_c) discussed above, and 2. the origin of pulsars with field strengths between 10^{11} and 10^{12} G, a group which we will henceforth refer to as the intermediate strength pulsars (see Figure 1). Bailes (1989) addressed these two issues, first arguing that the proper motion data were noisy and consistent with no field decay. However, in our opinion, the effect discussed above persists in the latest proper motion data (Harrison et al 1993) which has four times as many measurements as the Lyne et al (1982) sample. Next, even for $\tau_B = \infty$, τ_c is an overestimate of t_p if $P \sim P_i$ [see Equation (2.1)]. Thus Bailes argues, in the context of the recycling model, that the intermediate strength pulsars are the first-born pulsars, mildly spun up prior to the disruption of the system at the time of the second supernova. If so, then τ_c could be significantly overestimated, especially for those pulsars close to the rebirth line. Inspection of the five pulsars with largest τ_c in Figure 7 of Harrison et al (1993) shows that these pulsars have, for their observed period, an inferred magnetic field strength within a factor of two of the equilibrium value. Given the uncertainties in the constants in Equation (2.1), we conclude that most of these could well be recently recycled pulsars.

5.4 Tests for Field Constancy

The least model-dependent constraints on τ_B can be obtained by determining the fraction of pulsars returning to the Galactic plane and the fraction of pulsars above the interstellar electron layer, whose half thickness is estimated to be $h_e \lesssim 1$ kpc (Bhattacharya & Verbunt 1991). Pulsars will reach the death line shown in Figure 1 in time $t_D \sim 80/B_{12}$ My, where B_{12} is the field strength in units of 10^{12} G. Unless their luminosity drops on a timescale much shorter than t_D , pulsars with $B_{12} \lesssim 1$ will be able to reach the turning point ($|z| \sim 2$ kpc, for $v_z \sim 100$ km s $^{-1}$) and return to the Galactic plane within their radio lifetime. Bhattacharya et al (1992) present some evidence indicating that a small fraction of old pulsars lie above the electron layer. Harrison et al (1993) find 8 pulsars (out of $\lesssim 10^2$) returning to the Galactic plane. More high latitude pulsar searches and proper motion observations, especially of pulsars with $B \sim 10^{11}$ – 10^{12} G are needed to firmly constrain τ_B for single pulsars. Recently, Chen & Ruderman (1993) have argued that the location of the death line depends on the magnetic field topology. Thus pair production (and hence presumably radio emission) in pulsars with the same dipole field $P\dot{P}$ may cease at a range of periods, whose values depend on the pulsars' individual field topologies. They dub this range, the "Death Valley." Statistical analyses of the ongoing high latitude surveys will be most useful in demarcating the Death Valley in the B - P diagram.

5.5 Bimodal Field Distribution

In Section 3.1, we remarked that the binary pulsar systems with massive companions, the HMBPs 1913 + 16, 1534 + 12, and 0655 + 64, are supposed to emerge from massive binaries. These three pulsars have $B \sim 10^{10}$ G. In contrast, the magnetic field strengths of pulsars with low mass binary companions, the LMBPs such as 1855 + 09, etc have field strengths $\lesssim 10^9$ G. This led Kulkarni (1992) to propose that there is a “field gap” between $B = 10^9$ to $B = 10^{10}$ G since there is no known selection effect preventing the discovery of pulsars with field strengths in this range. Indeed, the selection effects favor the detection of pulsars with larger field strength.

Camilo et al (1994) argue that the field gap appears to persist in a larger sample. Correcting for the kinematic contributions to $\dot{P}$ (see Section 8), they find that for the LMBPs, $10^8 \lesssim B \lesssim 5 \times 10^8$ G. The discovery of an isolated pulsar, J2235 + 1506 with $B = 2.7 \times 10^9$ G (Camilo et al 1993), presumed to be a member of an HMBP system that was disrupted on the second explosion, suggests that the field gap is not due to some fundamental physics. Most likely, there are two distributions with two different mean values: one of intermediate field strength pulsars with B centered on $10^{11.5}$ G of which the HMBPs are the tail end, and one of low field strength with $B \sim 10^{8.5}$ G, constituting the rapidly rotating LMBPs. Note that LMBPs with slow rotation rates (e.g. PSR 0820+02) nominally belong to the intermediate group.

Note that there are no disk LMBPs with $B < 10^8$ G. However, the histogram of B for the rapidly rotating LMBPs shows that there are as many pulsars (six) with $B < 2 \times 10^8$ G as above this value, i.e. there is a steepening towards lower B . It is possible that the Galaxy contains a new class of neutron stars—low magnetic field strength neutron stars. It is unclear whether such objects would shine in the radio window at all (Section 6.2, Phinney 1994).

5.6 A Phenomenological Model

Following Bailes (1989), we find that the available data are consistent with the following picture:

1. Pulsars are born with Crab-pulsar-like properties: $B \sim 10^{12}$ G, $P \sim 20$ ms. The first half of the statement is motivated by the high field strengths of young pulsars in SNRs (Figure 1).
2. The magnetic field does not decay unless there is accretion. The age of the neutron star in Her X-1 is $\gtrsim 10^7$ y, given $z = 3$ kpc. The white dwarf in PSR 0820+02 is 2×10^8 y (Koester et al 1992). Thus, if there is any decay at all, $\tau_B \gtrsim 40$ My. A very gradual decay of the field strength over the age of the Galaxy, $\tau_D \sim 10^{10}$ y, cannot be ruled out. The highest B field of a cluster pulsar (PSR 1820–30B; Biggs et al 1993) is 10^{11} G (see also

Figure 1). Assuming that this pulsar has not been recycled, we conclude that neutron star magnetic fields cannot decay by a factor greater than ~ 30 over a timescale τ_D .

5.7 Models for Field Reduction

Above, we have interpreted the observations as if neutron star magnetic fields do not evolve unless there is accretion. The first part of the statement may have theoretical blessings. In a recent review, Goldreich & Reisenegger (1992) find no physical mechanism for fast field decay. They speculate that magnetic fields buoyed by Hall drift may undergo a turbulent cascade terminated by ohmic dissipation on a timescale $\sim 5 \times 10^8 / B_{12}$ y.

The second part of the above statement, the origin of accretion induced field decay, is a mystery and without a proper physical model. There are two qualitatively different models: 1. crustal models where ΔM —the amount of matter accreted (Taam & van den Heuvel 1986, Shibazaki et al 1989, Romani 1990)—or an inverse battery effect (Blondin & Freese 1986) leads to field reduction, and 2. interior models, where interaction of the magnetic flux tubes with angular momentum vortices in the interior leads to field expulsion (Srinivasan et al 1990).

The crustal models are attractive from the point of view of the bimodal field distribution. HMBPs with their smaller inferred ΔM have higher field strengths, whereas those LMBPs with large inferred ΔM have smaller B_s . However, observational data do not support a simple relation between accreted matter and field strength (Verbunt et al 1990). In addition, crustal models need to be fine tuned since it is the accretion process that causes both field reduction and spin-up [see also the discussion following Equation (3.2)].

In the interior model, the angular momentum vortices are assumed to be strongly coupled to the magnetic flux lines in the interior. Thus as the young pulsar slows down, magnetic flux lines are brought up to the crust where they are assumed to decay. In this model, low field pulsars, at an earlier phase in their life, were spun down to very long periods before being spun up by accretion. The major difficulty in this model is that it requires rapid decay in the crust (e.g. 1E2259+586, discussed above) which appears to be ruled out by theoretical considerations. In addition, the model provides no natural explanation for the bimodal field distribution discussed above.

In a series of papers, Ruderman (1991) has developed a variant of an interior model. Here, the rotational-magnetic-field coupling results in rearranging the magnetic flux lines, thus effecting changes in the dipole field strength. As the pulsar slows down, the dipole moment diminishes, explaining the intermediate strength field pulsars. There is one basic difficulty with this model: While magnetic pole migration can reduce the dipole field strength, it does not reduce strengths of individual field lines. However, during the last stages

of the spin-up process, when the pulsar is spinning at its equilibrium period, the inferred Alfvén radius must equal the Keplerian corotation radius, $R_{\text{eq}}/R_n \simeq 2(P/\text{ms})^{2/3}$, where R_n is the radius of the neutron star. Strong multipole field strength ($\sim 10^{12}$ G) would result in large R_{eq} (Arons 1993), and it may be difficult to obtain millisecond rotation periods. Furthermore, if the magnetic geometries of millisecond pulsars were very nondipolar, one would expect the pulse and polarization profiles to differ from those of ordinary pulsars. However, the pulse shapes of millisecond pulsars now appear to be no different from those of ordinary pulsars (Bailes et al 1994); the same appears to be true of the meager polarization data (Thorsett & Stinebring 1990). Further polarimetric observations of millisecond pulsars can potentially refute or confirm Ruderman's model.

Romani's (1993) model combines aspects of Ruderman's model and crustal models by having the overburden of the accreted matter push magnetic field lines below the neutron star surface and thereby reduce the external dipole. The model claims to predict a floor value, $B \sim 10^8$ G by having advection cease when the field goes below 10^8 G. The model is phenomenologically appealing in that it explains the field gap (a consequence of different ΔM in the HMBPs versus LMBPs) and the near constancy of field strength of millisecond pulsars. In contrast to the disk LMBPs, cluster pulsars have field strengths ranging from 10^8 to 10^{11} G; this is explained in this model by appealing to the diversity of the tidal products, from long-lived LMXBs to short-lived accretion tori resulting from star-destroying encounters. However, it is fair to say that we still have no satisfactory physical model that can explain the magnetic field spectrum of binary and millisecond pulsars.

6. PULSAR AND NEUTRON STAR PHYSICS

The ultimate importance of millisecond and binary pulsars lies not so much in their origin and evolution, but rather in the fundamental experiments in physics and astrophysics which nature performs for us with them. Some of these experiments involve the emissions and environmental impact of the pulsars themselves. The thermal and crustal histories of their neutron stars differ from those of ordinary pulsars, and their magnetospheres are much smaller. The differences between the emissions and relativistic winds of pulsars with spin periods and dipole fields differing by four orders of magnitude might be expected to provide clues to the physics of neutron star interiors, crusts, and magnetospheres. The energy stored in the rotational energy of a millisecond pulsar can be comparable to the nuclear energy released by its main sequence progenitor, and to the energy released in the accretion required to spin up the neutron star. The release of this energy in the pulsar's relativistic wind can have striking effects on the pulsar's stellar companions, and on its interstellar environment.

Other experiments make use of two properties of millisecond pulsars: 1. their excellence as clocks, accurate to about $1\ \mu\text{s}$ over decades, which allows changes in their distance from Earth to be determined to a precision of $c\ (1\ \mu\text{s}) = 30\ \text{m}$, and 2. the brevity and extremely high brightness temperature of the broad band radar pulses they emit. These powerful radar pulses act as both passive and active probes of the media through which they travel. We now review the many diverse experiments.

6.1 Calorimetry

One of the first compelling arguments that ordinary pulsars were neutron stars was the identification of the Crab nebula as a calorimeter for the Crab pulsar (cf Manchester & Taylor 1977). This showed that the Crab pulsar must have (and have had) a relativistic wind of luminosity $d(I\Omega^2)/dt$, where $I \simeq 10^{45}\ \text{g cm}^2$ is the moment of inertia of the neutron star and Ω its angular spin frequency. Millisecond pulsars have several other types of calorimeters.

6.1.1 WIND NEBULAE As a pulsar moves through the interstellar medium (ISM), its relativistic wind will form a bow shock around the pulsar. Ahead of the pulsar the contact discontinuity between the pulsar wind and the interstellar medium will lie at the point where the ram pressure (in the pulsar's rest frame) of the incoming ISM equals that of the pulsar's wind. Such a wind nebula is observed around PSR 1957+20. The ISM side of the contact discontinuity is an H α -emitting nebula of cometary form (Kulkarni & Hester 1988, Aldcroft et al 1992) aligned along the direction of the pulsar's proper motion μ (Ryba & Taylor 1991). The head of the nebula is projected $\theta_s = 4''$ ahead of the pulsar. A parabolic model of the bow shock predicts that independent of the (unknown) line-of-sight velocity,

$$\theta_s = \left(\frac{I\Omega\dot{\Omega}}{4\pi C^2 \rho_i c} \right)^{1/2} \frac{1}{D^2 \mu} = \frac{2.4\ I_{45}}{C D_{\text{kpc}}^2 n_0}, \quad (6.1)$$

where $C \sim 0.7$ if the ISM shock is adiabatic, $C \rightarrow 1$ if it is radiative, and the pre-shock ISM has hydrogen number density $1\ n_0\ \text{cm}^{-3}$. Observations of the companion star to 1957+20 (Djorgovski & Evans 1988) suggest that $0.7 < D_{\text{kpc}} < 1.3$ (see below). The dispersion measure distance (Taylor & Cordes 1993) is $1.5\ \text{kpc}$. Reasonable equations of state have $1 < I_{45} < 2$ (see Table 3). The fact that there is a Balmer-dominated shock in the ISM suggests that the pre-shock ISM is mostly neutral, so $n_0 \gtrsim 0.3$. The rough agreement of Equation 6.1 with the observed $\theta_s = 4''$ thus implies that the ISM encounters a (a) roughly isotropic pulsar wind, which (b) carries most of the spin-down luminosity, and (c) acts fluid-like on the $\sim 10^{17}\ \text{cm}$ scale of the nebula. If most of the wind energy is carried in ions, then (c) requires that the gyro radii of the ions be smaller than the nebula. This is true as long as the ions were accelerated

across no more than the voltage across a simple dipole polar cap (cf Arons & Tavani 1993).

6.1.2 COMPANION HEATING The companions of millisecond pulsars are bombarded by relativistic particles and electromagnetic waves from the pulsar. The constituents of the pulsar wind deposit their energy at energy-dependent column densities in the companion and its atmosphere: hundredths of g cm^{-2} for X rays to hundreds of g cm^{-2} for ultrarelativistic ions and gamma rays. The energy deposited below the photosphere will be thermalized and reradiated within a few minutes. The side of the companion illuminated by the pulsar wind will therefore be hotter than the “dark” side of the companion. The flux at the surface of a companion with orbital period P_b from an isotropic pulsar wind is

$$F = \frac{I \Omega \dot{\Omega}}{4\pi a^2} \\ = 4.2 \times 10^{20} I_{45} \left(\frac{\tau_c}{\text{yr}} \right)^{-1} \left(\frac{P_b}{\text{d}} \right)^{-4/3} \left(\frac{P}{\text{ms}} \right)^{-2} \text{erg cm}^{-2} \text{s}^{-1}. \quad (6.2)$$

For pulsar 1957+20, this, combined with models for the cooling of the companion, predicts (Phinney et al 1988) that the sub-pulsar part of the companion should have an effective temperature of ~ 8000 K, while the dark side should be cooler than 2000 K. These predictions have been confirmed by observations (Djorgovski & Evans 1988, Eales et al 1990, Callanan 1992). This does not directly demonstrate that the bulk of the pulsar’s spin-down energy is carried off in a form capable of penetrating below the $\sim 1 \text{ g cm}^{-2}$ photosphere of the companion. This is because in PSR 1957+20, the pulsar wind must be shocked far above the companion’s surface by the dense plasma in the companion’s wind or magnetosphere (Ryba & Taylor 1991; see Figure 1 of Phinney et al 1988). It could be that radiation from the shock, or particles accelerated there—and not the pulsar wind itself—impinge on the companion.

Cleaner diagnostics of the penetrating power of pulsar winds may be provided by observations of pulsars with more massive white dwarf companions. Applying Equation (6.2) to Table 1 reveals that besides 1957+20, the strongest companion heating should occur in PSRs J0034 – 0534, J2317 + 1439, and J0437 – 4715, the illuminated sides of whose companions should be respectively about 25%, 2%, and 1% hotter than the ~ 4000 K of their unirradiated sides. In addition to photometry, line spectra of these objects and 1957+20’s companion will prove interesting. Spectral line profiles, heights, and depths are sensitive to the temperature profile above the photosphere, and thus can be used to determine the heat deposition as a function of column density. The limits on emission lines from 1957+20’s companion (Aldcroft et al 1992) already rule out an X-ray heated wind from a compact companion (Levinson & Eichler 1991, Tavani & London 1993) as a significant source of mass loss in that system.

Heating of a convective companion can have a dramatic indirect effect on its structure. When one irradiates a star, which in isolation would have a low effective temperature, it initially develops a temperature inversion in its envelope. Heat is conducted inwards until, at the effective temperature set by the irradiation, an isothermal zone develops which extends inwards to the radius in the convective envelope at the same temperature. If this isothermal zone is at $\gtrsim 10^4$ K, its (hydrogen or helium ionization) opacity will be 1–2 orders of magnitude higher than it was at the unirradiated photosphere. In stars with deep convection zones, the isothermal layer can present the dominant bottleneck to the escape of radiation from the stellar interior and can therefore trap heat trying to escape from the stellar interior, bloating the star and causing it to expand on its thermal timescale. This effect, pointed out by Phinney et al (1988), has been studied in detail for isotropically irradiated stars by Podsiadlowski (1991), Harpaz & Rappaport (1991), and d'Antona & Ergma (1993). The star swells, and therefore transfers mass, on the thermal timescale, which is much shorter than the nuclear timescale. This results in short X-ray lifetimes, and rapid orbital evolution of low mass X-ray binaries, and expansion and perhaps destruction of close pulsar companions (Ruderman et al 1989, van den Heuvel & van Paradijs 1988, Phinney et al 1988).

In nature, however, the stars are irradiated on one side only. An isothermal zone with high opacity on one side of the star would simply force more of the star's internal luminosity to emerge on the unirradiated side—which would not necessitate much change in the convective envelope. Dramatic changes like those in isotropically irradiated stars would occur only if fast azimuthal winds, differential rotation, or asynchronous rotation carried the heat of irradiation to the unirradiated side of the companion in less than the cooling time of the isothermal zone. None of this is occurring, at least in PSR 1957+20's companion, otherwise the large temperature difference between its pulsar-illuminated face and its “dark” face would not be observed.

Furthermore, much of the observational evidence for irradiation-induced large $\dot{P}_b$ in X-ray binaries and pulsar companions has recently evaporated. Of the four cases adduced by Tavani (1991) as evidence, Cyg X-3's companion has been shown to be a luminous Wolf-Rayet star whose spontaneous wind naturally explains the magnitude and sign of the systems $\dot{P}_b$ (van Kerkwijk et al 1992, van Kerkwijk 1993); 4U1820 – 30's apparently large negative $\dot{P}_b$ (van der Klis et al 1993a) has recently been shown to be an artifact of changes in the shape of the X-ray light curve (van der Klis et al 1993b); and in both of the remaining systems, X1822 – 371 (Hellier et al 1990) and EX00748 – 676 (Parmar et al 1991), the total shift in the light curve on which the $\dot{P}_b$ is based is still a tiny fraction of an orbital period. Finally, what was once a large negative $\dot{P}_b$ of the eclipsing pulsar 1957+20 (Ryba & Taylor 1991) has changed sign since 1992 (Arzoumanian et al 1994), and is therefore probably a random walk in orbital

phase like those commonly observed in cataclysmic variables (Warner 1988), which are produced by magnetic cycles (Applegate 1992) or other torques from the companion star.

6.2 Diagnostics of Pulsar Magnetospheres

A pulsar of spin period P_{-3} ms has its light cylinder at $c/\Omega \sim 5P_{-3}$ neutron star radii, and the radius where the orbital frequency equals the spin frequency is $R_{\text{eq}} = (GM/\Omega^2)^{1/3} \sim 2P_{-3}^{2/3}$ neutron star radii. The spin-down torques (Krolik 1991) and equilibrium spin-up line (see Equation 2.1) in millisecond pulsars are thus much more sensitive to nondipolar magnetic multipoles than they are in slower pulsars. This has been used to show, for example, that the millisecond pulsars' surface magnetic fields are not dominated by a quadrupole, nor concentrated in a single polar clump (see Section 5.7; Arons 1993).

The above arguments suggest that millisecond pulsars have surface fields of roughly dipole strengths. It seems not to be generally appreciated that millisecond neutron stars with $\dot{P}$ s lower than the lowest observed $\dot{P}$ s would not then be able to initiate traditional magnetic e^+e^- pair cascades, and consequently might not be radio luminous (Phinney 1994; but note that other types of pair cascades may occur—see below). The predominance of 10^8 G fields in millisecond pulsars may thus not be a consequence of magnetic field decay, but simply a radio selection effect. The high brightness temperature of pulsar radio emission is hard to explain except as coherent emission from a highly relativistic electron-positron plasma. [Melrose (1992) gives a critical review of the models.] Such a plasma can be produced by pair cascades above a vacuum gap in the magnetosphere of a rotating neutron star (Ruderman & Sutherland 1975). A cascade requires two ingredients: particles that radiate photons, and conversion of the photons to more particle pairs. A gap of thickness h has an electric field $E \simeq \Omega Bh/c$, and potential drop $\Delta V = Eh/2$. This can accelerate electrons to a maximum Lorentz factor

$$\gamma_{\text{max}} = \min[e\Delta V/(m_e c^2), (E\rho^2/e)^{1/4}], \quad (6.3)$$

where the second limit is imposed by curvature radiation reaction when the radius of field line curvature is $\rho \equiv 10^6 \rho_6$ cm. The electrons radiate curvature photons of frequency $\omega_c \sim \gamma_{\text{max}}^3 c/\rho$, which can pair create on the magnetic field if (cf Ruderman & Sutherland 1975)

$$\frac{\hbar\omega_c}{2m_e c^2} \frac{B_{\perp}}{B_q} > \frac{1}{14}, \quad (6.4)$$

where $B_q = m_e^2 c^3 / e\hbar = 4.4 \times 10^{13}$ G. Except for a prescient paper by Goldreich & Keeley (1972), previous discussions of the pair creation limit, or “death line” (Chen & Ruderman 1993), have ignored the radiation reaction limit, and insert the first term on the right of (6.3) into (6.4), to derive death lines. These depend

on ρ , and therefore on the assumed magnetic field structure. Lines with small ρ are consistent with the boundary of high field radio pulsars in the B - P plane (see Figure 1, Chen & Ruderman 1993). But for millisecond pulsars, the appropriate limit is the second, radiation reaction term in (6.3). Inserting this into (6.4) with the maximum vacuum E , gives pair creation only if

$$B > 5 \times 10^7 P_{\text{ms}}^{9/14} \rho_6^{-2/7} G \quad (6.5)$$

(Phinney 1994; this conservative limit assumes a field line topology such that $B_{\perp} \sim B$), which for $\rho_6 \sim 1$ is plotted as the lower part of the “death boundary” in Figure 1. However, X rays from a heated polar cap (cf J0437–4715 discussed below) can pair create with curvature photons ($\gamma\gamma \rightarrow e^+e^-$) even for magnetic fields well below (6.5) (Phinney 1994); so it is unclear if this is a hard boundary. If magnetic pair creation were the only mechanism, pulsars with lower magnetic fields would be sources of GeV pulsed curvature gamma rays, and might be surrounded by bow shocks in the interstellar medium (Arons 1983), but would be very difficult to detect as pulsed sources unless they had outer gaps (Chen & Ruderman 1993, Halpern & Ruderman 1993) to produce X-ray or optical synchrotron emission.

Limits on gamma rays (Fichtel et al 1993) for many millisecond and binary pulsars from the EGRET sky survey (luminosity per octave of gamma ray energy E , $EL_E \lesssim 4 \times 10^{32} \text{ erg s}^{-1} \text{ sr}^{-1} D_{\text{kpc}}^2$ for $100 \text{ MeV} < E < 3 \text{ GeV}$ at distance D_{kpc} kpc) are already of order, or below the (crude) predictions of outer magnetosphere models ($L_{\gamma} \sim 10^{-2} I \Omega \dot{\Omega}$, Chen & Ruderman 1993), but do not constrain most polar cap models.

Kilovolt X rays have been detected from two millisecond pulsars: J0437–4715 (Becker & Trumper 1993) and 1957+20 (Kulkarni et al 1992b). In the former, the X rays are pulsed at the 5.75 ms radio period, and have $L_X \sim 4 \times 10^{-4} I \Omega \dot{\Omega}$. There is a hint that the spectrum may have both a thermal ($kT \sim 0.14 \text{ keV}$) and a power-law component. In the latter ($L_X \sim 5 \times 10^{-5} I \Omega \dot{\Omega}$), too few X-ray photons were detected with high time resolution (Fruchter et al 1992) to tell if the X rays are pulsed or modulated at the orbital frequency (Kulkarni et al 1992b). They might therefore be generated at the shock around the eclipsing companion, or at the reverse shock in the wind nebula (Kulkarni et al 1992b, Arons & Tavani 1993). The soft X-ray emission from these pulsars carries a fraction of $I \Omega \dot{\Omega}$ similar to that observed from some nearby ordinary pulsars (Ögelman 1993, Yancopoulos et al 1994). This might suggest that all pulsars have heated polar caps, though the ratio of X-ray to spin-down luminosity varies over several orders of magnitude.

6.3 Neutron Star Physics

Binary and millisecond pulsars provide some of the most rigorous constraints on nuclear equations of state. Neutron stars constructed with very soft equations of

Table 2 Pulsars with measured masses

Pulsar	M_p^a	M_c	$M_p + M_c$	Reference
1913+16	1.4411 (7)	1.3874 (7)	2.82843 (2)	Taylor 92
1534+12	1.34 (5)	1.34 (5)	2.6781 (7)	Taylor 92
2127+11C	—	—	2.712 (5)	Anderson 92
2303+46	1.2 (3)	1.4 (2)	2.53 (8)	Thorsett et al 93b
1855+09	1.5 (2)	0.26 (2)	—	Kaspi et al 94b
1802-07	1.4 (3)	0.33 (1)	1.7 (4)	Thorsett et al 93b

^aAll masses in units of $M_\odot$.

Notes: Pulsars in the top group have eccentric orbits and high-mass (probably neutron star) companions. PSR 2127+11C is in the globular cluster M15, and probably formed by exchange (Phinney & Sigurdsson 1991). In the bottom group, PSR 1855 + 09 has a low-mass helium white dwarf companion in a circular orbit; PSR 1802 - 07 is in the globular cluster NGC 6539, and its companion, in an eccentric orbit, probably results from exchange or a tidal 3-body encounter (see Phinney 1992).

state, including pion-condensate and the recently popularized Kaon-condensate models (Brown & Bethe 1994) give maximum stable neutron star masses of $\simeq 1.4\text{--}1.5 M_\odot$. This is because they have small radii, so the general relativistic contributions ($\sim GM/Rc^2$) to the radial stability criterion are large even for low masses. [Kunihiro et al 1993 have written a comprehensive monograph on recent work on pion and Kaon condensates; at sufficiently high densities in nuclear matter, the effective mass of the strange K^- meson can fall below that of the π^- meson and become an energetically favored constituent of neutrons star cores, reducing the pressure support and the neutron star radii (Kaplan & Nelson 1986, Politzer & Wise 1991, Brown et al 1992).] General relativistic effects in timing binary pulsars (see below) allow gravitational masses of the component stars to be measured, sometimes with high precision (see Table 2).

6.3.1 MASSES AND SOFTEQUATIONS OF STATE The high mass of PSR 1913+16 rules out, for example, the Reid soft-core equation of state of Pandharipande (1971) and that of Canuto & Chitre (1974), and is severely constraining to modern Kaon-condensate models. Even more severe constraints may well emerge in the future. The pulsars with the best determined masses so far are slowly rotating and in double neutron star binaries, for which standard evolutionary models (cf Verbunt 1993) do not require much accretion onto the pulsar. Nonstandard models with much higher accretion rates during common envelope evolution (Chevalier 1993) would turn most high mass X-ray binaries into black hole binaries, particularly if the equation of state is soft. However, even standard models for the evolution of many other binary pulsars predict large accreted masses, unfortunately all in systems where the neutron star masses are still ill-determined. Equation (3.2) shows that for a pulsar like 1957+20 to reach its

1.6 ms spin period, it must have accreted $\gtrsim 0.1 M_{\odot}$. Models of the formation of pulsars with low mass white dwarf companions (the large fourth group in Table 1) suggest that much of the $\sim 0.7 M_{\odot}$ lost by the progenitor of the white dwarf should have been accreted by the neutron star. Mass-conservative models are most compelling for the long-period binaries ($P_b > 50$ d) for which the predictions of mass and angular-momentum conserving models give reasonable accord between the birthrates of such pulsars (Section 3.3; Lorimer 1994) and their low-mass X-ray binary progenitors (Sections 10.1, 10.3, Verbunt & van den Heuvel 1994). Accretion of mass ΔM_0 onto an initially $1.3\text{--}1.4 M_{\odot}$ neutron star increases its gravitational mass by $\Delta M \simeq 0.74 \Delta M_0$. [See Woosley & Weaver's table 1 (1986), Woosley et al's table 2 (1993), and Weaver & Woosley (1993) for discussion of the iron core masses at the end of massive star evolution, and the neutron star masses expected to result after supernova explosion.] Thus conservative models for the formation of the neutron stars with white dwarf binaries predict that they should have gravitational masses of $\simeq 1.8\text{--}1.9 M_{\odot}$. These greatly exceed the maximum stable masses of equations of state with Kaon or pion condensates, and are in the midst of the range of maximum stable masses for modern equations of state without Kaon condensation (cf Table 3, and Cook et al 1994). It may prove possible to measure accurate masses for neutron stars in some of these binaries. This can be done in fortuitously edge-on systems such as 1855+09 (see Table 2), where relativistic time delays can be measured, and more generally by optical spectroscopy of the white dwarf companions, whose orbital velocities, combined with the mass function derived from pulsar timing, would determine a strict lower limit to M_p (e.g. the preliminary results on PSR1957+20 by Aldcroft et al 1992). Such measurements will allow us to decide which is wrong: conservative evolution models, or soft nuclear equations of state.

6.3.2 MINIMUM STABLE PERIODS AND EQUATIONS OF STATE Just as soft equations of state give small neutron stars with low maximum masses, hard equations of state give large neutron stars with high maximum masses. Large neutron stars cannot spin rapidly, or the speed ΩR of matter at their surface would exceed the escape speed $[(2GM)/R]^{1/2}$. If one spins up from rest a neutron star of mass M_s and areal radius R_s , the minimum spin period at which the star begins to shed mass from its equator is, to a good approximation (cf Figure 27 of Cook et al 1994, and Haensel & Zduk 1989)

$$P_{\min} = 0.77 \left(\frac{1.4 M_{\odot}}{M_s} \right)^{1/2} \left(\frac{R_s}{10 \text{ km}} \right)^{3/2} \text{ ms.} \quad (6.6)$$

If the star were a rigid Newtonian sphere, the coefficient of 0.77 would be 0.46—the difference is due to centrifugal distortion, which makes the equatorial radius of a spinning star larger than that of a static one, and to relativistic effects. Assuming the masses of the two fastest known pulsars, 1937+21 and 1957+20

Table 3 Neutron star properties for three nuclear equations of state^a

EOS	Model	M ($M_{\odot}$)	$P_{\min}$ (ms)	I ($10^{45} \text{ g cm}^{-2}$)	R_e (km)	ϵ	ρ_c ($10^{15} \text{ g cm}^{-3}$)
F (soft)	canonical	1.4	0.94	0.86	9.1	0.26	3.2
	$M_{\max}$	1.46	—	0.74	7.9	0.33	5.1
	$\Omega_{\max}$	1.67	0.50	—	10.9	—	4.4
FPS (med.)	canonical	1.4	0.88	1.2	10.8	0.21	1.3
	$M_{\max}$	1.80	—	1.4	9.3	0.35	3.4
	$\Omega_{\max}$	2.1	0.53	—	12.4	—	3.0
L (hard)	canonical	1.4	1.4	2.1	15.0	0.15	0.43
	$M_{\max}$	2.7	—	4.7	13.7	0.35	1.5
	$\Omega_{\max}$	3.3	0.76	—	18.2	—	1.3

^aEOS = equation of state; $P_{\min}$ is the minimum rotation period for a model of mass M with the EOS; I is the moment of inertia (for slowly rotating stars); R_e is the equatorial circumferential radius (for slowly rotating stars, it is the same as the Schwarzschild radius); $\epsilon = z/(1+z)$ is the fraction of rest mass energy released in dropping a particle onto the neutron star from infinity, where z is the surface redshift for slowly rotating stars; ρ_c is the central energy density/ c^2 , to be compared to the baryon density at nuclear saturation $\sim 0.17 \text{ fm}^{-3} \simeq 2.8 \times 10^{14} \text{ g cm}^{-3}$. Three models are given for each equation of state: one for a canonical neutron star of gravitational mass $1.4 M_{\odot}$, one for the slowly rotating star of the maximum stable mass $M_{\max}$, and one for the star with maximum spin frequency $\Omega_{\max}$ (nearly the same as the rotating star of maximum mass). Data are from Friedman & Ipser (1992), Cook et al (1994), and Arnett & Bowers (1977).

Equation of state F (Arponen 1972) is one of the softest not yet conclusively ruled out.

Equation of state FPS (Lorenz, Ravenhall & Pethick 1993) is a recent model without a pion or kaon condensate phase. Adding such a phase transition at a few times nuclear density would reduce $M_{\max}$ by $\simeq 0.2 M_{\odot}$, R_e by $\simeq 1 \text{ km}$, and $P_{\min}$ by $\sim 30\%$ (cf pp. 266–70 of Kunihiro et al 1993).

Equation of state L (Pandharipande & Smith 1975) is one of the hardest not yet conclusively ruled out.

to be $\sim 1.4 M_{\odot}$. Equation (6.6) would require the radii of static neutron stars of that mass to be $\lesssim 16 \text{ km}$. A precise mass for PSR 1957+20 may eventually be determined by optical spectroscopy of its companion (Aldcroft et al 1992).

Since reasonable equations of state give $P_{\min} \simeq 0.7 \text{ ms}$, and since pulsars with commonly observed dipole fields of $1\text{--}2 \times 10^8 \text{ G}$ have even shorter equilibrium periods at Eddington accretion rates in simple models (Figure 1), we are encouraged to hope that pulsars with much shorter spin periods may yet be found.

If such discoveries are not made, the result in isolation could be interpreted in several ways. An absence of shorter periods would naturally occur if neutron stars have a hard equation of state, and $P_{\min}$ is indeed $\sim 1.5 \text{ ms}$ (Friedman et al 1988). It might also occur if they have a very soft equation of state, and all neutron stars that accrete enough to spin them to $\lesssim 1.5 \text{ ms}$ exceed $M_{\max}$ and collapse to black holes. It might also occur if accretion cannot spin a neutron star up to breakup. Newtonian stars without magnetospheres can be spun nearly to breakup by accretion (Popham & Narayan 1991, Paczyński 1991). However, it

has been suggested that gravitational radiation from unstable nonaxisymmetric modes would remove angular momentum from rapidly spinning neutron stars so quickly that they could never be spun to periods less than 1.5 ms (Papaloizou & Pringle 1978, Wagoner 1984). These high- m modes have negative energy and in any dissipationless rotating star grow unstably through gravitational radiation reaction (Friedman & Schutz 1978). (In a frame rotating with the neutron star these modes are retrograde, and when excited, reduce the star's angular momentum; as measured by an inertial observer at infinity, they are prograde and thus carry positive angular momentum away, increasing their amplitude.). More recent calculations suggest that these modes are strongly damped: by bulk viscosity for $T > 10^{10}$ K (Cutler et al 1990, Friedman & Ipser 1992), and by scattering of normal particles on superfluid vortices for $T < 10^9$ K (Mendell 1991, Lindblom & Mendell 1992). If correct, and neutron stars are superfluid, this extinguishes hope for gravitational radiation reaction as a period-limiting device.

Another possibility is that the accretion disk itself removes more angular momentum, and therefore exerts less torque, than in simple models (Ghosh & Lamb 1992). It might appear that escaping photons could remove angular momentum from the disk effectively (Miller & Lamb 1993). This is not correct, however, since the maximum photon torque from a luminosity L_γ escaping from radius r , where the matter azimuthal velocity is v , is

$$\max \dot{J}_\gamma = \left(\frac{L_\gamma}{c} \right) r \left(\frac{v}{c} \right) = \left(\frac{L_\gamma}{\dot{M} c^2} \right) \dot{M} v r \equiv \epsilon \dot{J}_m, \quad (6.7)$$

where $\dot{J}_m = \dot{M} v r$ is the rate of advection of angular momentum by matter, and $\epsilon = L_\gamma / \dot{M} c^2 \sim 0.2$ (see Table 3) is the efficiency of accretion. A more physically acceptable way to remove angular momentum is via a magnetic wind from the inner accretion disk or neutron star surface. This could set a lower limit to the spin period if its relative importance increased as the spin period decreased.

If, in future, a pulsar is found with $P \lesssim 0.8$ ms, there are two ways it could be used to put strong constraints on the equation of state. First, if its mass could be measured (e.g. by optical spectroscopy of its companion), and was found to be $\sim 1.4 M_\odot$, it would suggest a pion or kaon condensed phase, because equations of state without those cannot reach such P at that mass (Table 3, and Figure 6-4 of Kunihiro et al 1993). If the equation of state were hard, the star's large moment of inertia would require it to have accreted enormously, and have $M \gtrsim 2 M_\odot$; such a measured mass would rule out both soft, condensed, and even "medium" equations of state. Second, if it had a large $\dot{P}$ and were in a short-period binary, it might prove possible to determine accurately the pulsar's moment of inertia (or more precisely, $dJ/d\Omega$), giving a quantitative constraint on the equation of state. The moment of inertia would be determined as follows. As the pulsar spins down, it loses mass-energy at a rate $\dot{M} = I\Omega\dot{\Omega}/c^2$. This

loss of gravitating mass causes the orbit to expand, and the orbital period to increase at a rate

$$\begin{aligned}\frac{\dot{P}_b}{P_b} &= \frac{-2\dot{M}_p}{M_p + M_c} = \frac{-2I\Omega\dot{\Omega}}{M_p c^2 + M_c c^2} \\ &\simeq \frac{I\Omega^2}{Mc^2} \frac{1}{\tau_c} \simeq 0.016 I_{45} M_{1.4}^{-1} \tau_c^{-1} P_{\text{ms}}^{-2},\end{aligned}\quad (6.8)$$

where $M_{1.4} = M/(1.4 M_\odot)$. To avoid confusion of the measured $\dot{P}_b/P_b$ by Galactic acceleration (see Section 8) would require $\tau_c P_{\text{ms}}^2 \leq 10^{8.5}$ y, hence the need for large $\dot{P}$ and short period. A pulsar similar to 1937+20 in a short-period binary would do.

There is also an interesting possibility that very massive pulsars with periods near P_{min} may have $\dot{P} < 0$, even though they are losing angular momentum. This is because although $\dot{J} = (dJ/d\Omega)\dot{\Omega} < 0$, and usually $dJ/d\Omega \simeq I$, so $\dot{\Omega} < 0$ and $\dot{P} > 0$, relativistic stars supported against collapse by rotation can have $dJ/d\Omega < 0$, so $\dot{\Omega} > 0$ when the star loses angular momentum (Cook et al 1992, 1994). The stars with $dJ/d\Omega < 0$ are those which are so massive that they are just barely supported by rotation against collapse to black holes. As such a neutron star loses angular momentum, its J approaches a few percent of the minimum value at which it can be supported at a large (quasi-Newtonian) radius. It then begins to evolve towards a second (unstable) equilibrium state of the same angular momentum, but with a smaller radius (confining by the enhanced gravitational attraction of strong-field relativity). As the star contracts at nearly constant J , its spin period must decrease by a small but finite amount, just before the pulsar collapses to a black hole. Negative observed $\dot{P}$ s can be produced by accelerating pulsars with positive $\dot{P}$ s (see Section 8), however, so caution would be required in interpreting any observed negative $\dot{P}$ as due to this manifestation of the effect of general relativity on neutron stars.

7. TESTING GENERAL RELATIVITY

Millisecond and binary pulsars have been used to perform several high-precision tests of general relativity, its axioms, and predictions. Because these have been the subject of several recent reviews (Backer & Hellings 1986, Damour & Taylor 1992, Taylor et al 1992, Taylor 1992, Damour 1992; the book by Will 1993 gives a pedagogical introduction), we confine ourselves to a brief summary.

A pulsar has a much larger gravitational (binding energy) contribution to its mass-energy than a white dwarf or main sequence star. In most theories of gravity other than Einstein's, a pulsar and its white dwarf companion would therefore fall through the Galactic gravitational field with different accelerations, so their relative orbit would be eccentric (Nordvedt 1968, Damour & Schäfer 1991). The low eccentricity of the binary pulsars 0820+02 and J1803 - 2712

(see Table 1) already allow one to deduce that the mass-equivalent of gravitational binding energy feels the same acceleration as do baryons to within 0.2%. This limit is comparable to the best Solar System limits for violations linear in M/R , and much better for nonlinear violations. The best similar test of the weak equivalence principle for weak interaction energy was provided by the simultaneous arrival of photons and neutrinos from supernova 1987A (Krauss & Tremaine 1988).

The weak-field effects of general relativity (which have been verified in Solar System experiments) are all measured in binary pulsars: gravitational redshift, Shapiro delay, and the advance of periastron. In some cases the amplitudes are spectacular: The periastron of the orbit of PSR 1913+16 has advanced by nearly 90 degrees since its discovery in 1974 (Hulse & Taylor 1975). By contrast, relativity has contributed only 10^{-2} degrees to the advance of the perihelion of Mercury since the non-Newtonian portion was identified by LeVerrier and Newcomb. If one assumes the correctness of relativity, the parameters of these weak-field effects can be used to solve for the Newtonian parameters—individual stellar masses and orbital inclination, which are ordinarily unmeasurable in a Newtonian single-line spectroscopic binary. In some cases, the Newtonian quantities are overdetermined by the relativistic parameters, and the agreement between different determinations can be used to provide precision tests of relativity, and to constrain the post-Newtonian parameters of alternative theories of gravity (Damour & Taylor 1992, Taylor et al 1992, Will 1993).

With the Newtonian parameters determined, the rates of more exotic relativistic effects can be predicted and compared with observation. In PSR 1913+16, the rate of decrease of the orbital period due to the emission of gravitational radiation is predicted by the quadrupole formula of general relativity (Peters 1964, Thorne 1980), $\dot{P}_b/P_b = -8.60924(14) \times 10^{-17} \text{ s}^{-1}$, and observed to be $\dot{P}_b/P_b = -8.63(4) \times 10^{-17} \text{ s}^{-1}$ (after correction for the Galactic acceleration: see Section 8; Taylor 1992, Damour & Taylor 1991). This agreement to within 0.5% is a stunning confirmation of Einstein's prediction of waves in a dynamical space-time. It will soon be possible to repeat this test in pulsars 1534+12 and 2127+11C, with respectively greater and lesser precision.

Another relativistic effect, which ought to be measurable in one of these pulsars, is geodetic precession. The spin axis of a pulsar is parallel transported around the curved spacetime of its companion (torques on its quadrupole moment by companions are negligible), and in relativity is therefore predicted to precess at a rate $d\hat{S}/dt = \Omega_p |\hat{L} \times \hat{S}|$, where $\hat{S}$ is the unit vector along the pulsar spin, $\hat{L}$ that along the normal to the orbit plane, and $\Omega_p \sim GM_c/(ac^2)$ is 1.2 degrees per year for PSR1913+16, and 0.4 degrees per year for PSR1534+12. Despite careful measurements, this has not been convincingly seen in PSR1913+16 (Cordes et al 1990), but the pulse properties of PSR1534+12 are more favorable, and its $|\hat{L} \times \hat{S}|$ appears larger.

Additional, and even more precise tests of general relativity would be possible in a binary system in which both components were millisecond pulsars. Such systems could form in a globular cluster by exchange, or by spin up from tidal debris in an encounter of two (neutron star–main sequence star) binaries (Sigurdsson & Hernquist 1992). Another type of binary system that ought to exist is a (young) pulsar orbiting a stellar mass black hole. Such black holes could either have formed directly in a supernova explosion, or could be neutron stars pushed beyond the maximum stable mass by accretion from the massive progenitor of the pulsar. If the orbital period were short, the black hole mass could be determined accurately by means of weak-field relativistic effects. However, unless the orbit happens to be improbably close to edge on, strong-field effects of the metric near the black hole horizon (e.g. faint “glory” pulses) are unlikely to be detectable.

8. LARGE SCALE ACCELERATIONS

8.1 *Newtonian Accelerations*

The spin and orbital periods we measure for pulsars are shifted from their values in a local Lorentz frame moving with the pulsar or its binary center mass, respectively. The largest effect is the first-order Doppler shift. A pulse period P measured at the Solar System barycenter (velocity $\mathbf{V}_b$) is thus related to a pulsar’s rest-frame pulse period P_0 by $P = P_0[1 + (\mathbf{V}_p - \mathbf{V}_b) \cdot \mathbf{n}/c]$, where $\mathbf{V}_p$ is the pulsar velocity and $\mathbf{n}$ is the unit vector pointing from the barycenter to the pulsar. If the Doppler shift were constant, the observed period would just differ from the true one by a small, uninteresting, and unmeasurable factor. However, the Doppler shift is never truly constant, because pulsars move ($\mathbf{n}$ varies) and are accelerated by gravitational forces ($\mathbf{V}_p$ varies). Differentiating the Doppler equation once more gives, to lowest order in v/c (Phinney 1992, 1993)

$$\frac{\dot{P}}{P} = \frac{\dot{P}_0}{P_0} + \frac{V_{\perp}^2}{cD} + \frac{(\mathbf{a}_p - \mathbf{a}_b) \cdot \mathbf{n}}{c} = \frac{1}{2\tau_{c0}} + \frac{1}{10^{10} \text{ y}} \left(\frac{V_{\perp}}{150 \text{ km s}^{-1}} \right)^2 D_{\text{kpc}}^{-1} + \frac{1}{4 \times 10^{10} \text{ y}} \frac{(\mathbf{a}_p - \mathbf{a}_b) \cdot \mathbf{n}}{A_{\odot}}, \quad (8.1)$$

where $\mathbf{a} = \dot{\mathbf{V}}$ is acceleration, D is the distance to the pulsar, $V_{\perp}$ is the transverse relative velocity μD , μ is the pulsar proper motion, and $A_{\odot}$ is the acceleration of the solar local standard of rest about the center of the Galaxy.

These Doppler corrections to (8.1) are not important for the young majority of pulsars ($\tau_{c0} \simeq 10^7 \text{ y}$), but they dominate the apparent $\dot{P}$ for many millisecond and binary pulsars, complicating determination of their true characteristic ages and magnetic field strengths. The second (centrifugal) term in (8.1) is, like the intrinsic $\dot{P}_0$, always positive, and is important for old, nearby pulsars with large

space velocities (see Table 1 and Camilo et al 1994). The third (acceleration) term, which can have any sign, is not important to the $\dot{P}$ s of Galactic pulsars (though it is important for their $\dot{P}_b$ s, and uncertainty in estimating it limits the precision of measurement of the rate of decay of orbits by gravitational radiation reaction). Equations for estimating the term for Galactic pulsars can be found in Damour & Taylor (1991) and Phinney (1992). The third term is, however, extremely important for pulsars in globular clusters, and in several cases makes negative the apparent period derivative. Methods for predicting this term are discussed at length by Phinney (1992, 1993), as are the use of pulsar positions and negative $\dot{P}$ to determine model-independent mass-to-light ratios and other constraints on the structure of globular clusters.

8.2 Gravitational Waves

Pulsars can be accelerated by non-Newtonian forces as well. If the Earth and the millisecond pulsars surrounding it were bobbing in a sea of gravitational waves of periods $\sim$ years, the waves would introduce irregularities in the timing of the pulsars (Detweiler 1979). As it passes the Earth (or the pulsar), a gravitational wave of frequency f and dimensionless amplitude h of wavelength c/f shorter than the distance between the pulsar and Earth will periodically modulate the spacetime, and thus the arrival times of pulses, by of order $h/(2\pi f)$. The energy density in the waves, of order $f^2 h^2 c^2 / G$, would be a fraction $\Omega_g \sim f^2 h^2 / H_0^2$ of the closure density of the universe, with Hubble constant H_0 . Thus the timing residuals produced by a gravitational wave background with Ω_g are of order $100 \Omega_g^{1/2} (f/1 \text{ y}^{-1})^{-2} \mu\text{s}$ (the numerical coefficient depends somewhat on the number of pulsar parameters fitted by timing, the wave spectrum assumed, and the duration of the observations, assumed longer than $1/f$; Blandford et al 1984). Limits on these, most strongly from PSRs 1937+21 and 1855+09, constrain at 95% confidence the energy density in such waves to a fraction $\Omega_g < 6 \times 10^{-8} h_0^{-2}$ of the closure density of the universe (Kaspi et al 1994b; here h_0 is the Hubble constant in units of $100 \text{ km s}^{-1} \text{ Mpc}^{-1}$). This is a severe constraint on the presence of cosmic strings; in the scale-free model of Bennett & Bouchet (1991), the string mass-energy per unit length μ is constrained to $G\mu/c^2 < 71 \Omega_g h^2 = 4 \times 10^{-6}$, while in the models of Caldwell & Allen (1992), which include additional modes of string excitation and emission by strings in the matter-dominated era and thus have more power at long periods, $G\mu/c^2 \lesssim 10^{-6}$. Pulsar timing is thus close to detecting, or ruling out, the presence of strings interesting for galaxy formation.

9. PLANETARY COMPANIONS

A planet of mass m orbiting a pulsar of mass M_p in an orbit of semimajor axis a and period P_y years moves the pulsar back and forth relative to Earth, and

thus produces a periodic timing residual of semiamplitude

$$\frac{a_p \sin i}{c} = \frac{m}{M_p + m} \frac{a \sin i}{c} \simeq 1200 \frac{m}{M_\oplus} \left(\frac{1.4 M_\odot}{M_p + m} \right)^{2/3} P_y^{2/3} \sin i \mu\text{s}. \quad (9.1)$$

For example, in orbits of one year period around a $1.4 M_\odot$ neutron star at $i = 60^\circ$, a Jupiter would produce a residual of semiamplitude 0.33 s, Earth of 1.0 ms, the Moon of 13 μs , and the asteroid Ceres 0.2 μs . Some millisecond pulsars can be timed to accuracies $\lesssim 1 \mu\text{s}$, so if they were surrounded by planetary systems, even bodies as small as the moon would be potentially detectable. The precision of such measurements is some 2000 times higher than radial velocity measurements with precisions of 10 m s^{-1} (about the best obtained so far in searches for planetary systems about ordinary stars using stabilized I_2 absorption cells as wavelength references—cf Cochran et al 1991; Jupiter moves the Sun at 10 m s^{-1} through a displacement of 3 light seconds, which would require an angular resolution of $6 \mu\text{arcsec}$ to detect at 1 kpc).

Before a neutron star forms, its planetary environment is subjected to the sublimating heat of a red giant star and a supernova blast, and whatever is left at large radii will generally be left behind as the newborn neutron star recoils at the $\gtrsim 100 \text{ km s}^{-1}$ speed typical of pulsars. Only in carefully contrived circumstances (Wijers et al 1992, Thorsett & Dewey 1993)—“Salamander scenarios” in the notation of Phinney & Hansen (1993)—might primordial planets survive around pulsars. Many pulsars clearly do not have any planetary companions (Thorsett & Phillips 1992, Thorsett et al 1993c).

It was therefore something of a surprise when the first unambiguous evidence for a planetary system outside our Sun’s was found around the 6 ms pulsar 1257+12 (Wolszczan & Frail 1992). Two planets circle this pulsar. Both planets are a few times the mass of the Earth, and have nearly circular ($e = 0.02$) orbits of period $P_{\text{in}} = 66.6 \text{ d}$ and $P_{\text{out}} = 98.2 \text{ d}$ (Table 1). The timing evidence for the planets of PSR 1257+12 seems secure, since its position determined by interferometry agrees with the position derived from timing, and the timing has by now been confirmed on several telescopes, with different timing hardware, and analyzed with independent software and ephemerides (Backer et al 1992).

PSR 1257 + 12’s high spatial velocity (290 km s^{-1} , Wolszczan 1993) combined with the circularity of its planets’ orbits, make it very unlikely that its planets are survivors from an era when the neutron star had not yet been formed (“Salamanders”). The planets must instead have formed *after* the neutron star’s progenitor metamorphosed into a neutron star (“Memnonides scenarios,” in the notation of Phinney & Hansen 1993). Since the pulsar has the short period and small dipole field characteristic of recycled pulsars, the most natural scenarios for planet formation are those in which the planets formed from the debris of a companion star. This may have been destroyed gradually (Banit et al 1993) or catastrophically (e.g. Phinney & Hansen 1993). Most of the mass would

accrete, but to conserve angular momentum, a small fraction must be excreted outwards; planets could form in this cool excretion disk (Ruden 1993, Phinney & Hansen 1993). For a review of these, and many other proposed scenarios, the reader is referred to the proceedings *Planets Around Pulsars* (Phillips et al 1993), particularly the summary by Podsiadlowski (1993).

Like the Solar System, the planetary system of PSR1257+12 will provide entertainment for generations of dynamicists. The two planets are close to a 3:2 resonance: The inner planet revolves $2.95 \simeq 3$ times for every two revolutions (196.4 d) of the outer planet. Since the relative configurations of the planets repeat almost exactly every 196.4 days, their mutual effects on each others' orbits (e.g. their longitudes of periastron ω and orbital eccentricities e) can build up over many orbital periods. The planets' masses were initially known only from the Keplerian mass function—i.e. only the $m \sin i$ were known. If we had been viewing the planetary system close to face-on ($\sin i < 0.1$), the masses of the planets would have had to have been so large that their mutual interactions would have been strong enough to make their angles librate, locking them in resonance (Malhotra et al 1992). This would already have been noticed in the pulsar timing (Wolszczan 1993), so $\sin i > 0.1$ and the planets are not locked in the 3:2 resonance. Consequently, the difference of the planets' phases with respect to the resonance rotates cyclically with period P_d given by $1/P_d = 2/P_{\text{in}} - 3/P_{\text{out}} = 1/(5.3 \text{ y})$. Thus the planets' ω , e , and a should vary almost periodically with period P_d , with its amplitude $\propto m \propto 1/\sin i$ (Rasio et al 1992). Timing residuals of about the amplitude expected (Malhotra 1993, Peale 1993) for $\sin i \sim 1$ are already detected at about the 2σ level (Wolszczan 1993).

PSR1257 + 12 may not be the only millisecond pulsar with planetary companions. The 11 ms pulsar 1620 – 26 in the globular cluster M4 has a $\ddot{P}$ (Backer et al 1993a, Thorsett et al 1993a) much larger than could be produced by the jerking of randomly passing cluster stars (Phinney 1993). One plausible interpretation is that the jerk is produced by a companion with a mass 0.1–10 times that of Jupiter in a very long period orbit (Backer et al 1993a, Phinney 1993, Sigurdsson 1993), a hypothesis readily testable by further timing. Unlike PSR 1257 + 12, PSR 1620 – 26 also has a stellar mass companion with orbital period $P_b = 191.4 \text{ d}$. The mass function, period, and circular orbit are all consistent with this companion being the $\sim 0.4 M_\odot$ helium white dwarf remains of the red giant, the accretion of whose envelope spun up the pulsar (see Phinney 1992, and Section 10). Though some material might have been excreted, the low metallicity of stars in M4 makes it difficult to form such a massive planet in such a wide orbit (Sigurdsson 1992). It is more likely that the planet is a primordial planet which originally orbited a main sequence star, and was retained as a bystander during the exchange of partners which put the pulsar in orbit about the main sequence star (Sigurdsson 1993). This star then evolved into the current white dwarf companion, spinning up the pulsar in the

process. In this model, the planet is predicted to have an eccentric orbit. If further observations reveal the orbit to be circular, the first ("Memnonides") scenario would be implicated, though in this case too the orbit is more likely to be eccentric, due to perturbations of passing cluster stars.

10. DYNAMICAL FOSSILS OF THE SPIN-UP ERA

As discussed in Section 2 and reviewed at length by Verbunt (1990, 1993) and van den Heuvel (1992), most millisecond and binary pulsars are believed to be products of mass-transfer, and their predecessors should have been X-ray binaries. But direct evidence for a connection between millisecond pulsars and low-mass X-ray binaries is still tenuous, given the apparent discrepancy between their birthrates (Section 3.3) and that searches have failed to reveal millisecond pulsations in LMXB (Wood et al 1991, Kulkarni et al 1992a). For evidence of the connection, one must search for dynamical fossils in the orbits of binary pulsars. Such fossil evidence is clearest among the binary pulsars with low mass companions and orbital periods $\gtrsim 1$ d. The two fossils are the $P_b(M_c)$ relation (predicted by Refsdal & Weigert 1971), and the $e(P_b)$ relation (predicted by Phinney 1992), illustrated respectively in Figures 2 and 3.

10.1 Core Mass–Period Relation

The $P_b(M_c)$ relation arises when a neutron star is in a wide orbit with a companion star of $\lesssim 2 M_\odot$. Such a binary begins mass transfer when the $\lesssim 2 M_\odot$ companion, with its electron-degenerate core surrounded by a thin hydrogen burning shell, evolves off the main sequence. The high gravity of the compact core requires a steep pressure gradient across the burning shell. Thus the properties of the shell are almost independent of the envelope above it, and are little affected by the mass loss that begins far above. As the shell burns outwards, its helium ash joins the degenerate core, increasing its mass and decreasing its radius. The temperature of the shell increases to remain in hydrostatic equilibrium; because of the steep temperature dependence of nuclear reaction rates, the shell's luminosity is thus a rapidly increasing function of core mass (Refsdal & Weigert 1970). For giants of solar metallicity, $L = [M_c/(0.16 M_\odot)]^8 L_\odot$. Because the decreasing thermal conductivity of the increasingly large and opaque envelope ultimately forces the envelope to be convective (Renzini et al 1992), the giant lies on the Hayashi track $T_{\text{eff}}(L, M)$. As $L = 4\pi R^2 \sigma T_{\text{eff}}^4$, these relations give the radius R of the giant as a function of M_c :

$$R \simeq 1.3 \left(\frac{M_c}{0.16 M_\odot} \right)^5 R_\odot \quad (10.1)$$

for solar metallicity. During mass transfer, the giant of mass M must fill its Roche lobe, whose volume-equivalent radius $R_{L,1}$ is given for a giant less

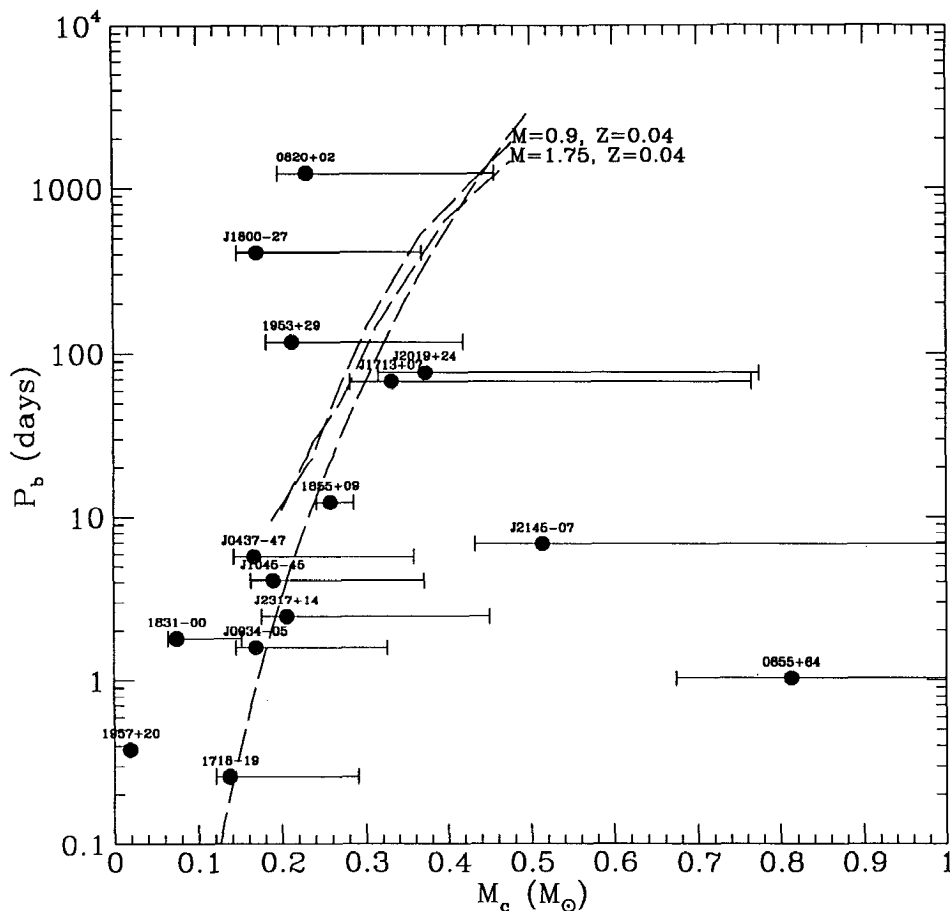


Figure 2 The predicted relation between core mass (i.e. final white dwarf mass) and orbital period for binaries in which a $\approx 1 M_{\odot}$ giant star fills its Roche lobe with a $1.4 M_{\odot}$ neutron star (solid curve, Refsdal & Weigert 1971; dashed curves: data from Sweigart & Gross 1978). The filled circles with error bars give most probable, and 90% confidence estimates of the masses of the white dwarf companions of the indicated pulsars (assumed, when not known, to be $1.4 M_{\odot}$). All Galactic binary pulsars with orbital eccentricities < 0.1 are shown. Note that hydrogen shell burning giants exist only for $0.16 < M_c < 0.45 M_{\odot}$. White dwarf companions of mass outside this range (e.g. those of PSRs 0655 + 64, J2145 – 0750, 1957 + 20, etc) had different evolutionary histories (see notes to Table 1 and text).

massive than the neutron star, within a few percent, by

$$\frac{GM}{R_{L1}^3} = 10 \Omega_b^2, \quad (10.2)$$

where $\Omega_b = 2\pi/P_b$ is the orbital frequency. Towards the end of transfer, when $M \simeq M_c$, (10.1) and (10.2) therefore give

$$P_b \simeq 1.3 \left(\frac{M_c}{0.16 M_\odot} \right)^7 \text{ d.} \quad (10.3)$$

This is valid for $0.16 M_\odot < M_c < 0.45 M_\odot$, where the lower limit is the helium core mass at the end of main sequence evolution, and the upper limit is the core mass at the helium flash, for which $P_b \simeq 2000$ d. A more accurate version of Equation (10.3), derived from computed stellar evolutionary models, is plotted in Figure 2. Other approximations are given in Refsdal & Weigert (1971) (see also Joss et al 1987 and Verbunt 1993).

During stable mass transfer, the expanding giant will always just fill its Roche lobe, with radius given by (10.1), until the envelope has been reduced to a few times the mass of the burning shell ($\lesssim 10^{-2} M_\odot$). At that point, the envelope will become radiative (cf Renzini et al 1992) and begin to shrink on its thermal timescale (Refsdal & Weigert 1970, Taam 1983), and mass transfer will cease. As the burning shell consumes the shrinking envelope, it eventually has too little overburden to confine it, hydrogen burning ceases, and the core cools, becoming a helium white dwarf companion to the spun-up neutron star. The orbit should be nearly circular because tidal dissipation in the giant would have rapidly circularized the orbit (see Section 10.2).

The fossil evidence for stable mass transfer from a Roche-lobe filling giant star is therefore: a helium white dwarf companion of mass M_c between $0.16 M_\odot$ and $0.45 M_\odot$, in a circular orbit with period (between 1 and 2000 days) given by the P_b – M_c relation [Equation (10.3) or the more accurate versions plotted in Figure 2]. It is evident from Figure 2 that all 10 of the binary pulsars in the fourth group of Table 1 are consistent with this prediction. The other binary pulsars in the table, with companion masses less than $0.16 M_\odot$, or greater than $0.45 M_\odot$, cannot have had this evolutionary history, and therefore are not expected to obey the P_b – M_c relation (10.3). PSRs 0655 + 64 and J2145 – 0750 have white dwarf companions with masses greater than the $\sim 0.45 M_\odot$ core mass at helium flash, and therefore cannot be the result of mass transfer from a red giant. Their properties are, however, just what would be expected from the evolution of a wide binary containing a neutron star and a main sequence companion more massive than $2 M_\odot$ (see note to Table 1 and Table 4; Iben & Livio 1993 review the physics of the common envelope phase expected in these circumstances, cf also van den Heuvel 1992).

10.2 *Eccentricity-Period Relation*

Another fossil of the mass transfer phase was recently pointed out by Phinney (1992). As a function of orbital phase, the difference in pulse arrival times

between a pulsar in a circular orbit and one with a small eccentricity e will be

$$\delta t = \Delta t(t) - \Delta t_{\text{circ}}(t) = \frac{ea_p \sin i}{2c} [\sin \omega + \sin(2\pi t/P_b - \omega)]. \quad (10.4)$$

Thus for a typical binary pulsar with $a_p \sin i/c = 10$ s, timing accuracy of $1 \mu\text{s}$ allows us to detect e as small as $1 \mu\text{s}/10 \text{ s} = 10^{-7}$.

After the supernova (or accretion-induced collapse) which creates a neutron star in a low mass binary, the orbital eccentricity will be high (>0.1). When the companion evolves to become a red giant, the time-dependent tides induced in the giant by the neutron star will be exponentially damped by convective eddy viscosity (Zahn 1966, 1977), on a timescale $\sim 10^4$ y, much shorter than the lifetime of the giant $\sim 10^7$ – 10^8 y. If this were the entire story, the orbital eccentricities would be predicted to be of order $\exp(-1000)$. As is clear from Table 1 and Figure 3, the observed eccentricities are small, but measurably nonzero, in the range 10^{-6} – 10^{-2} . Perturbations by passing stars are inadequate to induce such eccentricities (Phinney 1992).

However, the fluctuation-dissipation theorem reminds us that the tidal dissipation cannot be the whole story. The fluctuating density of the convection cells in the convective giant star produces time-dependent moments of quadrupole and higher order. The neutron star in its orbit feels the noncentral forces produced by these fluctuating multipoles, and these randomly pump the eccentricity of the orbital motion. Dissipation of the time-dependent tide always tends to damp the eccentricity of the orbit. The resulting epicyclic motion is thus like that of a pendulum in air, which is continually bumped by air molecules. The pendulum is excited into Brownian motion—but if the amplitude becomes large, air drag slows it down. In equilibrium, a pendulum in air has energy that random walks on the drag timescale around a mean value of kT .

Phinney (1992) proves an analogous theorem for the binary star system: If convection is confined to a single thin layer, a (statistical) equilibrium eccentricity is reached when the energy of the orbital epicyclic motion is equal to the energy in a single convective eddy. The driving and damping forces can be summed over the convective layers of model red giants as a function of time during the mass transfer. The squared eccentricity random walks about its equilibrium value until the end of mass transfer, when the giant envelope begins to shrink as it becomes radiative (see Section 10.1). At that point, the tidal damping time rapidly increases, becoming longer than the evolution time, and the current (random) value of the eccentricity is frozen into perpetuity (Phinney 1992) as the star becomes a white dwarf. The predictions of this theory (Figure 3) are in good agreement with the measured eccentricities of the orbits of pulsars with white-dwarf companions—a remarkable result, since the theory has no adjustable parameters! It is curious that the orbit of PSR J2145 – 0750, which clearly must have undergone spiral-in during unstable mass transfer (see Figure 2), lies in the predicted band along with all the other binaries. This

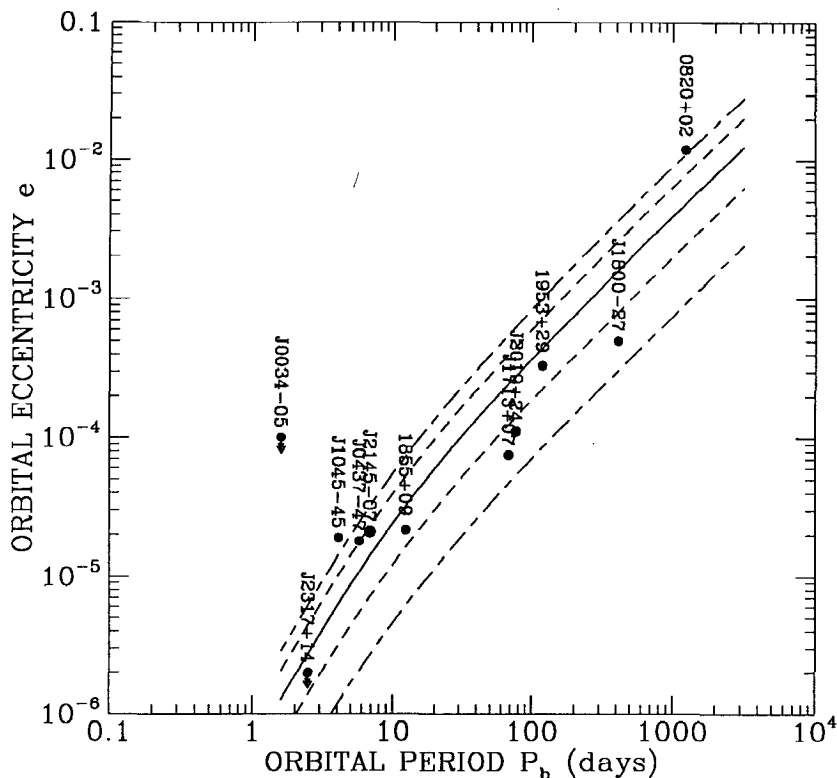


Figure 3 Orbital eccentricity vs orbital period for the pulsars whose companion masses and periods are consistent with their having formed by stable mass transfer from a Roche-lobe filling red giant (i.e. along the line of Figure 2). The solid line is the median eccentricity predicted by the convective fluctuation-dissipation theory of Phinney (1992; see also text). The inner and outer dashed lines are predicted to contain 68% and 95% of the final eccentricities.

suggests that the entire red giant envelope was not ejected during the common-envelope phase, and the companion, of reduced mass, subsequently filled, or nearly filled its Roche lobe for $\gtrsim 10^5$ y.

10.3 Puzzles in Mass Transfer

As we have seen, conservative Roche lobe transfer models of binary evolution are successful at explaining the periods, masses, eccentricities, and, to some extent birthrates (Section 3.3) of LMBPs. However, some puzzles and issues of self-consistency remain.

The Eddington rate for accretion of a cosmic hydrogen/helium mixture onto a $1.4 M_{\odot}$ neutron star is $1.9 \times 10^{-8} (0.2/\epsilon) M_{\odot} \text{ y}^{-1}$, where ϵ is the surface binding energy given in column 7 of Table 3. Because large giant stars have nuclear evolution times shorter than 10^8 y , mass transfer on their nuclear timescale can lead to super-Eddington accretion rates. For standard conservative models of transfer, this occurs (see e.g. Figure 8 of Verbunt 1993) in binaries with initial orbital periods exceeding $\sim 10 \text{ d}$ (hence final periods $\gtrsim 100 \text{ d}$). The evolution of such systems is therefore unlikely to be conservative of mass. If matter is ejected only from the regions of the accretion disk near the neutron star, tidal torques (Priedhorsky & Verbunt 1988 and references therein) on the outer part of the disk may still keep the evolution conservative of angular momentum. The evolution will then have the same stability properties as conservative transfer (see Table 4, case of a convective mass loser), and the evolution of the binary will not be qualitatively affected. If, however, angular momentum were lost (e.g. by ablation of the giant star), the evolution would be dramatically different, leading to unstable transfer on a dynamical timescale. The existence of pulsars with white dwarf companions near the predicted $M_c - P_b$ relation for $P_b > 100 \text{ d}$, and the rough agreement for long-period systems between the pulsar and the X-ray binary birthrates (inferred assuming evolution on the nuclear timescale), suggests that such angular momentum loss is *not* common. For pulsars with initial orbital periods between 2 and 10 d (final periods between 20 and 100 d), simple conservative models do not have any obvious physical contradictions, and the final period is almost linearly proportional to the initial period.

Table 4 Stability of mass loss in binary stars

CONVECTIVE LOSER	conservative transfer	nonconservative (of M , J) transfer	partly noncons. (conserve J , not M)
$M_{\text{loser}} \gg M_{\text{gainer}}$	unstable	stable	unstable
$M_{\text{loser}} \ll M_{\text{gainer}}$	$M_{\text{loser}} = \frac{2}{3} M_{\text{gainer}}$ stable	$M_{\text{loser}} = M_{\text{gainer}}$ unstable	stable
RADIATIVE LOSER	conservative transfer	nonconservative (of M , J) transfer	partly noncons. (conserve J , not M)
$M_{\text{loser}} \gg M_{\text{gainer}}$	delayed dynamically unstable $M_{\text{loser}} = 2 M_{\text{gainer}}$ thermally unstable	stable	delayed weakly unstable
$M_{\text{loser}} \ll M_{\text{gainer}}$	$M_{\text{loser}} = 1.3 M_{\text{gainer}}$ stable	stable	stable

A different problem exists for short-period systems (initial period ~ 0.7 d, final periods $1 \lesssim P_b \lesssim 6$ d for a $1 M_\odot$ donor). For these, the companion comes into contact with its Roche lobe just as it is evolving off the main sequence, while it is still mostly radiative. For such stars, mass loss causes the star to shrink within its Roche lobe (see Table 4, and discussion in Hjellming & Webbink 1987). Thus the mass transfer will cease unless there is an angular momentum loss mechanism (e.g. magnetic braking) to shrink the orbit. The models thus predict a bifurcation, wherein systems with initial periods slightly less than 0.7 d spiral together to very short orbital periods, while systems with initial periods slightly greater spiral out to final periods $\gtrsim 6$ d (Pylyser & Savonije 1988). For pulsars with initial periods between ~ 0.7 d and 2 d, these models (see also Coté & Pylyser 1989) predict a steep dependence of the final period (0.7 to 20 d) on the initial period. Intermediate final periods are therefore expected to be very rare, since they require a fine-tuning of the initial period. The range of rare final periods can be reduced somewhat by increasing the donor mass into the range where mass transfer is initially unstable (Pylyser & Savonije 1988). Examining the list of the pulsars in Table 1, we see that the pulsars listed between PSR J0437 – 4715 and PSR 1831 – 00, inclusive, fall within this difficult category of intermediate periods. Furthermore, the birthrate inferred from these pulsars' discovery (Lorimer 1994; see also Section 3.3) seems substantially larger than the birthrate of X-ray binaries inferred in these models (Coté & Pylyser 1989), which suggests that we do not fully understand the origin of these systems.

The difficulties with the birthrates of these short-period systems may be related to the even greater difficulties with the birthrates of the "black widow" pulsars (PSRs 1957+20 to J0322+2057 in Table 1). These have companions of zero or very low mass ($< 0.02 M_\odot$), yet have clearly been spun up by accretion from a companion now digested or destroyed. Despite initial enthusiasm (van den Heuvel & van Paradijs 1988, Phinney et al 1988, Ruderman et al 1989) following the discovery of the wind from the companion of the eclipsing pulsar PSR 1957 + 20 (Fruchter et al 1988), it now seems that on both theoretical (Levinson & Eichler 1991) and observational grounds (Fruchter & Goss 1992) that PSR 1957 + 20 is not significantly ablating its companion. If so, it makes unlikely the otherwise attractive model of van den Heuvel & van Paradijs (1988), in which a low mass companion has its orbit shrunk by magnetic braking to the "period gap," and is then ablated. The model proposed by Phinney et al (1988), in which the nonconservative mass loss from a Roche-lobe filling companion becomes unstable as its mass drops to the point that it becomes convective (see Table 4), is perhaps still viable, though it is doubtful if the remnant of the unstable mass transfer would be as small as $0.02 M_\odot$. The X-ray lifetimes in these models may be long or short, depending on the transport of the pulsar's heating (see Section 6.1.2).

Another process is the recoil of a newly formed neutron star into, or nearly into, its main-sequence companion. This would result respectively in the disruption, or severe bloating and loss of mass of the companion. The former could result in single pulsars; planets may form from the debris as in PSR1257 + 12 (Phinney & Hansen 1993). The tidal heating of near-misses could result in systems like PSRs 1831 – 00 and 1718 – 19 [if the latter is, as seems likely, not in the globular cluster NGC 6342; Wijers & Paczyński (1993) present cluster models]. Better aimed neutron stars might sink into their companions and form a single pulsar via a Thorne-Żytkow object (Leonard et al 1994). All versions of disruptive recoil will certainly result in a very short-lived X-ray phase, and thus avoid difficulty with the large birthrate of these types of pulsars relative to the number of X-ray sources (Section 3.3).

11. PULSARS AS PLASMA PROBES

The radio emission from several binary pulsars (PSRs 1957 + 20, 1718 – 19, 1744 – 24A and 1259 – 63) is delayed, pulse-smeared, and even eclipsed by plasma emitted from their companion stars. Thompson et al (1994) give a comprehensive and critical analysis of mechanisms for the pulse-smearing and eclipses. Remarkably, it appears that because of its high brightness temperature, pulsar emission can actively modify the plasma through which it passes (as lasers do to hydrogen pellets in inertial-confinement fusion experiments), and the eclipses of PSR1744 – 24A are most likely due to stimulated Raman scattering (Thompson et al 1994).

Pulsar radio waves propagating through the interstellar medium are more passive. Fluctuations in the electron density on scales $\sim 10^9$ cm, smaller than the Fresnel scale $(\lambda D/2\pi)^{1/2} \sim 10^{11}$ cm, contribute to the (observationally irritating) diffractive scintillation of pulsars (reviewed by Rickett 1990, Narayan 1992), while fluctuations on larger scales ($\sim 10^{13}$ cm) are probed by the long-term intensity fluctuations associated with refractive scintillation. Density fluctuations on still larger scales ($\sim 10^{13}$ – 10^{15} cm) can be probed by timing millisecond pulsars at multiple frequencies, and measuring the variations in the pulsar's dispersion measure as its motion changes our line of sight to it. Such precision measurements (Backer et al 1993b) complement low-frequency measurements on ordinary pulsars (Phillips & Wolszczan 1991, 1992), and tell us about the spectrum of typical fluctuations on the same scales as the rare "extreme scattering events" discovered by Fieldler et al (1987), and recently seen in both flux and delay in the millisecond pulsar 1937+21 (Cognard et al 1993). Differences in the dispersion measure of pulsars in globular clusters probe fluctuations on still larger scales ($\sim 10^{17}$ cm) (Anderson 1992), and the cluster pulsars also tell us that the scale height of ionized gas in the Galaxy is ~ 0.6 – 0.8 kpc (Bhattacharya & Verbunt 1991, Nordgren et al 1992).

12. WHAT NEXT?

The discovery of millisecond and binary pulsars has revitalized the pulsar field. We expect that searches for millisecond pulsars will continue into the next millennium. Undoubtedly there will be some delights (a nearby long-period pulsar with a low-mass main sequence companion; a millisecond pulsar with an infrared-luminous asteroid belt and nine planets; a pulsar interacting with a magnetic white dwarf companion; a millisecond pulsar in a triple star system; a source that switches between an LMBP and an LMXB; and perhaps even a pulsar in a black hole binary) and surprises. The searches will improve our knowledge of the variety and demography of millisecond pulsars and their binary companions. Our present knowledge of the luminosity function, crucial to a quantitative understanding of the birthrate problem and estimation of the pulsar population in clusters, is in a state of some confusion. The Arecibo data suggest that unlike ordinary pulsars, millisecond pulsars may have a high minimum luminosity, ~ 10 mJy kpc². However, the detection efficiency at Arecibo is surprisingly higher than that of the Parkes survey, inconsistent with the previous conclusion.

As discussed in Section 5, the origin of the low magnetic field strengths of millisecond pulsars is an outstanding mystery. The bimodal field strength distribution might be interpreted to mean that LMBPs are a new class of neutron stars, which are born as millisecond pulsars (Arons 1983, Pacini 1983), e.g. born via the AIC mechanism. However, the standard recycling model is still more attractive. The high incidence of binarity, the P_b – M_c relation (Section 10.1) and the P_b – e relation (Section 10.2) clearly indicate that millisecond pulsars emerge from binary systems that had substantial mass transfer after the neutron star formed—consistent with the standard model, but inexplicable in AIC models if accretion ceased after formation of the neutron star. In models in which mass-accretion reduces the dipole field (Section 5.7), the observed bimodal distribution of magnetic field strengths would result from the very different histories of mass transfer of LMBPs and HMBPs. It will clearly be important to identify physical mechanisms by which accretion could lead to field reduction.

Accurate measurement of white dwarf masses, and the pulsar masses and kinematics offer the best quantitative checks of the formation scenarios. The present sample of LMBPs has $v_{\text{rms}} \sim 75$ km s^{−1}, which suggests that LMBPs suffered natal velocity kicks. It is important to measure velocities for as large a sample as possible and see if the predicted anti-correlation between P_b and v is seen (Bailes 1989, Johnston 1992). Finally, detection of millisecond pulsations (in radio or X rays) from LMXBs would provide direct evidence for the standard recycling model, in which millisecond pulsars were spun up by accretion.

It is of some importance to see if the histogram of magnetic field strength towards small B continues as the present (meager) data suggest. If so, there

could be genuinely a large number of neutron stars with field strengths smaller than the weakest field LMBP, $B < 5 \times 10^7$ G. Such a hypothetical class would worsen the birthrate problem. A better understanding of the death line of such objects is needed as well as searches for sub-millisecond pulsars using coherent dedispersion techniques. X-ray and γ -ray detections will give insights into magnetospheric physics.

Finally, we predict that millisecond and binary pulsars will continue to perform serendipitous and exciting physics experiments. Just as commercial applications of the terrestrial Global Positioning System (GPS) are expanding exponentially, so applications of the pulsars' equally precise Galactic Positioning System are likely to expand, beyond gravitational wave detection, globular cluster dynamics, planetary dynamics, and convection in red giants. More systems in which the pulsar beams pass through companions and their winds will doubtless be found, as will systems in which a pulsar's wind had, and continues to have effects on its companion and environment. We have learned to expect unexpectedly exotic physics.

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