Applied Mathematical Sciences, Vol. 3, 2009, no. 48, 2369 - 2381

A Centroid-Based Performance Evaluation Using Aggregated Fuzzy Numbers

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Abstract

In recent years, some methods have been proposed in solving performance evaluation issues by using fuzzy set theory. In this paper, an improvised method for performance evaluation under fuzzy environment is presented. Fuzzy linguistic variables are used throughout the process and the aggregated fuzzy numbers which are based on the standard score concept are employed in aggregating the fuzzy assessment of the decision makers. The centroid indices such as distance index, area index, score index and index based on standard deviation are used in calculating the ranking order of the alternatives. A study has been carried out in evaluating lecturers' teaching performance at one of the public universities in the East Coast of Malaysia. This method is capable to provide consistent, effective and precise results and may also give great satisfaction to all parties involved in the decision-making process. It can also be a valuable tool in solving a variety of decision-making problems.

Mathematics Subject Classification: 90C70

Keywords: Aggregated fuzzy numbers, centroid index, performance evaluation, ranking fuzzy numbers, standard score

1 Introduction

In recent years, a number of researchers have focused on solving performance evaluation issues by using fuzzy set theory. Chang and Sun [2] presented a method for performance assessment of junior high school students based on the fuzzy max-min composition operations. Fourali [11] highlighted the relevance of fuzzy concept in the assessment of portfolios in which he stated that the fuzzy concept can provide reliable and rational decision. In the same year, Chiang and Lin [9] presented fuzzy statistical method in teaching performance assessment which improves the classical statistical method. Weon and Kim [18] presented a new learning achievement evaluation strategy in students' learning performance namely, fuzzy evaluation. The inverse sigmoid function, fuzzy concentration function, fuzzy dilution function and fuzzy square method were used in the procedure. Along the same line, Lee et al. [13] employed fuzzy measures and fuzzy integral concept in analyzing the performance of vocational and technological education in Taiwan. Their method is more applicable and reasonable compared to the traditional evaluation process. Wu [19] proposed a new method in evaluating students' learning performance based on the fuzzy set theory and the item response theory. In another study, Cheng et al. [8] established the criteria of evaluation for high school teachers using fuzzy linguistic questionnaires and Chen and Cheng's [5] method in ranking the fuzzy grades.

In a recent study by Wang and Chen [16], a model on evaluation of high school teachers based on fuzzy number arithmetic operations and fuzzy ranking method by centroid points from Chen and Chen [3] is presented. However, in Wang and Chen's [16] method , the ratings of alternatives and the weights of criteria are presented as crisp numbers where the total of the related crisp numbers must be equal to 100% which can be cumbersome to the decision makers. Besides that, in Wang and Chen's [16] procedure, the data with the smallest and largest values were trimmed off which is similar to Lin and Chang's [14] approaches. Due to Lin and Chang [14], the aggregated fuzzy assessment should not be influenced by the extreme data and, thus, they proposed the aggregated fuzzy assessment as $\tilde{g}_{ij} = \frac{\sum_{k=1}^{K} \tilde{x}_{ij}^{k} - \max_k \tilde{x}_{ij}^{k} - \min_k \tilde{x}_{ij}^{k}}{K-2}$ where $\tilde{x}_{ij}^{k}$ is the rating of the k-th decision maker. However, the extreme data do not necessarily become the outliers that will affect the results of the data analysis. Barnett and Lewis [1] defined the outliers as the data having values which are very different from the data values of the majority of the cases in the data set. Thus, the aggregated fuzzy assessment presented in Lin and Chang's [14] and Wang and Chen's [16] study may not be precise as the data which were trimmed off might not be the outliers.

In order to solve the issue of presenting the ratings of alternatives and the weights of criteria as crisp numbers and trimming off the non-outliers data, this paper presents an improvised method for performance evaluation in fuzzy environment. This method improves the Wang and Chen's method [16] by using the fuzzy linguistic variables throughout the process instead of using crisp number in some steps. The outliers which are detected by using the standard score are trimmed off and the improvised aggregated fuzzy numbers are applied in aggregating the fuzzy assessments of the decision makers. Wang et al.'s [17] centroid corrected formula, Cheng's [7] distance method, Chu and Tsao's [10] area method and Chen and Chen's [3, 4]indices are applied in calculating the ranking order of the alternatives. The current study has been conducted at one of the universities in the East Coast of Malaysia where the evaluation of lecturers' teaching performance is one of the factors considered in the rewarding process. Furthermore, the need of a fair and simple way for performance appraisal is very crucial for future decision-making. Thus, the proposed method is capable to provide a consistent and effective result and may also give great satisfaction to all parties involved in the decision-making process. It can also be a valuable tool in solving a variety of decision-making problems.

2 Preliminaries

2.1 Fuzzy Number

A fuzzy number is a fuzzy subset in the universe discourse that is both convex and normal. The membership function of a fuzzy number $\tilde{A}$ can be defined as

$$f_{\tilde{A}}(x) = \begin{cases} f_A^L(x) & , a \leq x \leq b \\ 1 & , b \leq x \leq c \\ f_A^R(x) & , c \leq x \leq d \\ 0 & , \texttt{otherwise} \end{cases}$$

where $f_A^L : [a, b] \to [0, 1]$, $f_A^R : [c, d] \to [0, 1]$, f_A^L and f_A^R are the left and the right membership functions of the fuzzy number $\tilde{A}$ respectively. Trapezoidal fuzzy numbers are denoted as (a,b,c,d) and triangular fuzzy numbers which are special cases of trapezoidal fuzzy numbers with b=c are denoted as (a,b,d).

2.2 Operations on Triangular Fuzzy Numbers

Let $\tilde{X}$ and $\tilde{Y}$ be two triangular fuzzy numbers parameterized by the triplets (x_1, x_2, x_3) and (y_1, y_2, y_3) respectively. The fuzzy number arithmetic operations between $\tilde{X}$ and $\tilde{Y}$ as presented in Chen and Hwang [6] are as follows:

Addition operation : $\tilde{X} \oplus \tilde{Y} = (x_1 + y_1, x_2 + y_2, x_3 + y_3)$

Subtraction operation : $\tilde{X} \ominus \tilde{Y} = (x_1 - y_1, x_2 - y_2, x_3 - y_3)$

Multiplication operation: $\tilde{X} \otimes \tilde{Y} = (x_1 y_1, x_2 y_2, x_3 y_3)$

Division operation : $\tilde{X} \oslash \tilde{Y} = (x_1/y_3, x_2/y_2, x_3/y_1)$

3 Centroid-based Ranking Methods

The method of ranking fuzzy numbers has been proposed firstly by Jain [20]. Since then, a large variety of methods have been developed ranging from the trivial to the complex and from one fuzzy number attribute to many fuzzy number attributes. According to Chen and Hwang [6], the ranking methods can be classified into four major classes which are preference relation, fuzzy mean and spread, fuzzy scoring and linguistic expression. However, this section only presents the ranking methods of fuzzy numbers based on centroid index which is a sub-class of fuzzy scoring.

Yager [20], the first researcher who contributed the centroid concept in the ranking method, only used the horizontal coordinate x as the ranking index which is defined as $x_{\tilde{A}} = \frac{\int_0^1 x f_{\tilde{A}}(x)\,dx}{\int_0^1 f_{\tilde{A}}(x)\,dx}$. The larger the value of $x_{\tilde{A}}$, the higher the ranking of the fuzzy number $\tilde{A}$. In 1983, Murakami et al. [15] presented both the horizontal x and vertical y coordinates of the centroid point as the ranking index. The $x_{\tilde{A}}$ is similar to the $x_{\tilde{A}}$ in Yager [20] but the $y_{\tilde{A}}$ is defined as $y_{\tilde{A}} = \frac{\int_0^1 x f_{\tilde{A}}(x)\,df_{\tilde{A}}(x)}{\int_0^1 f_{\tilde{A}}(x)\,dx}$. The larger the value of $x_{\tilde{A}}$ and (or) $y_{\tilde{A}}$, the better the ranking of $\tilde{A}$.

Cheng [7] proposed a distance method based on the distance between the centroid point and original point. For a trapezoidal fuzzy number $\tilde{A} = (a, b, c, d; w)$, the distance index can be defined as

$$R(\tilde{A}) = \sqrt{\tilde{x}_{\tilde{A}}^2 + \tilde{y}_{\tilde{A}}^2} \tag{1}$$

with $\tilde{x}_{\tilde{A}} = \frac{\int_a^b x f_A^L\,dx + \int_b^c x\,dx + \int_c^d x f_A^R\,dx}{\int_a^b f_A^L\,dx + \int_b^c dx + \int_c^d f_A^R\,dx}$, $\tilde{y}_{\tilde{A}} = w.\frac{\int_0^1 y g_A^L\,dy + \int_0^1 y g_A^R\,dy}{\int_0^1 g_A^L\,dy + \int_0^1 g_A^R\,dy}$, f_A^R and f_A^L are the right and left membership functions of $\tilde{A}$ respectively, g_A^R and g_A^L are the inverse of f_A^R and f_A^L respectively.

However, Chu and Tsao [10] found out Cheng's [7] distance method is inconsistent to rank some fuzzy numbers and their images. Therefore, they proposed a new method based on the area between centroid point $(\tilde{x}_{\tilde{A}}, \tilde{y}_{\tilde{A}})$ and original points (0,0) that is defined as

$$S(\tilde{A}) = \tilde{x}_{\tilde{A}} . \tilde{y}_{\tilde{A}} \tag{2}$$

where $\tilde{x}_{\tilde{A}}$ is similar to $\tilde{x}_{\tilde{A}}$ in Cheng [7] and $\tilde{y}_{\tilde{A}} = \frac{\int_0^w y g_A^L\,dy + \int_0^w y g_A^R\,dy}{\int_0^w g_A^L\,dy + \int_0^w g_A^R\,dy}$. The larger the value of $S(\tilde{A})$, the better the ranking of $\tilde{A}$.

In another study by Wang et al. [17], the centroid formulae proposed by Cheng [7] and Chu and Tsao [10] were shown to be incorrect. This is because the centroid point formulae used by both researchers do not satisfy the properties for a correct centroid formulae have to possess. Therefore, to avoid more misapplications, Wang et al. [17] presented the correct centroid formulae as, $\tilde{x}_{\tilde{A}} = \frac{\int_a^b x f_A^L \, dx + \int_b^c x w \, dx + \int_c^d x f_A^R \, dx}{\int_a^b f_A^L \, dx + \int_b^c w \, dx + \int_c^d f_A^R \, dx}$ and $\tilde{y}_{\tilde{A}} = \frac{\int_0^w y(g_A^R - g_A^L) \, dy}{\int_0^w (g_A^R - g_A^L) \, dy}$. They also simplified the corrected formulae from the viewpoint of analytical geometry. The correct centroid point formulae from the viewpoint of analytical geometry for a trapezoidal fuzzy number, $\tilde{A} = (a, b, c, d; w)$ is given as,

$$\tilde{x}_{\tilde{A}} = \frac{a + b + c + d - \frac{dc - ab}{(d+c)-(a+b)}}{3} \text{ and } \tilde{y}_{\tilde{A}} = \frac{w}{3}.[1 + \frac{c - b}{(d + c) - (a + b)}]. \quad (3)$$

In a study conducted by Chen and Chen [3], they found that Cheng's [7], Murakami et al.'s [15] and Yager's [20] methods cannot rank correctly different fuzzy numbers in some situations and cannot calculate the ranking order of a crisp number. Hence, to overcome the problems of incorrect ranking order, Chen and Chen [3] derived a new method on ranking generalized trapezoidal fuzzy numbers based on centroid point and standard deviations. The ranking value for a generalized trapezoidal fuzzy number $A = (a_1, a_2, a_3, a_4; w)$ is defined as

$$Rank(A) = x_A + (w - y_A)^s.(y_A + 0.5)^{1-w} \quad (4)$$

where for $a_1 \neq a_4$, $y_A = \frac{w}{6}.(\frac{a_3 - a_2}{a_4 - a_1} + 2)$ and for $a_1 = a_4$, $y_A = \frac{w}{2}$, $x_A = \frac{y_A(a_3+a_2)+(a_4+a_1)(w_A-y_A)}{2w_A}$, $s = \sqrt{\frac{\sum_{i=1}^4 (a_i - \bar{a})^2}{3}}$ and $\bar{a} = \frac{a_1+a_2+a_3+a_4}{4}$. The conclusion is similar to the distance index by Cheng [7] where the larger the value of $Rank(A)$, the better the ranking of A.

In 2007, Chen and Chen [4] again found that Chu and Tsao's [10] area method also contains shortcomings as in Cheng's [7], Murakami et al.'s [15] and Yager's [20] methods. Thus, Chen and Chen [4] proposed the score value of the fuzzy numbers as the ranking method which is defined as

$$Score(\tilde{A}_j) = \sqrt{(\hat{x}_{\tilde{A}_j} - min_{j=1,2,\ldots,n}[\hat{x}_{\tilde{A}_j}])^2 + (\hat{y}^s_{\tilde{A}_j})^2} \quad (5)$$

with $(\hat{x}_{\tilde{A}_j}, \hat{y}_{\tilde{A}_j})$ similar to Chen and Chen's [3] centroid point, $\hat{y}^s_{\tilde{A}_j} = \frac{w_{\tilde{A}_j}}{2} - (\hat{y}_{\tilde{A}_j} \times s_{\tilde{A}_j})$ where $s_{\tilde{A}_j} = \sqrt{\frac{\sum_{i=1}^4 (a_{ij} - \bar{a_j})^2}{3}}$ and $\bar{a_j} = \frac{a_{1j}+a_{2j}+a_{3j}+a_{4j}}{4}$. Similar to Chen and Chen's [3] conclusion, the higher the value of $Score(\tilde{A}_j)$, the better the ranking of the fuzzy number $\tilde{A}_j$.

4 A Review of Wang and Chen's [16] Method

In this section, Wang and Chen's [16] method for appraising the high school teachers is briefly reviewed. Table 1 shows fuzzy linguistic questionnaire used by Wang and Chen [16] in evaluating the importance levels of each criterion and the satisfaction levels of each alternative.

Table 1: Fuzzy Linguistic used in [16]

Criterion	Very Low	Low	Medium	High	Very High
X_i	0%	0%	20%	60%	20%

The procedure by Wang and Chen [16] is presented as follows:

Step 1: The fuzzy weight $W(\tilde{x}_i)$ of each criterion is calculated using $W(\tilde{x}_i) = \frac{\sum_{k=1}^{5} \tilde{x}_{ik} f(\tilde{x}_{ik})}{\sum_{k=1}^{5} f(\tilde{x}_{ik})}$ where i denotes the index of the criterion, k denotes the index of linguistic levels, $\tilde{x}_{ik}$ denotes the k-th importance level of the criterion X_i, $1 \leq k \leq 5$, $\tilde{x}_{ik} \in \{\tilde{1}, \tilde{2}, \tilde{3}, \tilde{4}, \tilde{5}\}$, $f(\tilde{x}_{ik})$ denotes the degree of percentage the criterion X_i satisfies the k-th importance level and $\sum_{k=1}^{5} f(\tilde{x}_{ik}) = 1$. The fuzzy weight will be represented by a triangular fuzzy number *(a, b, c)* and must satisfy the rules: " If $a < 1$, then let $a = 1$; if $c > 5$, then let $c = 5$".

Step 2: For each criterion X_i $(1 \leq i \leq 3)$ evaluated by the students, drop the fuzzy weight with the smallest ranking value and the largest ranking value. Then, calculate the average of the remaining fuzzy weight $\tilde{w}_i$ using addition and division operations of fuzzy numbers.

Step 3: The fuzzy grade $G(\tilde{x}_{ij})$ of each sub-criterion of each lecturer evaluated by each student is calculated using $G(\tilde{x}_{ij}) = \frac{\sum_{k=1}^{5} \tilde{x}_{ijk} f(\tilde{x}_{ijk})}{\sum_{k=1}^{5} f(\tilde{x}_{ijk})}$ where $\tilde{x}_{ijk}$ denotes the k-th linguistic satisfaction level of the sub-criterion X_{ij}, $\tilde{x}_{ijk} \in \{\tilde{1}, \tilde{2}, \tilde{3}, \tilde{4}, \tilde{5}\}$, $f(\tilde{x}_{ijk})$ denotes the degree the lecturer satisfies the k-th satisfaction level and $\sum_{k=1}^{5} f(\tilde{x}_{ijk}) = 1$. The fuzzy grade also satisfies the rules in *Step 1.*

Step 4: For each sub-criterion of each lecturer evaluated by each student, drop the fuzzy grades with the smallest ranking value and the largest ranking value. Then, calculate the average of the remaining fuzzy grades $\tilde{g}_{ij}$ using addition and division operations of fuzzy numbers.

Step 5: Build the fuzzy grade matrix $\tilde{G}$ defined as

$$\tilde{G} = \begin{pmatrix} \tilde{g}_{11} & \tilde{g}_{12} & \cdots & \tilde{g}_{1k} \\ \tilde{g}_{21} & \tilde{g}_{22} & \cdots & \tilde{g}_{2k} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{g}_{n1} & \tilde{g}_{n2} & \cdots & \tilde{g}_{nk} \end{pmatrix},$$

where $\tilde{g}_{ij}$ denotes the fuzzy grade of the i-th lecturer A_i with respect to the j-th criterion X_j, n denotes the number of lecturers and k denotes the number of criteria.

Step 6: Calculate the total fuzzy grade vector $\tilde{R}$ with

$$\tilde{R} = \tilde{G} \otimes \tilde{W} = \begin{pmatrix} \tilde{g}_{11} & \tilde{g}_{12} & \cdots & \tilde{g}_{1k} \\ \tilde{g}_{21} & \tilde{g}_{22} & \cdots & \tilde{g}_{2k} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{g}_{n1} & \tilde{g}_{n2} & \cdots & \tilde{g}_{nk} \end{pmatrix} \otimes \begin{pmatrix} \tilde{w}_1 \\ \tilde{w}_2 \\ \vdots \\ \tilde{w}_k \end{pmatrix} = \begin{pmatrix} \tilde{R}_1 \\ \tilde{R}_2 \\ \vdots \\ \tilde{R}_k \end{pmatrix}$$

where $\tilde{R}_i$ denotes the total fuzzy grade of the i-th lecturer A_i and $1 \leq i \leq n$.

Step 7: Based on (4), calculate the ranking value, $Rank(\tilde{R}_i)$.

5 Proposed Methodology

Two sets of fuzzy linguistic questionnaires comprising of the importance levels of each criterion and the satisfaction levels of each sub-criterion are used in the evaluation process. The fuzzy linguistic variables can be expressed as triangular fuzzy numbers as in Table 2.

Table 2: Linguistic Variable for Importance and Satisfaction Levels

Linguistic Variables	Fuzzy Numbers
Very Low	(0,0,3)
Low	(0,3,5)
Medium	(2,5,8)
High	(5,7,10)
Very High	(7,10,10)

The proposed procedure which improves Wang and Chen's [16] method can be expressed in a series of steps:

Step 1: For K decision makers, the fuzzy weight $\tilde{w}_j$ of each criterion can be calculated by using aggregated fuzzy assessment. According to Lin and Chang [14], the aggregated fuzzy assessment should not be influenced by the extreme data but the extreme data are not necessarily the outliers that can change the results of the data analysis. Thus, in this study the outliers are detected based on the standard score. For small sample size which are 80 or fewer cases, a case is an outlier if its standard score is ± 2.5 or beyond. However, for larger sample size with more than 80 cases, a case is an outlier if its standard score is ± 3 or beyond. Based on the standard score, the outliers were trimmed off and the importance weight $\tilde{w}_j$ of each criterion is defined as

$$\tilde{w}_j = \frac{\sum_{k=1}^{K} \tilde{w}_j^k - \sum_{n=1}^{s} \tilde{w}_{j_{(n)}}}{K - s}$$

where $\tilde{w}_j^k$ is the importance weight of the k-th decision maker, $\tilde{w}_{j_{(n)}}$ is the n-th outlier of the importance weight and s is the number of outliers. The fuzzy weighted vector for three criteria can be represented as $W = \begin{bmatrix} \tilde{w}_1 & \tilde{w}_2 & \tilde{w}_3 \end{bmatrix}^T$.

Step 2: The fuzzy grade $\tilde{g}_{ij}$ of each sub-criterion of each alternative can be calculated using aggregated fuzzy assessment. However, using the same method in *Step 1*, the outliers of the linguistic judgment values of the decision maker for evaluating the i-th alternative with respect to the j-th criterion are trimmed off. Therefore, the fuzzy grade $\tilde{g}_{ij}$ of each sub-criterion of each alternative is defined as

$$\tilde{g}_{ij} = \frac{\sum_{k=1}^{K} \tilde{x}_i^k j - \sum_{n=1}^{s} \tilde{x}_{ij_{(n)}}}{K - s}$$

where $\tilde{x}_i^k j$ are the ratings of the k-th decision maker, $\tilde{x}_{ij_{(n)}}$ is the n-th outlier of the rating and s is the number of outliers.

Step 3: Build the fuzzy grade matrix $\tilde{G}$ defined as

$$\tilde{G} = \begin{pmatrix} \tilde{g}_{11} & \tilde{g}_{12} & \cdots & \tilde{g}_{1k} \\ \tilde{g}_{21} & \tilde{g}_{22} & \cdots & \tilde{g}_{2k} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{g}_{n1} & \tilde{g}_{n2} & \cdots & \tilde{g}_{nk} \end{pmatrix},$$

where $\tilde{g}_{ij}$ denotes the fuzzy grade of the i-th alternative A_i with respect to the j-th criterion X_j, n denotes the number of alternatives and k denotes the number of criteria.

Step 4: Calculate the total fuzzy grade vector $\tilde{R}$ with

$$\tilde{R} = \tilde{G} \otimes \tilde{W} = \begin{pmatrix} \tilde{g}_{11} & \tilde{g}_{12} & \cdots & \tilde{g}_{1k} \\ \tilde{g}_{21} & \tilde{g}_{22} & \cdots & \tilde{g}_{2k} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{g}_{n1} & \tilde{g}_{n2} & \cdots & \tilde{g}_{nk} \end{pmatrix} \otimes \begin{pmatrix} \tilde{w}_1 \\ \tilde{w}_2 \\ \vdots \\ \tilde{w}_k \end{pmatrix} = \begin{pmatrix} \tilde{R}_1 \\ \tilde{R}_2 \\ \vdots \\ \tilde{R}_k \end{pmatrix}$$

where $\tilde{R}_i$ denotes the total fuzzy grade of the i-th alternative A_i and $1 \leq i \leq n$.

Step 5: Based on the distance index (1), the area index (2), the corrected formula (3) and indices (4) and (5), calculate the ranking order.

6 Numerical Example

The present study was carried out in one of the universities in the East Coast of Malaysia. Eleven items of the fuzzy linguistic questionnaire on lecturers' appraisal are categorized into three basic criteria that are teaching, class management and professional and motivational attitude. Each main criterion is divided into several sub-criteria which are preparation of delivering teaching materials, knowledge ability in using teaching materials, presenting well-organized teaching materials and ability to keep students' attention throughout the class for teaching criteria. While for the class management criteria, the sub-criteria are giving opportunities for questions and discussion and evaluating assignments, tests, quizzes fairly according to the standard of the course. As for the professional and motivational attitude criteria, it can be divided into five sub-criteria which are always attending classes, coming to class on time, showing interest and enthusiasm during teaching, showing concern on student's attendance and treating students fairly. The questionnaires were distributed to students and the data were analyzed by using the proposed procedure.

7 Result and Discussion

The ranking values of lecturer's teaching performance are shown in Table 3 and 4. All the four ranking indices produce the same ranking results as $A_5 > A_3 > A_6 > A_4 > A_2 > A_1$. The ranking result for Chen and Chen's [3] index gives the ranking value exactly the same (at least up to 2 decimal places) with the distance method by Cheng [7]. However, these two indices will not produce the same ranking values for all cases of fuzzy number. For instance, let $A = (0.1, 0.3, 0.5; 1)$, the distance is 0.45 while by using Chen and Chen's [3] index, the value of index is 1.24. Thus, it is expected that for smaller values of maximum, minimum and modal of the fuzzy number, the ranking values between the two indices will be different and the bigger the value of the maximum, minimum and modal of the fuzzy number, the ranking indices start to approach the same value.

Table 3: Ranking Values

Alternatives	Proposed Method				Rank
	Value of x	Value of y	Index [3]	Index [7]	
$\tilde{A}_1$	185.15	$\frac{1}{3}$	185.15	185.15	6
$\tilde{A}_2$	190.85	$\frac{1}{3}$	190.85	190.85	5
$\tilde{A}_3$	211.58	$\frac{1}{3}$	211.58	211.58	2
$\tilde{A}_4$	203.23	$\frac{1}{3}$	203.23	203.23	4
$\tilde{A}_5$	216.36	$\frac{1}{3}$	216.36	216.36	1
$\tilde{A}_6$	208.97	$\frac{1}{3}$	208.97	208.97	3

Table 4: Ranking Values

Alternatives	Proposed Method				Rank
	Index [7]	Index [10]	Index [3]	Index [4]	
$\tilde{A}_1$	185.15	61.72	185.15	25.38	6
$\tilde{A}_2$	190.85	63.62	190.85	26.0	5
$\tilde{A}_3$	211.58	70.53	211.58	35.92	2
$\tilde{A}_4$	203.23	67.74	203.23	30.82	4
$\tilde{A}_5$	216.36	72.12	216.36	39.77	1
$\tilde{A}_6$	208.97	69.66	208.97	34.1	3

Table 5: Comparative Results between Traditional Evaluation, [16] and Ordinary Aggregated Fuzzy Numbers

Alternatives	Traditional (Mean)	[16]	Ordinary Aggregated Fuzzy Number		
			Index [7]	Index [10]	Index [4]
$\tilde{A}_1$	3.83	50.30	179.31	59.77	24.91
$\tilde{A}_2$	4.06	52.38	187.64	62.55	26.48
$\tilde{A}_3$	4.41	55.49	202.36	67.45	33.09
$\tilde{A}_4$	4.23	53.87	197.11	65.70	30.30
$\tilde{A}_5$	4.45	56.34	205.52	68.51	35.56
$\tilde{A}_6$	4.45	54.72	205.43	68.48	35.61
Rank	$A_5 \approx A_6 > A_3 > A_4 > A_2 > A_1$	$A_5 > A_3 > A_6 > A_4 > A_2 > A_1$	$A_5 > A_6 > A_3 > A_4 > A_2 > A_1$		$A_6 > A_5 > A_3 > A_4 > A_2 > A_1$

Besides that, as the alternatives are represented with normal triangular fuzzy numbers, the value of y is the same ($y = \frac{1}{3}$) for all sets of fuzzy numbers. Thus, if considering Cheng's [7] and Chen and Chen's [3] method, the value of x is exactly the same between the two ranking indices and it can be concluded that the value of x is sufficient in determining the ranking results.

This paper also compares the ranking values with the ordinary aggregated fuzzy numbers, Wang and Chen's [16] method and the traditional evaluation which used the mean values (Table 5). The ordinary aggregated fuzzy numbers produce two different ranking results with $A_5 > A_6 > A_3 > A_4 > A_2 > A_1$ using Cheng's [7] and Chu and Tsao's [10] method while Chen and Chen's

[4] gives ordering as $A_6 > A_5 > A_3 > A_4 > A_2 > A_1$. This reveales that for this study, the ordinary aggregated fuzzy numbers yields inconsistent ranking result. However, by looking in detail at alternatives A_5 and A_6 in the case of ordinary aggregated fuzzy numbers, the difference values between the two alternatives are very small which is less than 0.14%. Thus, from the decision-maker's point of view, the ranking values of the two alternatives can be considered as equal which resulted in the ranking as $A_5 \approx A_6 > A_3 > A_4 > A_2 > A_1$ and is also similar to the mean value by the traditional calculation. Therefore, it can be concluded that the ordinary aggregated fuzzy numbers and the traditional mean calculation cannot discriminate the ranking between two different fuzzy numbers. The result also shows that when the outliers were trimmed off, the ranking result changed to $A_5 > A_3 > A_6 > A_4 > A_2 > A_1$ which is consistent with Wang and Chen's [16] method. This means that the outliers had affected the ranking process and by taking them away, consistent and effective results are obtained.

8 Conclusion

This paper presents a new method for ranking alternatives under fuzzy multiple criteria. It improves Wang and Chen's [16] approaches by using fuzzy linguistic variables throughout the process and the outliers are detected by using the standard score. The improvised aggregated fuzzy numbers are applied in aggregating the fuzzy assessments of the decision makers which improves Lin and Chang's [14] approach. Surprisingly, Chen and Chen's [3] index produces exactly the same ranking values with Cheng's [7] distance method, though it is not true for all cases of fuzzy numbers. The ranking result is obviously affected by the outliers, thus, by taking them away, consistent and effective results are obtained which is consistent with Wang and Chen's [16] method. This method is not a burden to the decision makers since they only have to give the ratings depending on the linguistic variables without having any pressure of satisfying any requirement. Instead of using crisp numbers in some steps, this study had used fuzzy linguistic variables throughout the process which is the rationale of using fuzzy method. The trimming off outliers which are detected by using the standard score had minimized the variation within the data and thus makes this study more precise. Furthermore, this method can also give great satisfaction to all parties involved in the decision-making process and can be a valuable tool in solving a variety of decision-making problems.

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Received: March, 2009